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On the coexistence of different licensing schemes

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Abstract

We consider patent licensing under a simple model of asymmetric information, where an outsider innovator of a cost-reducing innovation interacts with a monopolist, whose cost is private information. When the innovator is endowed with combinations of fixed fee and royalty, in any optimal menu, the low-cost monopolist is always offered a pure fixed fee contract, while for the contract offered to the high-cost monopolist, the royalty rate is always positive. Moreover, there are cases where it is a pure royalty contract. This provides an explanation of royalty licensing, in particular, and the coexistence of different licensing schemes, in general.

JEL classification: D45; D82
Keywords: Licensing; Cost-reducing innovation; Fixed fee plus royalty; Asymmetric information; Individual rationality; Incentive compatibility

1. Introduction

In practice, licensing contracts of technological innovations take various diverse forms, but they can be classified into three broad categories: licensing by means of an output-based royalty, a lump-sum upfront fee, and combination of the two. The theoretical literature on patent licensing has mainly considered outsider innovators, namely, innovators who are not one of the producers in the industry. Under such a scenario, the theoretical conclusion is that licensing by means of royalty is never the optimal policy for the innovator. In particular, royalty licensing is inferior to other policies like auction or charging only a fixed fee, and it is certainly inferior to policies that combine both fixed fee and royalty (Kamien & Tauman, 1984, 1986;

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Kamien, Oren, & Tauman, 1992; see Kamien, 1992 for a survey). Nevertheless, royalty policy is observed commonly in practice. In fact, empirical studies point out the coexistence of all three standard policies (e.g., Firestone, 1971; Taylor & Silberstone, 1973; Rostoker, 1984; Macho-Stadler, Martinez-Giralt, & Pérez-Castrillo, 1996). This paper seeks to provide a theoretical explanation of this phenomenon of coexistence, in general, and the optimality of royalty licensing, in particular. We consider a simple model of asymmetric information, where an innovator has a cost-reducing innovation that is intended to be sold to a monopolist. The magnitude of the innovation is common knowledge. The source of asymmetry is the cost of the monopolist, which is private information and can take either a low or high value. The licensing contracts available to the innovator is the set of all contracts that include a fixed fee and a per-unit linear royalty. When the innovator offers a single nondiscriminatory licensing contract, we show that there are cases where the uninformed innovator finds it optimal to charge only royalty and no fixed fee to the incumbent firm, although the former has the bargaining power and can offer any combination of fixed fee and royalty. It is further shown that the parameter space can be divided into three regions that correspond to three policies, viz., fixed fee, royalty, and combination of the two, so that each policy is optimal in the corresponding region. This conclusion is sustained under the more general approach, where the innovator can offer any menu of licensing contracts. We show that in any optimal menu, the low-cost monopolist is always offered a pure fixed fee contract, while the contract offered to the high-cost monopolist always involves a positive royalty. Moreover, there are regions in the parameter space in which the high-cost monopolist is offered a pure royalty contract. Thus, we provide an explanation of royalty licensing, in particular, and the coexistence of different licensing schemes, in general, as observed in practice.[1] In what follows, we discuss our results in relation to the existing literature of patent licensing under asymmetric information.

Gallini and Wright (1990) have considered a model of licensing where the value of the innovation is private information to the innovator, who signals the value through the contract offer. If a potential licensee accepts the offer, a fixed fee is paid upfront and the innovation is received. After that, the licensee decides whether to imitate the innovation, and then production is carried out. In case there is no imitation, the licensee pays the innovator the output-based royalty as specified in the licensing contract. Considering separating equilibrium contracts, Gallini and Wright have shown that a fixed fee contract will be offered for innovations with a low value, while for high-value innovations, the innovator will offer an output-based royalty contract. In our paper, the magnitude of the cost-reducing innovation is common knowledge, while the cost of the potential licensee is his private information. Because the innovator is the uninformed party, there is no signaling game. Furthermore, there is no imitation, and once the licensee accepts a contract, he has to pay both the fixed fee upfront and the output-based royalty. Thus, asymmetry of information can explain the optimality of royalty licensing and coexistence of different licensing policies under a relatively simple setting with almost no additional requirement.

Macho-Stadler and Pérez-Castrillo (1991)[2] have considered a model of asymmetric information where an innovator interacts with a monopolist who is privately informed of the value of the innovation. Because the asymmetry of information arises from the innovation, the pre-innovation payoff of the monopolist is common knowledge. In contrast, the source of asymmetry in our paper is a characteristic of the monopolist, namely, his cost. Consequently, the payoff of the monopolist (pre-innovation as well as post-innovation) is

[1] In the oft-quoted survey of Rostoker (1984), royalty licensing was observed in 39% of cases, fixed fee was observed in only 13% of cases, while 46% of cases were contributed to combinations of fixed fee and royalty.

[2] See Macho-Stadler and Pérez-Castrillo (2001), (pp. 149–153), for a discussion of this work. I thank Vincenzo Denicòlo for bringing these references to my notice.

always private information. Macho-Stadler and Pérez-Castrillo have shown that the optimal menu of contracts proposed by the innovator is separating. The contract for the good innovation involves only fixed fee, while for the bad innovation, the contract is a combination of fixed fee and royalty. To certain extent, our conclusions are consistent with theirs. One difference is that in their analysis, an optimal menu never contains a pure royalty contract, while we show that there are cases where such a contract is offered.

Beggs (1992) has considered a model of asymmetric information where the innovator is the uninformed party, while the buyer knows the true value of the innovation, and it is the buyer who makes the offer. He has shown that royalty licensing can make a separating equilibrium possible, and doing so, may ensure that trade takes place in cases where it fails with fixed fee licensing. However, with royalty licensing, at least one type of buyer is worse off. Furthermore, to convince the innovator of the low quality of the innovation, production may be inefficient. Thus, the primary justification of royalty licensing is that it ensures trade. In contrast, we consider licensing schemes as outcomes that result from optimizing behavior of agents involved in a strategic interaction: The innovator chooses a licensing scheme to maximize her rents, given the fact that for any licensing contract, the decision to accept it or not and the output to be produced is chosen by the monopolist to maximize his own profit. In contrast to Beggs, it is the innovator who makes the offer for the license, which is arguably the more natural case.[3] Moreover, the licensing policies we consider are more general because we allow the innovator to offer combinations of fixed fee and royalty.

From our discussion of the literature, one can observe the underlying common theme: In one form or another, an output-based royalty is used as an effective separating device under asymmetry of information. The present paper complements the earlier works by maintaining this basic theme. It should be mentioned that while asymmetry of information is one plausible explanation of royalty, it is certainly not the only one. Several other, often overlapping, approaches have been taken to justify the use of royalty licensing. In particular, it has been shown that royalty can be explained by variation in the quality of innovation (Rockett, 1990), product differentiation (Muto, 1993; Wang & Yang, 1999; Stamatopoulos & Tauman, 2003), moral hazard (Macho-Stadler et al., 1996; Choi, 2001), risk aversion (Bousquet, Cremer, Ivaldi, & Wolkowicz, 1998), incumbent innovator (Shapiro, 1985; Wang, 1998, 2002; Kamien & Tauman, 2002; Sen, 2002; Sen & Tauman, 2003), leadership structure (Filippini, 2001; Kabiraj, 2002, 2004), or strategic delegation (Saracho, 2002).

The rest of the paper is organized as follows. We present the model in Section 2. Section 3 discusses about nondiscriminatory licensing contracts. In Section 4, we discuss about menus of contracts. All proofs have been relegated to Appendix A.

2. The model

We consider a monopolist (M), who faces the demand $q=a-p$ and produces with a constant marginal cost c, where $0<c<a$. The cost c of M is private information and it can take two values: c_L and c_H, where $0<c_L<c_H<a$. For $t\in\{L,H\}$, we say that M is of type t if the cost of M is c_t. The innovator (I), has a cost-reducing innovation that reduces the cost from c to $c-\varepsilon$, where $\varepsilon>0$ is common knowledge. One can interpret this by assuming that I reduces the cost of one component of the production process that is common to both technologies, L and H, so that both c_L and c_H are reduced by the same magnitude, viz., ε.

[3] This is consistent with the way the interaction is usually modeled in the literature, where the innovator acts as a Stackelberg leader and makes the offer (e.g., Gallini, 1984; Kamien & Tauman, 1984, 1986; Gallini & Winter, 1985; Katz & Shapiro, 1985, 1986; Kamien et al., 1992). However, see also Katz and Shapiro (1987), where the licensing fee is determined through negotiation between two parties.

The innovator, I, assigns probability λ to the event that $c=c_L$ and 1-λ to $c=c_H$, and seeks to maximize the expected payoff. We assume that $0<\varepsilon<c_L$. The interaction between I and M can be modeled as an extensive-form game, which we call the licensing game. In the beginning, nature chooses the cost c to be c_L or c_H, with probability λ and 1-λ, respectively. The innovator, not knowing the realization of c, offers a licensing contract or a menu of contracts to M, who is informed about the realization. If M rejects, he produces with his old cost and earns the corresponding monopoly profit, while I earns zero. If I offers a single licensing contract and M accepts, he produces with the reduced cost and pays I in accordance with the licensing contract. If I offers a menu of contracts and M accepts, he chooses one contract from the menu, produces with the reduced cost, and pays I in accordance with the chosen licensing contract. It is assumed that outputs produced by M are observable and there is no renegotiation.

3. Nondiscriminatory contracts

The set of nondiscriminatory licensing contracts available to I is the set of all contracts that specify a combination of a fixed fee and a per-unit linear royalty. In particular, this set includes all fixed fees and all royalties separately. Thus, a typical contract is given by (r, f), where $r\in\Re$ is the per-unit linear royalty and $f\in\Re$ is the fixed fee.[4] If M accepts the contract (r, f) and produces q, he pays $f+rq$ to I. The analysis of nondiscriminatory contract is of some independent interest because in practice, offering a menu of discriminatory licensing contracts might be sometimes prohibited. Moreover, the result on the optimal nondiscriminatory contract will be used to determine the more general optimal menu of contracts.

3.1. The optimal nondiscriminatory contract

Let us denote by $\Pi_M\,(c_t,r)$ the monopoly profit of M of type t when he has the reduced cost, and the rate of royalty is r. Note that when M of type t operates under a contract (r, f), the fixed fee f can be viewed as his fixed cost, while the rate of royalty r enters into the marginal cost, so that his effective marginal cost is $c_t-\varepsilon+r$. Hence, his payoff is given by $\Pi_M(c_t, r)-f$. If ~M of type t does not have the reduced cost, his payoff is the pre-innovation monopoly profit, given by $\Pi_M(c_t)=(a-c_t)^2/4=\Pi_M\,(c_t,\varepsilon)$. Thus, M of type t will accept a contract (r, f) only if $\Pi_M\,(c_t,r)-f\geq\Pi_M(c_t,\varepsilon)$.

Proposition 1. *For $t\in\{L,H\}$, let $\Delta_t(r)\equiv\Pi_M(c_t,r)-\Pi_M(c_t,\varepsilon)$. There exists $0<\bar{\varepsilon}<c_H-c_L$ such that in the unique subgame-perfect equilibrium of the licensing game, the following hold:*

1. *For $\varepsilon\in[0,\ \bar{\varepsilon}]$, there is $\tilde{\lambda}(\varepsilon)\in[\varepsilon/(c_H-c_L),1]$ such that the contract is $(\lambda(c_H-c_L),\Delta_H\,(\lambda(c_H-c_L)))$ (which is a combination of a nonnegative royalty and a nonnegative fixed fee) when $\lambda\in[0,\ \varepsilon/(c_H-c_L)]$; it is the royalty contract with rate of royalty ε when $\lambda\in[\varepsilon/(c_H-c_L),\ \tilde{\lambda}(\varepsilon)]$ and it is the fixed fee contract with nonnegative fee $\Delta_L(0)$ when $\lambda\in[\tilde{\lambda}(\varepsilon),1]$.*
2. *For $\varepsilon\geq\bar{\varepsilon}$, there is $\bar{\lambda}(\varepsilon)\in(0,\ 1)$ such that $(\lambda(c_H-c_L),\ \Delta_H\,(\lambda(c_H-c_L)))$ is the licensing contract when $\lambda\in[0,\ \bar{\lambda}(\varepsilon)]$, and it is the fixed fee contract with fee $\Delta_L(0)$ when $\lambda\in[\bar{\lambda}(\varepsilon),1]$.*
3. *Both types of M accept the offer unless it is the fixed fee contract with fee $\Delta_L(0)$, in which case, only type L accepts.*

[4] We put no restriction on either royalty or fixed fee; in particular, both are allowed to take negative values, although offering a negative fixed fee or royalty is not optimal for the innovator. However, in an oligopoly, the innovator might find it optimal to charge a negative royalty for relatively insignificant innovations (see Liao & Sen, 2003).

See Appendix A for the Proof of Proposition 1.

Remark. It can be noted that

$$\bar{\varepsilon}=\sqrt{(a-c_H)(a+c_H-2c_L)}-(a-c_H) \text{ and } \tilde{\lambda}(\varepsilon)=\frac{2(a-c_H)}{2(a-c_H)+\varepsilon}.$$

The expression of $\bar{\lambda}(\varepsilon)$ is not simple. However, $\bar{\lambda}(\varepsilon)=\tilde{\lambda}(\varepsilon)=\varepsilon/(c_H-c_L)$ when $\varepsilon=\bar{\varepsilon}$. Define the continuous function $\rho(\varepsilon)$ as follows:

$$\rho(\varepsilon)=\begin{cases}\tilde{\lambda}(\varepsilon) & \text{for } \varepsilon\leq\bar{\varepsilon},\\ \bar{\lambda}(\varepsilon) & \text{for } \varepsilon>\bar{\varepsilon}.\end{cases}$$

Noting the fact that $\varepsilon\in[0,c_L]$, depending on the relative magnitudes of $\bar{\varepsilon}$, c_H-c_L, and c_L, we illustrate the proposition on the (ε,λ) plane in Figs. 1–3. In the figures, the line OE and the curve AD correspond to $\lambda=\varepsilon/(c_H-c_L)$and $\lambda=\rho(\varepsilon)$, respectively. Under complete information, when the licensing contract is a combination of fixed fee and royalty, for any given royalty, the innovator chooses the fixed fee to extract full surplus from the monopolist; thus, the latter is left with the preinnovation monopoly profit. However, under incomplete information, there is a trade-off in that the extraction of full surplus from type L results in rejection from type H. When the probability that M is type L is above certain threshold level, the innovator prefers to extract full surplus from type L. Otherwise, the innovator prefers acceptance from both types, the rate of royalty is positive, and type L is left with some surplus. Observe that a pure royalty contract is optimal only if $\varepsilon\leq\lambda(c_H-c_L)$ (equivalently, $\lambda\geq\varepsilon/(c_H-c_L)$). Noting that $\lambda(c_H-c_L)<a-c_L$, we conclude that a pure royalty is optimal only if the innovation is nondrastic with respect to the cost c_L.[5] Thus, as in the case of licensing under complete information, the notion of drastic innovation plays an important role in determining optimal licensing policies under asymmetric information.[6]

Proposition 1 rationalizes royalty contract and shows the coexistence of the three standard licensing schemes. However, the nondiscriminatory analysis is not entirely satisfactory because, although both types get the same contract, the innovator can infer perfectly about the cost of the monopolist from the royalty payments. This motivates us to adopt a more general approach in the next section, where, instead of a single nondiscriminatory contract, the innovator is allowed to offer a menu of contracts. It is shown that in the optimal menu, full separation is achieved; that is, two types get two different contracts, thus avoiding the uncomfortable nature of the static, nondiscriminatory analysis of this section.[7]

4. Menus of contracts

For this section, we take the more general approach, where the innovator can offer a menu of contracts. Since there are only two types of the monopolist, it is sufficient to consider the set of all

[5] A cost-reducing innovation is said to be *drastic* (Arrow, 1962) if the monopoly price under the new technology does not exceed the competitive price under the old technology; otherwise, it is *nondrastic*. For the demand $q=a-p$ and constant marginal cost c, a cost-reducing innovation of magnitude ϵ is drastic if $a-c\leq\varepsilon$, and it is nondrastic if $a-c>\varepsilon$.

[6] I thank an anonymous referee for bringing this to my notice. Under complete information, irrespective of the industry size, the optimal policy of an outsider innovator of a drastic innovation is to sell the license to only one firm, who becomes a monopolist with the reduced cost and the innovator collects the entire monopoly profit through a fee.

[7] This motivation for the analysis of menus of contracts was suggested by an anonymous referee.

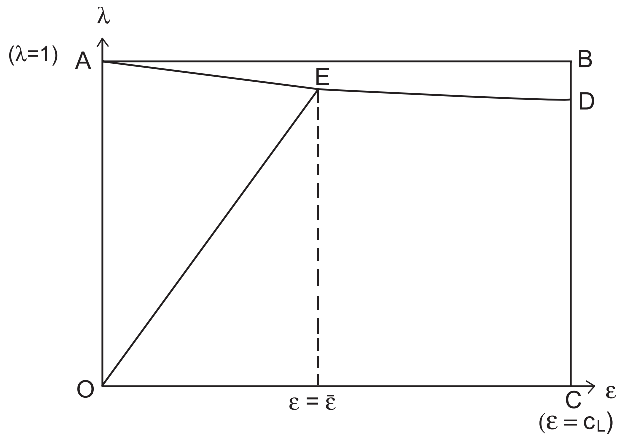

Fig. 1. $\bar{\varepsilon} < c_H - c_L \leq c_L$. Region OAE: royalty; Region ADB: fixed fee; Region OCDE: fixed fee plus royalty.

pairs of contracts of the form $\langle (r_L, f_L), (r_H, f_H) \rangle$, where $r_L, r_H, f_L, f_H \in \Re$. Observe that the set of all pairs of contracts include the set of all nondiscriminatory contracts because any nondiscriminatory contract (r, f) is equivalent to the pair of contracts $\langle (r, f),(r, f) \rangle$. The licensing game for this case can be described as follows. In the beginning, nature chooses the cost c of M to be c_L or c_H, with probability λ and 1-λ, respectively. The innovator (I), not knowing the realization of c, offers a menu of contracts $\langle (r_L, f_L),(r_H, f_H) \rangle$ to M, who is informed about the realization. If M rejects the menu of contracts, he produces with his old marginal cost and earns the corresponding monopoly profit, while I earns zero. If M accepts, then M has to choose a specific contract from the pair of contracts $\langle (r_L, f_L),(r_H, f_H) \rangle$. When M accepts, he produces with the reduced cost. For $j \in \{L,H\}$, if he chooses the contract (r_j, f_j) and produces q, he pays $r_j q + f_j$ to I. It is assumed that outputs produced by M are observable and there is no renegotiation.

In what follows, we shall determine the optimal menu of contracts for the innovator. Note that by our convention, when a typical menu $\langle (r_L, f_L),(r_H, f_H) \rangle$ is offered, the contract (r_t, f_t) is meant for type t for $t \in \{L,H\}$. Recall that Π_M (c_t, r) denotes the monopoly profit of M of type t when

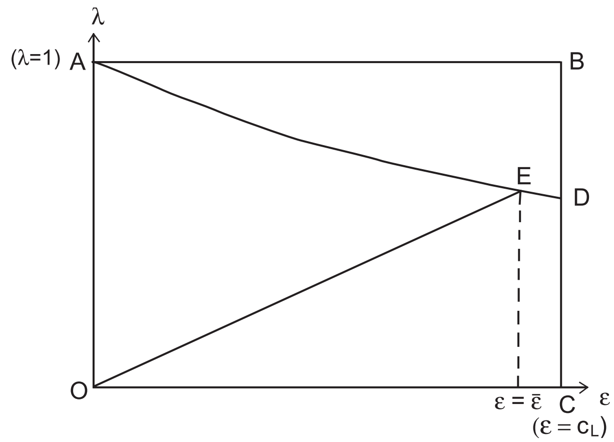

Fig. 2. $\bar{\varepsilon} \leq c_L \leq c_H - c_L$. Region OAE: royalty; Region ADB: fixed fee; Region OCDE: fixed fee plus royalty.

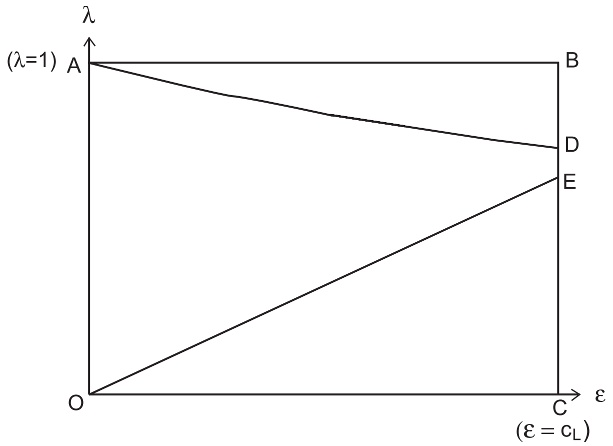

Fig. 3. $c_L \leq \bar{\varepsilon} < c_H - c_L$. Region OADE: royalty; Region ADB: fixed fee, Region OCE: fixed fee plus royalty.

he has the reduced cost and the rate of royalty is r and the preinnovation monopoly profit of M of type t is $\Pi_M(c_t,\varepsilon)$. Thus, M of type t will choose the contract (r_t,f_t) only if it satisfies $\Pi_M(c_t, r_t) - f_t \geq \Pi_M(c_t,\varepsilon)$.

Definition 1. We say that a menu of contracts $\langle (r_L, f_L),(r_H, f_H)\rangle$ satisfies the individual rationality constraints for both types if

$$\Pi_M(c_L, r_L) - f_L \geq \Pi_M(c_L, \varepsilon) \text{ and } \Pi_M(c_H, r_H) - f_H \geq \Pi_M(c_H, \varepsilon).$$

When the menu $\langle (r_L,f_L),(r_H,f_H)\rangle$ is offered, for $t \in \{L,H\}$, M of type t will choose the contract (r_t, f_t) only if his payoff from (r_t, f_t) is at least as high as his payoff from $(r_{t'}, f_{t'})$ for $t \neq t'$, that is, $\Pi_M(c_t, r_t) - f_t \geq \Pi_M(c_t, r_{t'}) - f_{t'}$.

Definition 2. We say that a menu of contracts $\langle (r_L, f_L),(r_H, f_H)\rangle$ satisfies the incentive compatibility constraints for both types if

$$\Pi_M(c_L, r_L) - f_L \geq \Pi_M(c_L, r_H) - f_H \text{ and } \Pi_M(c_H, r_H) - f_H \geq \Pi_M(c_H, r_L) - f_L.$$

The next observation follows from noting that if one of the constraints above is violated for a menu of contracts, that menu results in the same outcome as some nondiscriminatory contract. Since we have already determined the optimal nondiscriminatory contract in the previous section, it is thus enough to consider menus that satisfy these constraints.

Observation 1. *To determine the optimal menu of contracts, it is sufficient to consider menus that satisfy individual rationality and incentive compatibility constraints for both types.*

4.1. The optimal menu of contracts

In view of Observation 1, first, we find the optimal menu of contracts for the set of menus that satisfy individual rationality and incentive compatibility constraints. Then, we determine the optimal menu over all menus by comparing the payoff of the innovator from this contract with the corresponding payoff from the optimal nondiscriminatory contract that has been obtained in Proposition 1. It is shown that the optimal menu is always separating; that is, two types get two different contracts.

Proposition 2. *For* $t \in \{L,H\}$, *let* $\Delta_t(r) \equiv \Pi_M(c_t,r)\ \Pi_M(c_t,\varepsilon)$ *and* $\Psi_t(r_L,r_H) \equiv \Pi_M(c_t,r_H) - \Pi_M(c_t,r_L)$. *Suppose that the innovator (I) can offer any menu of contracts. Then, in the unique subgame-perfect equilibrium of the licensing game, the following holds:*

1. *I offers the menu* $\langle(0,\Delta_L(0)),(\varepsilon,\ 0)\rangle$ *when* $\lambda \geq \varepsilon/(c_H - c_L + \varepsilon)$.
2. *I offers the menu* $\langle(0,\ -\Psi_L(0,\bar{r}(\lambda)) + \Delta_H\ (\bar{r}(\lambda))),(\bar{r}(\lambda)\Delta_H(\bar{r}(\lambda)))\rangle$, *when* $\lambda \leq \varepsilon/(c_H - c_L + \varepsilon)$, *where* $\bar{r}(\lambda) = \lambda(c_H - c_L)/(1-\lambda)$.
3. *Both of the above menus of contracts satisfy individual rationality and incentive compatibility constraints. In both menus, individual rationality binds for type H and incentive compatibility binds for type L. Moreover, for the menu* $\langle 0,\Delta_L(0)),(\varepsilon,0)\rangle$, *individual rationality also binds for type L.*

See Appendix A for the Proof of Proposition 2.

The result of Proposition 2 can be illustrated on the (ε,λ) plane in Fig. 4.

4.2. Discussion on the optimal menu of contracts

The intuition behind Proposition 2 can be developed in particular clarity when the analysis is carried out in terms of iso-payoff curves of the two types of the monopolist. For the sake of completeness, we discuss the properties of iso-payoff curves in some detail. It should be mentioned that many of these properties are standard in the literature of contract theory (see, e.g., pp. 21–26 of Salanié, 1998 or pp. 110–116 of Macho-Stadler & Pérez-Castrillo, 2001).

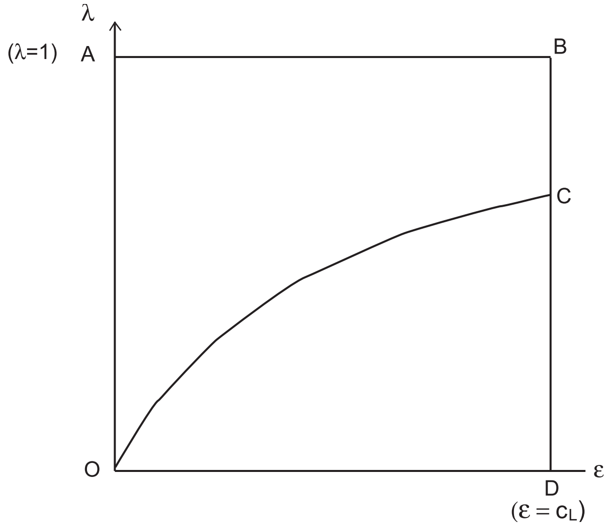

Fig. 4. Optimal menu of contracts. Region OABC—Type L: only fixed fee; Type H: only royalty. Region OCD—Type L: only fixed fee; Type H: fixed fee plus royalty.

• *It does not pay the innovator* (I) *to subsidize the monopolist* (M): Observe that I has two instruments to design a contract royalty and fixed fee. I can charge a negative royalty ($r<0$) to collect a high fixed fee; on the other hand, a high rate of royalty ($r>\varepsilon$) will be accepted by M only if the fixed fee is negative. However, the gain from a high fixed fee (royalty) is outweighed by the loss from negative royalty (fixed fee). [See Appendix A for the proof]. Hence, we can restrict our attention to contracts where r$\in[0,\varepsilon]$.

• *For both types, an iso-payoff curve of M is decreasing and strictly convex in r, and the "single-crossing property" is satisfied*: The iso-payoff curve of M of type t corresponding to the payoff $\bar{\Pi}$of is the locus of (r, f) such that $\Pi_M(c_t,r)-f=\bar{\Pi}$, or $f=\Pi_M(c_t, r)\Pi$. Since $\bar{\Pi}_M(c_t,r)$ is decreasing and strictly convex in r for $r\in[0,\varepsilon]$, so is any iso-payoff curve. Also note from the relation $f=\Pi_M(c_t,r)-\bar{\Pi}$ that for any r, a higher value of $\bar{\Pi}$ implies a lower value of f. Hence, for any two iso-payoff curves, the one that lies lower will have a higher value of $\bar{\Pi}$. We also note that the single-crossing property holds: An iso-payoff curve of type L intersects an iso-payoff curve of type H exactly once (the intersection can occur outside the interval $[0,\varepsilon]$). In Fig. 5, the curves AC and BC correspond to the individual rational levels of payoffs of types L and H [$\Pi_M(c_t, \varepsilon)$ for type t], respectively, and they intersect at the Point C (where $r=\varepsilon$).

• *For any* $r\in[0, \varepsilon]$, *an iso-payoff curve of type L is steeper than an iso-payoff curve of type H*: Note that for any $r\in[0, \varepsilon]$, the output of type L is higher than the output of type H, so that for a change in r, the change in royalty payment is also higher for type L. Thus, to keep the level of payoff fixed after a

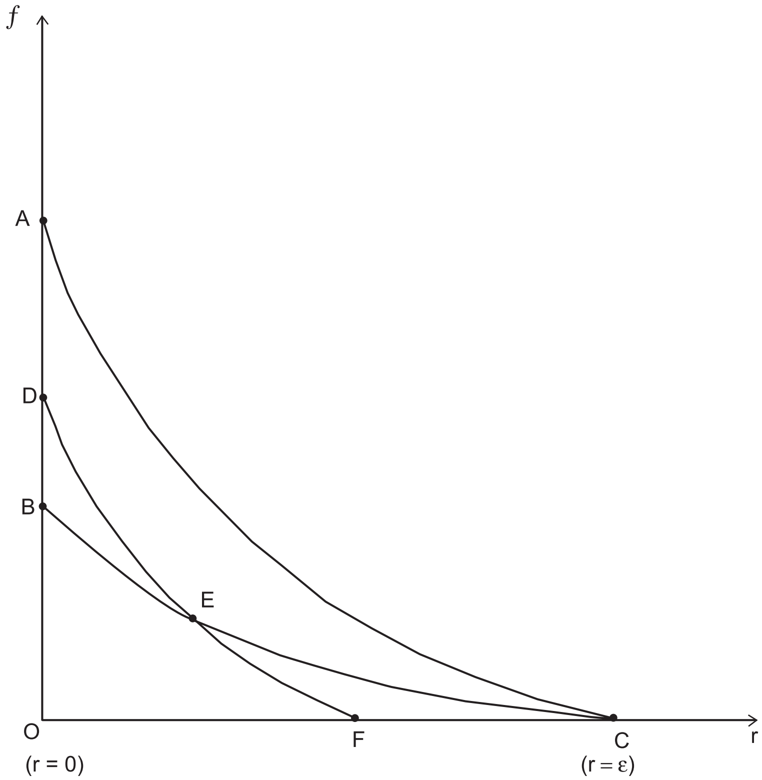

Fig. 5. Iso-payoff curves.

change in r, the change in fixed fee f must be higher for type L compared with type H; that is, the "marginal rate of substitution" between r and f is higher for type L, causing its iso-payoff curve to be steeper. As a consequence, for two different iso-payoff curves of type L, the one that intersects an iso-payoff curve of type H at a lower rate of royalty lies lower (and corresponds to a higher payoff for type L). For example, in Fig. 5, the iso-payoff curve DF of type L intersects the iso-payoff curve BC of type H at E, which corresponds to a rate of royalty $r<\varepsilon$, while the iso-payoff curve AC (that corresponds to the individual rational level of type L) intersects BC at $r=\varepsilon$.

• *In an optimal menu, there must be acceptance from both types*: If a contract (r, f) lies outside the region OAC in Fig. 5, it will be rejected by both types; thus, it is dominated for I to include such a contract in a menu. If a contract lies in ABC (excluding the curve BC), it will be accepted only by type L. When only type L accepts, the maximum payoff of I is achieved at the pure fixed fee contract $(0,\Delta_L(0))$ (Point A), where I extracts full surplus from type L. However, this is clearly dominated by the menu $\langle(0,\Delta_L(0)),(\varepsilon, 0)\rangle$ (the pair of points (A,C)), where, in addition to the fixed fee from type L, I gets a royalty payment from type H (note that type L is indifferent between points A and C, while H prefers C to A). Thus, in an optimal menu, there must be acceptance from both types, so that the contract of type L must lie in the region OAC (including AC), while that of type H must lie in OBC (including BC).

• *To determine an optimal menu, it is sufficient to consider menus that satisfy individual rationality and incentive compatibility for both types*: Because an optimal menu must be accepted by both types, individual rationality constraints will hold. If, for a menu, incentive compatibility is violated for both types, I can offer a new menu by simply switching the contracts: The new menu will satisfy incentive compatibility without changing the payoff of I. If for a menu, incentive compatibility is violated for only one type, I can offer a new (pooling) menu by offering the same contract to both types; this will again satisfy incentive compatibility without changing the payoff of I.

• *In an optimal menu, the incentive compatibility constraint of type L must bind*: Suppose incentive compatibility does not bind for type L. If individual rationality also does not bind for type L, then I can increase the fixed fee of the contract of type L and improve the payoff without affecting any constraints. Hence, consider a menu where individual rationality of type L binds; that is, the contract of type L lies on AC in Fig. 5. Note that AC intersects any iso-payoff curve of type H in the region OBC at $r\geq\varepsilon$. If incentive compatibility of type L does not bind, then the contract of type H must lie on the right of the point of intersection of AC and the iso-payoff curve of the contract type H. This will correspond to a royalty rate $r>\varepsilon$, which can be shown to be suboptimal (see Appendix A). Thus, in an optimal menu, the incentive compatibility constraint of type L must bind so that the contract of type H must be the point of intersection of the iso-payoff curves of contracts of L and H.

• *In an optimal menu, the individual rationality constraint of type H must bind*: Suppose individual rationality does not bind for type H. If individual rationality does not bind also for type L, then I can improve the payoff by increasing the fixed fee of both contracts by a small amount. Hence, consider a menu where individual rationality of type L binds; that is, the contract of type L lies on AC in Fig. 5. Note that any iso-payoff curve of type H that lies below BC (and corresponds to a payoff higher than the individual rational level) intersects AC at $r>\varepsilon$, which is suboptimal. Thus, in an optimal menu, the individual rationality constraint of type H must bind so that the contract of type H is the point of intersection of BC and the iso-payoff curve of the contract of type L.

• *In an optimal menu, the contract of type L must be a pure fixed fee contract*: This can be illustrated using the relative steepness of iso-payoff curves of two types. Suppose DF is the iso-payoff curve that corresponds to the contract of type L (see Fig. 5; the same argument will hold for AC or any other iso-payoff curve of type L). Because DF intersects BC (the individual rational level iso-payoff curve of type H) at E, by our discussion from the last paragraph, we conclude that the contract of type H is E. Observe that the contract of type L must lie on the part DE; otherwise, the incentive compatibility of type H will be violated. Out of all contracts of type L that lie on the part DE, the payoff of I is maximized at the pure fixed fee contract given by the point D. This is because any positive royalty results in a distortion of the innovation and reduces the surplus. By charging zero royalty, I allows M of type L to operate in the most efficient way so that maximum surplus is generated, which is then collected through a fixed fee (as in complete information). This shows that in an optimal menu, the contract of type L must be a pure fixed fee contract.[8]

• *The royalty rate charged to type H completely determines an optimal menu*: In view of our discussion so far, we conclude that for an optimal menu, (i) the contract of type H lies on the curve BC and (ii) the contract of type L is the fixed fee contract that lies on the iso-payoff curve of L, which intersects BC at the contract of type H. This proves that the royalty rate charged to type H completely determines a menu, and I will choose this royalty to maximize her payoff. Suppose the rate of royalty is r. Then the fixed fee to type H, $f_H(r)$, can be determined from the individual rational level of payoff of H and is given by

$$f_H(r) = \Pi_M(c_H, r) - \Pi_M(c_H, \varepsilon). \tag{1}$$

The fixed fee to type L, $f_L(r)$, can be determined from the incentive compatibility of L and it satisfies $\Pi_M(c_L,0) - f_L(r) = \Pi_M(c_L, r) - f_H(r)$, so that for a suitably chosen constant k_1,

$$f_L(r) = \Pi_M(c_H, r) - \Pi_M(c_L, r) + k_1. \tag{2}$$

Let $p_t(r)$ and $q_t(r)$ be the monopoly price and output, respectively, for type t under reduced cost when the rate of royalty is r. Then, the payoff of I is given by $\Pi_I(r) = \lambda f_L(r) + (1-\lambda)[rq_H(r) + f_H(r)]$. From Eqs. (1) and (2), after slightly rearranging the terms, we then have the following for a suitably chosen constant k_2.

$$\Pi_I(r) = [p_H(r) - (c_H - \varepsilon)]q_H(r) - \lambda[\Pi_M(c_L, r) + rq_H(r)] + k_2. \tag{3}$$

• *In an optimal menu, the contract of type H always involves positive royalty*: This is shown by observing that $\Pi_I(r)$ is increasing in r for small positive values of r. Let us denote $R_1(r) \equiv [p_H(r) - (c_H - \varepsilon)]q_H(r)$, $R_2(r) \equiv \theta rq_H(r)$, and $R_3(r) \equiv -[\lambda \Pi_M(c_L, r) + (\lambda + \theta)rq_H(r)]$. Then, from Eq. (3), we have $\Pi_I(r) = R_1(r) + R_2(r) + R_3(r) + k_2$. Observe that $R_3(r)$ is increasing in r for small positive values of r when $\theta = 0$. Hence, the same is true when θ is sufficiently small. Let us consider such a small value of θ satisfying $0 < \theta < 1$. In what follows, we will show that $R_1(r) + R_2(r)$ is also increasing in r for small values of θ. It will be convenient to carry out the analysis in terms of a scaled-down version of the output. Let us denote $q(r) \equiv \theta q_H(r)$. Since $p_H(r) = a - q_H(r)$, we have

$$R_1(r) = \left[(a - c_H + \varepsilon)/\theta - q(r)/\theta^2\right]q(r) \text{ and } R_2(r) = rq(r).$$

Note that for the revenue R_1, the average revenue and marginal revenue for output q are given by

$$AR_1(q) = (a - c_H + \varepsilon)/\theta - \left(1/\theta^2\right)q \text{ and } MR_1(q) = (a - c_H + \varepsilon)/\theta - \left(2/\theta^2\right)q.$$

[8] Note that in Fig. 5, if D and E coincide, the menu will be pooling, and both types will get a pure fixed fee contract; in fact, type H can get a pure fixed fee contract only if the menu is pooling. However, offering such a pooling contract is not optimal for I.

Noting that $q(r)=\theta(a-c_H+\varepsilon\text{-}r)/2$ when $r\in[0,\varepsilon]$, for the revenue R_2 (here, the rate of royalty r is the "price"), we have

$$AR_2(q) = a - c_H + \varepsilon - (2/\theta)q \text{ and } MR_2(q) = a - c_H + \varepsilon - (4/\theta)q.$$

In Fig. 6, taking the rate of royalty and price on the vertical axis and quantity on the horizontal axis, the lines MR_1, AR_2, and MR_2 have been drawn. For any rate of royalty i, the quantity $q(r)$ is identified from AR_2. Then, for $i=1$, 2, the area under MR_i until the point $q(r)$ presents $R_i(r)$. When $r=0$, $q(0)=\theta(a-c_H+\varepsilon)/2$ (given by OB in Fig. 6). Then, $R_1(0)=\Delta$OAB and R_2 $(0)=\Delta\text{OCD}-\Delta\text{DBE}=0$, so that R_1 $(0)+R_2$ $(0)=\Delta$OAB. Now, consider a positive value of r such that $r<(a-c_H+\varepsilon)/2$ so that $q(r)>\theta(a-c_H+\varepsilon)/4$, that is, $q(r)$ lies on the right of D in Fig. 6 [OR is such a value of r, and the corresponding $q(r)$ is given by OG]. Then, compared with $r=0$, the loss in revenue from R_1 is given by ΔGHB, while the gain in revenue from R_2 is given by the shaded region BGFE. While deciding the rate of royalty, the innovator thus faces a trade-off.

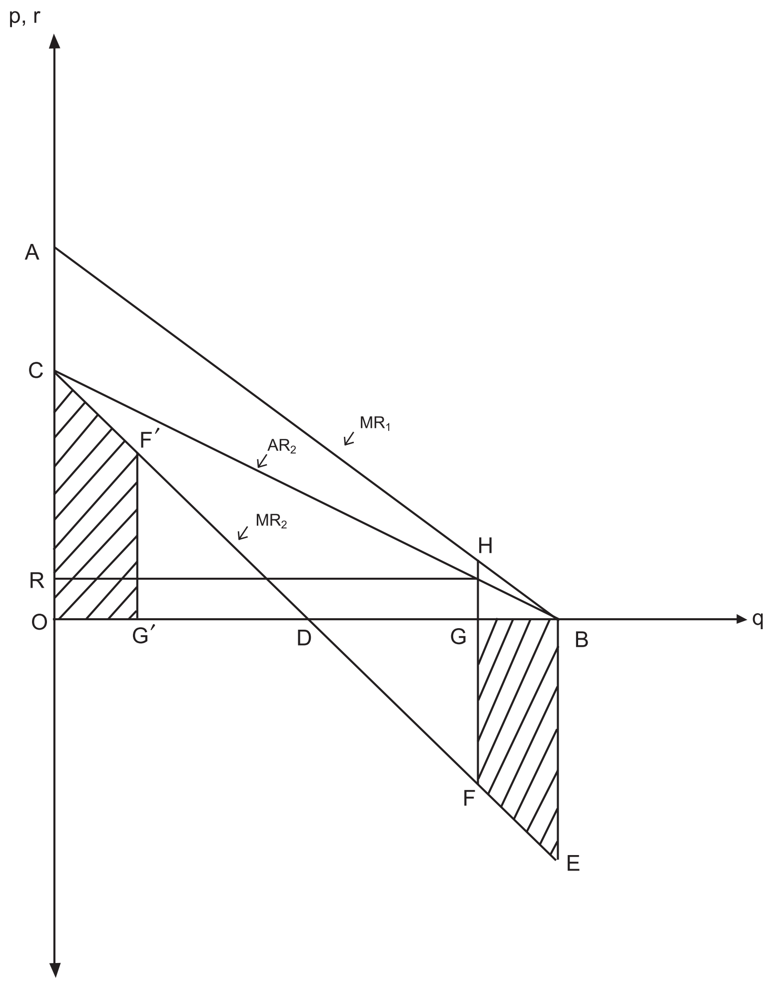

Fig. 6. The role of the elasticity of demand.

On the one hand, a low rate of royalty leads to higher revenue for M and contributes more to the fixed fee [$R_1(r)$], but then, the revenue from royalty [$R_2(r)$] is less. The elasticity of demand plays a crucial role in settling this trade-off.[9] To see this, let us compare ΔGHB with the region BGFE. Observe that the region BGFE has the same area as the shaded region OG′F′C. We know that for linear demand, elasticity falls in quantity.[10] Comparing ΔGHB with OG′F′C, it can be seen that when G′ is sufficiently close to O (equivalently, G is sufficiently close to B, which holds when r is sufficiently small), the elasticity of AR_2 at output OG′ is more than the elasticity of AR_1 at output OG, and as a consequence, OG′F′C is more than ΔGHB. This can be also seen by observing that

$$\text{BGFE} = OG'F'C = \theta r\left(a - c_H + \varepsilon - r\right)/2 \text{ and } \Delta GHB = r^2/4,$$

so that for any $\theta > 0$, there is $0 < r(\theta) < (a - c_H + \varepsilon)/2$ such that

$$\text{BGFE} > \Delta GHB \text{ for } 0 < r < r(\theta).$$

This shows that $R_1(r)+R_2(r)$ is also increasing in r for small values of r, and then we conclude that the optimal value of r is positive, implying that the contract of type H always involves positive royalty.

• *The contract of type H is a pure royalty contract when the magnitude of the innovation is sufficiently small*: This follows immediately from our discussion in the last paragraph. Because the payoff of I, $\Pi_I(r)$, is increasing in r for small values of r and the optimal value of r is, at most, ε, when ε is sufficiently small, $\Pi_I(r)$ would be increasing in r throughout the interval $[0,\varepsilon]$. Then, it is maximized at $r = \varepsilon$; that is, the contract offered to M of type H would be a pure royalty contract with rate of royalty ε.[11] One reason behind this could be that when ε is small, the post-innovation monopoly profit is not significantly higher than the pre-innovation profit, and as a consequence, the fixed fee (that depends on the difference between these two) does not contribute significantly towards the payoff of the innovator. Then, the loss of revenue from fixed fee due to high royalty is outweighed by the gain in revenue from royalty payments (i.e., gain in R_2 is more than the loss in R_1). Now, the exact threshold level of the innovation that renders a pure royalty to be optimal will, of course, depend on the parameters of the model (in particular, the relative steepness of AR_1 and AR_2 depends on θ, which, in turn, depends on λ). Specifically, a pure royalty contract is offered to type H when $\varepsilon \leq \lambda(c_H - c_L)/(1-\lambda)$, which is equivalent to the condition $\lambda \geq \varepsilon/(c_H - c_L + \varepsilon)$, as stated in Proposition 2.

To conclude, we have shown that asymmetry of information can clearly explain the optimality of royalty licensing and the coexistence of different licensing schemes. While standard properties of contracts drive the results to some extent, in this specific context of licensing of a cost-reducing innovation, the elasticity of market demand and the magnitude of the innovation play crucial roles in determining the optimal configuration of licensing policies.

[9] I am grateful to an anonymous referee for this precise intuition. It should be mentioned that elasticity of demand also plays important role in the models of Beggs (1992) and Choi (2001).

[10] We define elasticity as rate of change of output/rate of change of price; thus, for the inverse demand $p = a – bq$, elasticity at output q is given by $e(q) = (\Delta q/q)/(\Delta p/p) = (a – bq)/bq$. Thus, $e(q)$ is always positive, equals one when $MR(q) = 0$, more than one when $MR(q) = 0$, and less than one when $MR(q) < 0$.

[11] When type H gets a pure royalty contract, the menu is given by the pair of points (A,C) in Fig. 5. When type H gets a combination of fixed fee and royalty, the menu is the pair (D,E) for some iso-payoff curve DF of type L that lies below AC.

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Appendix A

Notations. For $t \in \{L,H\}$, we denote by $\Pi_M(c_t,r)$ the monopoly profit of M of type t with the reduced cost $c_t - \varepsilon$ when the rate of royalty is r. Observe that the preinnovation monopoly profit of M of type t is given by $\Pi_M(c_t,\varepsilon)$. We denote $\Delta_t(r) \equiv \Pi_M(c_t,r) - \Pi_M(c_t\varepsilon)$.

Proof of Proposition 1. Consider $t \in \{L,H\}$ and let M be of type t. The payoff of M of type t when he accepts the contract (r,f) and produces output q is given by $(a-q)q-(c_t-\varepsilon+r)q-f$. Let $q_M(c_t,r)$ denote the optimal output of M of type t when the rate of royalty is r. Standard optimization conditions yield

$$q_M(c_t,r) = \max\{(a - c_t + \varepsilon - r)/2, 0\} \text{ and } \Pi_M(c_t,r) = (q_M(c_t,r))^2. \tag{4}$$

Observation 1.1. *When the rate of royalty is r, the maximum fixed fee that M of type t is willing to pay is $\Delta_t(r)$, so that a contract (r,f) is rejected by M of type t when $f > \Delta_t(r)$.*

To simplify the analysis, it is implicitly assumed that for any type, when M is indifferent between accepting or rejecting a contract, he chooses the action where the payoff of I is more. The proof proceeds in the following way. For any given rate of royalty r, we determine the fixed fee (as a function of r) such that the payoff of I is maximum for that given value of r. Then, the optimal contract is found by considering all possible values of r. We consider the following possible cases.

Case 1. $r \geq a - c_L + \varepsilon$. Using the fact that $c_L < c_H$, it follows from Eq. (4) that I cannot earn a positive payoff for this case.

Case 2. $r \in [a - c_H + \varepsilon, a - c_L + \varepsilon]$. For this case, again from Eq. (4), it follows that $q_L(r) = (a - c_L + \varepsilon - r)/2$ and $q_H(r) = 0$.

Observation 1.2. *Let $r \in [a - c_H + \varepsilon, a - c_L + \varepsilon]$. Then $\Delta_L(r) < \Delta_H(r)$.*

From Observations 1.1 and 1.2, it can be seen that when $f > \Delta_L(r)$, the payoff of I is, at most, zero. For Case 2, when $f \leq \Delta_L(r)$, for any given rate of royalty r, the optimal fixed fee is $\Delta_L(r)$. Then, it can be shown from certain standard optimization conditions that for Case 2, the maximum payoff of I is attained at $r = a - c_H + \varepsilon$ for all $\lambda \in [0,1]$. From Cases 1 and 2, we conclude the following.

Conclusion 1.1. *When $r \in [a - c_H + \varepsilon, a - c_L + \varepsilon]$, the maximum payoff of I is either zero or the maximum is attained at $r = a - c_H + \varepsilon$.*

Observation 1.3. *When $r \in (\varepsilon, a - c_H + \varepsilon]$, then $\Delta_L(r) < \Delta_H(r) < 0$. When $r < \varepsilon$, then $\Delta_L(r) > \Delta_H(r) > 0$. Moreover, $\Delta_H(\varepsilon) = \Delta_L(\varepsilon) = 0$.*

Case 3. $r \in [\varepsilon,\ a - c_H + \varepsilon]$. For this case, using Observation 1.3 and certain standard optimization conditions, it can be shown that the payoff of I is maximized at the royalty contract $(\varepsilon, 0)$ and is given by $\Pi_I(\varepsilon, 0) > 0$, where

$$\Pi_I(\varepsilon, 0) = \lambda\varepsilon(a - c_L)/2 + (1 - \lambda)\varepsilon(a - c_H)/2. \tag{5}$$

Because $r = a - c_H + \varepsilon$ is feasible for Case 3, from Conclusion 1.1, we have the following.

Conclusion 1.2. *For $r \geq \varepsilon$, the payoff of I is maximized at the royalty contract $(\varepsilon, 0)$.*

Case 4. $r \leq \varepsilon$. From Observation 1.3, we have $\Delta_L(r) \geq \Delta_H(r) \geq 0$ for this case.

Subcase 4.(a). *$f > \Delta_L(r)$.* For this case, both types of M will reject the offer, so that the payoff of I is zero.

Subcase 4.(b). $f \in (\Delta_H(r), \Delta_L(r)]$. For this case, only M of type L will accept the offer. When I maximizes, $f = \Delta_L(r)$ is chosen, so that the payoff is $\Pi_I(r, \Delta_L(r)) = \lambda[q_L(r) + \Delta_L(r)]$, which is maximized at $r = 0$ (a fixed fee contract), and the payoff is given by

$$\Pi_I(0, \Delta_L(0)) = \lambda[(a - c_L + \varepsilon)^2/4 - (a - c_L)^2/4]. \tag{6}$$

Subcase 4.(c). $f \leq \Delta_H(r)$. For this case, both types accept the offer. When I maximizes, $f = \Delta_H(r)$ is chosen, and the payoff is given by $\Pi_I(r, \Delta_H(r)) = \lambda r q_L(r) + (1-\lambda) r q_H(r) + \Delta_H(r)$. The unconstrained maximizer is $r = \lambda(c_H - c_L)$.

(i) $\lambda(c_H - c_L) \geq \varepsilon$. Then, the maximum is attained at $r = \varepsilon$. Since $\Delta_L(\varepsilon) = 0$, the payoff of I is $\Pi_I(\varepsilon, 0)$, given by Eq. (5).

(ii) $\lambda(c_H - c_L) \leq \varepsilon$. Then, the maximum is attained at $r = \lambda(c_H - c_L)$ and the payoff of I is

$$\Pi_I(\lambda(c_H - c_L), \Delta_H(\lambda(c_H - c_L))) = \lambda^2(c_H - c_L)^2/4 + \varepsilon[2(a - c_H) + \varepsilon]/4. \tag{7}$$

Now, we are in a position to determine the optimal licensing contract for I. Recall from Conclusion 1.2 that for $r \geq \varepsilon$, the payoff of I is maximized at $r = \varepsilon$, which is feasible for Case 4. Thus, it is enough to consider Case 4; that is, the case where $r \leq \varepsilon$. Since the payoff of I is zero when $f > \Delta_L(r)$ Subcase 4.(a), the relevant cases are Subcases 4.(b) and 4.(c). In what follows, we compare the payoffs in these cases to find the optimal contract.

[1] When $\varepsilon \geq c_H - c_L$, we have $\lambda(c_H - c_L) \leq \varepsilon$ for all $\lambda \in [0,1]$ so that Item (ii) of Subcase 4.(c) holds. Comparing Eq. (6) from Subcase 4.(b) and Eq. (7) from Item (ii) of Subcase 4.(c), we have

$$\Pi_I(0, \Delta_L(0)) \geq \Pi_I(\lambda(c_H - c_L)), \Delta_H(\lambda(c_H - c_L)) \Leftrightarrow h(\lambda) \geq 0 \text{ where} \tag{8}$$

$h(\lambda) = -(c_H - c_L)^2\lambda^2 + \varepsilon(2a - 2c_L + \varepsilon)\lambda - \varepsilon(2a - 2c_H + \varepsilon)$. Before proceeding further, we state the following observations regarding the function $h(\lambda)$, which follow from certain standard properties of quadratic functions.

Observation 1.4. *Fix $\varepsilon > 0$. If $h(k(\varepsilon)) \geq 0$ for some number $k(\varepsilon) > 0$, then there is $\bar{\lambda}(\varepsilon) \in (0, k(\varepsilon)]$ such that $h(\bar{\lambda}(\varepsilon)) = 0$, $h(\lambda) < 0$ for $\lambda \in [0, \bar{\lambda}(\varepsilon)]$ and $h(\lambda) > 0$ for $\lambda \in (\bar{\lambda}(\varepsilon), k(\varepsilon)]$. In particular, $\bar{\lambda}(\varepsilon) = k(\varepsilon)$ iff $h(k(\varepsilon)) = 0$.*

Observation 1.5. *$h(\lambda)$ is increasing in λ for $\lambda \in [0, \varepsilon/(c_H - c_L)]$.*

Note that $h(1) = (c_H - c_L)[2\varepsilon - (c_H - c_L)]$. Hence, $h(1) > 0$ when $\varepsilon \geq c_H - c_L$. From Eq. (8) and Observation 1.4, we conclude that when $\varepsilon \geq c_H - c_L$, there is $\bar{\lambda}(\varepsilon) \in (0,1)$ such that the optimal contract is the fixed fee contract $\langle 0, \Delta_L(0) \rangle$ for $\lambda \in [\lambda(\varepsilon), 1]$ and it is $(\lambda(c_H - c_L), {}_H(\lambda(c_H - c_L)))$ [a combination of fixed fee $\Delta_H(\lambda(c_H - c_L))$ and royalty $\lambda(c_H - c_L)$][12] for $\lambda \in [0, \bar{\lambda}(\varepsilon)]$. The result for this case can be summarized as follows (Table 1).

Table 1

	$\lambda \in [0, \bar{\lambda}(\varepsilon)]$	$\lambda \in [\bar{\lambda}(\varepsilon), 1]$
$\varepsilon \geq c_H - c_L$	Fixed fee plus royalty	Fixed fee

[2] When $\varepsilon \leq c_H - c_L$ and $\lambda(c_H - c_L) \leq \varepsilon$, again Item (ii) holds for Subcase 4.(c), and we need to compare Eqs. (6) and (7). Consider again the function $h(\lambda)$. Evaluating $h(\lambda)$ at $\lambda = \varepsilon/(c_H - c_L)$, we get $h(\varepsilon/(c_H - c_L)) = \varepsilon g(\varepsilon)/(c_H - c_L)$, where

$$g(\varepsilon) = \varepsilon^2 + 2(a - c_H)\varepsilon - 2(a - c_H)(c_H - c_L). \tag{9}$$

Using the fact that $g(\varepsilon)$ is a quadratic function in ε, from standard properties, we conclude that there exists $\varepsilon \in (0, c_H - c_L)$ such that $g(\varepsilon) \leq 0$ for $\varepsilon \in [0, \bar{\varepsilon}]$ and $g(\varepsilon) \geq 0$ for $\varepsilon \in [\bar{\varepsilon}, c_H - c_L]$ with equality iff $\varepsilon = \bar{\varepsilon}$.

First, consider the case when $\varepsilon \in [0, \varepsilon]$. Since $h(\varepsilon/(c_H - c_L)) \leq 0$ for this case, from Eq. (8) and Observation 1.5, it follows that $(\lambda(c_H - c_L), \Delta_H(\lambda(c_H - c_L)))$ is the optimal contract for all $\lambda \in [0, \varepsilon/(c_H - c_L)]$. The result for this case can be summarized as follows (Table 2).

Table 2

	$\lambda \in [0, (\varepsilon)/(c_H - c_L)]$
$\varepsilon \in [0, \bar{\varepsilon}]$	Fixed fee plus royalty

Next, consider $\varepsilon \in [\bar{\varepsilon}, c_H - c_L]$. Since $h(\varepsilon/(c_H - c_L)) \geq 0$ for this case, from Eq. (8) and Observation 1.4, it follows that $(\lambda(c_H - c_L), \Delta_H(\lambda(c_H - c_L)))$ is the optimal contract for $\lambda \in [0, \lambda(\varepsilon))]$ and it is $(0, \Delta_L(0))$ for $\lambda \in [\lambda(\varepsilon), \varepsilon/(c_H - c_L)]$. In particular, evaluating $\lambda(\varepsilon)$ at $\varepsilon = \varepsilon$, we have $\lambda(\varepsilon) = \varepsilon/(c_H - c_L)$. We can summarize the result for this case as follows (Table 3).

Table 3

	$\lambda \in [0, \bar{\lambda}(\varepsilon)]$	$\lambda \in [\bar{\lambda}(\varepsilon), \varepsilon/(c_H - c_L)]$
$\varepsilon \in [\bar{\varepsilon}, c_H - c_L]$	Fixed fee plus royalty	Fixed fee

[12] When $\lambda = 0$, this contract reduces to the fixed fee contract $\langle 0, \Delta_H(0) \rangle$, which is the optimal contract in case of complete information, where M is of type H for sure.

[3] When $\varepsilon \leq c_H - c_L$ and $\lambda(c_H - c_L) \geq \varepsilon$, Item (i) of Subcase 4.(c) holds, and we need to compare Eqs. (6) and (5). We have

$$\Pi_I(\varepsilon, 0) \geq \Pi_I\big(0, \Delta_L(0)\big) \Leftrightarrow \lambda \leq 2\big(a - c_H\big)/\big[2(a - c_H) + \varepsilon\big] \equiv \tilde{\lambda}(\varepsilon). \tag{10}$$

Since $\lambda \geq \varepsilon/(c_H - c_L)$, from Eq. (10), we conclude that $\Pi_I(0,\Delta_L(0)) \geq \Pi_I(\varepsilon,0)$ when $\varepsilon/(c_H - c_L) \geq \lambda(\varepsilon)$. Next, we note that $\varepsilon/(c_H - c_L) \geq \lambda(\varepsilon)$ iff $g(\varepsilon) \geq 0$, where $g(\varepsilon)$ is given by Eq. (9).

First, consider $\varepsilon \in [0,\varepsilon]$. For this case, $g(\varepsilon) \leq 0$ so that $\varepsilon/(c_H - c_L) \geq \lambda(\varepsilon)$. Then, from Eq. (10), we conclude that the optimal contract is the royalty contract $(\varepsilon,0)$ when $\lambda \in [\varepsilon/(c_H - c_L),\lambda(\varepsilon)]$ and it is the fixed fee contract $(0,\Delta_L(0))$ when $\lambda \in [\lambda(\varepsilon),1]$. It can be noted that

$$\tilde{\lambda}(\bar{\varepsilon}) = \bar{\varepsilon}/(c_H - c_L) \text{ and } \bar{\varepsilon} = \sqrt{(a - c_H)(a + c_H - 2c_L)} - (a - c_H). \tag{11}$$

The result for this case can be summarized as follows (Table 4).

Table 4

	$\lambda \in [\varepsilon/(c_H - c_L),\tilde{\lambda}(\varepsilon)]$	$\lambda \in [\tilde{\lambda}(\varepsilon),1]$
$\varepsilon \in [0,\bar{\varepsilon}]$	Royalty	Fixed fee

When $\varepsilon \in [\bar{\varepsilon},c_H - c_L]$, we have $g(\varepsilon) \geq 0$, implying that $\varepsilon/(c_H - c_L) \geq \tilde{\lambda}(\varepsilon)$. Hence, for this case, $\Pi_I(0,\Delta_L(0)) \geq \Pi_I(\varepsilon,0)$ for all $\lambda \geq \varepsilon/(c_H - c_L)$ and the optimal contract is the fixed fee contract $(0,\Delta_L(0))$. The result for this case can be summarized as follows (Table 5).

Table 5

	$\lambda \in [\varepsilon/(c_H - c_L),1]$
$\varepsilon \in [\bar{\varepsilon},c_H - c_L]$	Fixed fee

Combining Tables 2 and 4, we get the following (Table 6).

Table 6

	$\lambda \in [0,(\varepsilon)/(c_H - c_L)]$	$\lambda \in [(\varepsilon)/(c_H - c_L),\tilde{\lambda}(\varepsilon)]$	$\lambda \in [\tilde{\lambda}(\varepsilon),1]$
$\varepsilon \in [0,\bar{\varepsilon}]$	Fixed fee plus royalty	Royalty	Fixed fee

Combining Tables 3 and 5, we get the following (Table 7).

Table 7

	$\lambda \in [0,\bar{\lambda}(\varepsilon)]$	$\lambda \in [\bar{\lambda}(\varepsilon),1]$
$\varepsilon \in [\bar{\varepsilon},c_H - c_L]$	Fixed fee plus royalty	Fixed fee

Now combining Tables 1 and 7, we get the following (Table 8).

Table 8

	$\lambda \in [0,\bar{\lambda}(\varepsilon)]$	$\lambda \in [\bar{\lambda}(\varepsilon),1]$
$\varepsilon \geq \bar{\varepsilon}$	Fixed fee plus royalty	Fixed fee

Then, Parts 1 and 2 of Proposition 1 follows from Tables 6 and 8. Part 3 follows by noting that for the royalty contract (ε,0) and fixed plus royalty contract ($\lambda(c_H - c_L)$,$\Delta_H(\lambda(c_H - c_L))$), both types of M accept the offer, while for the fixed fee contract (0,$\Delta_L(0)$), type L accepts and type H rejects. □

Proof of Proposition 2. We find the optimal menu of contracts in the following way. Using the conclusion of Observation 1, first we restrict ourselves to the set of menus that satisfy individual rationality and incentive compatibility constraints for both types and find the optimal menu within this set. Then, we compare the payoff from this menu with the payoff of the optimal nondiscriminatory contract that is obtained in Proposition 1 and determine the optimal menu of contracts in the set of all menus of contracts.

A menu of contracts $\langle(r_L, f_L),(r_H, f_H)\rangle$ satisfies individual rationality constraints for both types if

$$\Pi_M(c_L, r_L) - f_L \geq \Pi_M(c_L, \varepsilon), \tag{12}$$

$$\Pi_M(c_H, r_H) - f_H \geq \Pi_M(c_H, \varepsilon). \tag{13}$$

The menu satisfies incentive compatibility constraints for both types if

$$\Pi_M(c_L, r_L) - f_L \geq \Pi_M(c_L, r_H) - f_H, \tag{14}$$

$$\Pi_M(c_H, r_H) - f_H \geq \Pi_M(c_H, r_L) - f_L. \tag{15}$$

The following observation follows from certain standard comparisons.

Observation 2.1. *Consider any menu of contracts $\langle(r_L, f_L),(r_H, f_H)\rangle$ such that Eqs. (12)–(15) hold. If it does not satisfy both $r_L \leq a - c_H + \varepsilon$ and $r_H \leq a - c_H + \varepsilon$, then offering such a menu is a dominated strategy for the innovator.*

In view of Observation 2.1, it is sufficient to consider menus of contracts that, apart from satisfying Eqs. (12)–(15), also satisfy $r_L \leq a - c_H + \varepsilon$ and $r_H \leq a - c_H + \varepsilon$. For such a menu, from Eq. (4), we have

$$\Pi_M(c_t, r_{t'}) = (a - c_t + \varepsilon - r_{t'})^2/4 \text{ for } t, t' \in \{L, H\}.$$

For $t \in \{L,H\}$, let us denote $\Psi_t(r_L,r_H) \equiv \Pi_M(c_t,r_H) - \Pi_M(c_t,r_L)$. For the contracts under consideration, we have

$$\Delta_t(r) = (a - c_t + \varepsilon - r)^2/4 - (a - c_t)^2/4, \tag{16}$$

$$\Psi_t(r_L, r_H) = (a - c_t + \varepsilon - r_H)^2/4 - (a - c_t + \varepsilon - r_L)^2/4. \tag{17}$$

Then, the Constraints (12)–(15) can be written as

$$f_L \leq \Delta_L(r_L), f_H \leq \Delta_H(r_H), \Psi_L(r_L, r_H) \leq f_H - f_L \leq \Psi_H(r_L, r_H). \tag{18}$$

Observation 2.2. *Consider any menu of contracts $\langle(r_L, f_L),(r_H, f_H)\rangle$ such that $r_L \leq a - c_H + \varepsilon$ and $r_H \leq a - c_H + \varepsilon$. Such a menu satisfies Eq. (18) only if $r_L \leq r_H$.*

In view of Observation 2.2, we can restrict ourselves to menus satisfying Eq. (18) and $r_L \leq r_H \leq a - c_H + \varepsilon$. The payoff of I from such a menu, $\Pi_I\langle(r_L, f_L),(r_H, f_H)\rangle$, is given by

$$\lambda[r_L(a - c_L + \varepsilon - r_L)/2 + f_L] + (1 - \lambda)[r_H(a - c_H + \varepsilon - r_H)/2 + f_H]. \tag{19}$$

We find the optimal menu of contracts in the following way. First, for given values of r_L and r_H, we determine the optimal values of f_L and f_H as functions of r_L and r_H. Then, we determine the optimal values of r_L and r_H.

Observation 2.3. *Let $0<c_L<c_H<\varepsilon$ and $r_L\leq r_H\leq a-c_H+\varepsilon$. Then, the following holds:*

$$\Psi_L(r_L,r_H)\leq\Psi_L(r_L,r_H)\leq 0,$$

$$\Delta_L(r_L)+\Psi_H(r_L,r_H)\geq\Delta_H(r_H)\Leftrightarrow r_L\leq\varepsilon,$$

$$\Delta_L(r_L)+\Psi_L(r_L,r_H)\geq\Delta_H(r_H)\Leftrightarrow r_H\leq\varepsilon.$$

Case 1. $r_L\leq\varepsilon\leq r_H$. For this case, from Observation 2.3, it follows that for given values of r_L and r_H , the optimal values of f_L and f_H are $\Delta_L(r_L)$ and $\Delta_H(r_H)$, respectively. Replacing these values in Eq. (19), by standard optimization conditions, it follows that the optimal values of r_L and r_H satisfy $r_L\leq r_H=\varepsilon$, which are feasible for Case 3 [$r_L\leq r_H\leq\varepsilon$].

Case 2. $\varepsilon\leq r_L\leq r_H$. For this case, for given values of r_L and r_H, the optimal values of f_L and f_H are $\Delta_L(r_L)$ and $\Psi_H(r_L,r_H)+\Delta_L(r_L)$, respectively. Replacing these values in Eq. (19), by standard optimization conditions, it can be shown that the optimal values of r_L and r_H satisfy $r_L=r_H=\varepsilon$. As before, these values are feasible for Case 3. Thus, it will be sufficient to consider.

Case 3. $r_L\leq r_H\leq\varepsilon$. For given values of r_L and r_H, the optimal values of f_L and f_H are $-\Psi_L(r_L,r_H)+\Delta_H(r_H)$ and $\Delta_H(r_H)$, respectively. Replacing these values in Eq. (19), and maximizing the payoff with respect to r_L, for given value of r_H, it can be shown that the unconstrained maximum is attained at $r_L=0$. Since we are considering $r_L\leq r_H\leq\varepsilon$, the maximum is attained at $r_L=r_H$, when $r_H\leq 0$, and at $r_L=0$, when $r_H\geq 0$. We consider the following subcases.

Subcase 3.(a). $r_H\leq 0$. For this case, the maximum with respect to r_L (for given value of r_H) is attained at $r_L=r_H$. Replacing $r_L=_H$, noting that $\Psi_L(r_H,r_H)=0$, from Eq. (19), we conclude that the payoff of I is increasing in r_H for $r_H\leq\lambda(c_H-c_L)$. Since $\lambda(c_H-c_L)\geq 0\geq r_H$, the maximum is attained at $r_H=0$. Because this is feasible for the case $r_H\geq 0$, it is thus sufficient to consider the next case.

Subcase 3.(b). $0\leq r_H\leq\varepsilon$. For this case, the maximum with respect to r_L is attained at $r_L=0$. Taking $f_L=-\Psi_L(0,r_H)+\Delta_H(r_H), f_H=\Delta_H(r_H)$ and $r_L=0$ in Eq. (19), we note that the derivative of this function with respect to r_H is $[\lambda(c_H-c_L)-(1\text{-}\lambda)r_H]/2$. When $\lambda=1$, the payoffis strictly increasing in r_H. Since $r_H\leq\varepsilon$, the maximum is attained at $r_H=\varepsilon$, when $\lambda=1$. Noting that $\Delta_H(\varepsilon)=0$ and $-\Psi_L(0,\varepsilon)+\Delta_H(\varepsilon)=\Delta_L(0)$, the optimal menu of contracts is given by $\langle(0,\Delta_L(0)),(\varepsilon,0)\rangle$.[13] When $\lambda<1$, the unconstrained maximum is attained at $r_H=\lambda(c_H-c_L)/(1-\lambda)\equiv\bar{r}(\lambda)$. Since $r_H\leq\varepsilon$ for the case under consideration, to determine the optimal solution, we consider the following cases.

(i) $\lambda\geq\varepsilon/(c_H-c_L+\varepsilon)$. For this case, $\bar{r}(\lambda)\geq\varepsilon$, so that the maximum is attained at $r_H=\varepsilon$. Then the optimal menu of contracts is given by $\langle(0,\Delta_L(0)),(\varepsilon,0)\rangle$.

[13] When $\lambda=1$, we have the complete information case where M is of type L for sure. Note that the contract offered to type L is the fixed fee contract $\langle 0,\Delta_L(0)\rangle$, which is indeed the optimal contract under complete information.

(ii) $\lambda \leq \varepsilon/(c_H - c_L + \varepsilon)$. For this case, $\bar{r}(\lambda) \leq \varepsilon$, so that the maximum is attained at $r_H = \bar{r}(\lambda)$. Then, $\langle(0, -\Psi_L(0,\bar{r}(\lambda)) + \Delta_H(\bar{r}(\lambda))),(\bar{r}(\lambda),\Delta_H(\bar{r}(\lambda)))\rangle$ is the optimal menu.[14]

We have now determined the optimal menu of contracts in the set of menus that satisfy individual rationality and incentive compatibility constraints for both types.

Conclusion 2.1. *Consider the set of all menus of contracts that satisfy individual rationality and incentive compatibility constraints for both types.*
The following hold in this set:

(i) When $\lambda \geq \varepsilon/(c_H - c_L + \varepsilon)$, *the optimal menu of contracts is given by* $\langle(0,\Delta_L(0)),(\varepsilon,0)\rangle$.
(ii) When $\lambda \leq \varepsilon/(c_H - c_L + \varepsilon)$, *the optimal menu of contracts is given by* $\langle(0, -\Psi_L(0,\bar{r}(\lambda)) + \Delta_H(\bar{r}(\lambda))),(\bar{r}(\lambda),\Delta_H(\bar{r}(\lambda)))\rangle$, *where* $\bar{r}(\lambda)$ *is a function of* λ, *given by* $\lambda(c_H - c_L)/(1-\lambda)$.

To complete the Proof of Proposition 2, we have to show that the payoff of the innovator from Conclusion 2.1 is more than the payoff from the optimal nondiscriminatory contract. Note that a nondiscriminatory contract (r,f) is equivalent to the menu of contracts $\langle(r,f),(r,f)\rangle$. If for the contract (r,f), both types of M accept the offer, then, clearly, for the menu $\langle(r,f),(r,f)\rangle$, individual rationality constraints are satisfied for both types, while incentive compatibility constraints are satisfied trivially. From Proposition 1, we know that in the set of nondiscriminatory contracts, there are three potential candidates for the optimal contract: $(\varepsilon,0)$, $(\lambda(c_H - c_L),\Delta_H(\lambda(c_H - c_L)))$, and $(0,\Delta_L(0))$. Because the first two contracts are accepted by both types, the payoff of the innovator from Conclusion 2.1 is more than these contracts. Noting that the contract $(0,\Delta_L(0))$ is dominated by the menu $\langle(0,\Delta_L(0)),(\varepsilon,0)\rangle$, the proof is complete. □

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[14] Observe that $\bar{r}(0)=0$, so for the complete information case $\lambda=0$ (i.e., when M is of type H for sure), the contract offered to M is the fixed fee contract $(0,\Delta_H(0))$.

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