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General licensing schemes for a cost-reducing innovation

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Abstract

Optimal combinations of upfront fees and royalties are considered for a cost-reducing innovation in a Cournot oligopoly for both outside and incumbent innovators. It is shown that for any nondrastic innovation (a) the license is practically sold to all firms, ensuring full diffusion of the innovation, (b) consumers are better off, firms are worse off and the social welfare is improved, (c) the optimal licensing policy involves positive royalty for relatively significant innovations, (d) compared to an incumbent firm, an outsider invests more in R&D and has higher incentive to innovate and (e) as a function of the magnitude of the innovation, the industry size that provides the highest incentive to innovate is U-shaped.

JEL classification: D21; D43; D45; L13

Keywords: Cost-reducing innovation; Outside innovator; Incumbent innovator; AR policy

1. Introduction

A patent confers an innovator monopoly right over an innovation for a given period of time. It is intended to provide incentive to innovate as well as to disseminate the innovation. Licensing is a profitable way for the innovator to diffuse the innovation. This paper considers the licensing of a cost-reducing innovation by means of combinations of upfront fees and royalties.

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The formal analysis of patent licensing was initiated by Arrow (1962) who argued that a perfectly competitive industry provides a higher incentive to innovate than a monopoly. Licensing under oligopoly was first analyzed by Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986). The early literature has mainly considered innovators who are outsiders to the industry. Comparing licensing by means of royalty, upfront fee and auction, it has concluded that for an outside innovator, royalty licensing is inferior to both auction and upfront fee[1] (Kamien and Tauman, 1984, 1986; Kamien et al., 1992; see Kamien, 1992 for a survey). The literature has been since extended in several directions,[2] but it has been restrictive by not allowing the innovator to realize the full potential of the innovation: either the licensing policies are confined to pure upfront fee or pure royalty, or the number of firms is considered to be very large (perfect competition) or very small (monopoly or duopoly), thus limiting the extent of strategic interaction. The motivation of the present paper is twofold. First, we seek to fill the gap in the literature by studying licensing in an oligopoly of general size where the innovator uses combinations of upfront fees and royalties. This is a natural generalization over both upfront fee and royalty.[3] Second, we consider two possible cases, where the innovator is either an outsider (e.g., an independent research lab), or one of the incumbent producers in the industry. The existing literature has mainly focused on outside innovators, but in actual licensing practices, the innovator is often one of the incumbent firms.[4] While an outsider is interested only in the rents from licensing, an incumbent innovator has to also take into account the effect of licensing on its own profit. This difference in their objectives potentially leads to different optimal licensing policies and incentives to innovate. By analyzing both of these cases, we can have a more complete understanding of patent licensing and related issues.

[1] See, however, Sen (2005a).

[2] Some of the issues addressed by the later papers on licensing include the role of informational asymmetry (Gallini and Wright, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Macho-Stadler et al., 1996; Choi, 2001; Poddar and Sinha, 2002; Schmitz, 2002; Sen, 2005b), bargaining (Katz and Shapiro, 1987; Sempere-Monerris and Vannetelbosch, 2001), possibility of relicensing (Muto, 1987), variation in the quality of innovation (Rockett, 1990), product differentiation (Muto, 1993; Wang and Yang, 1999; Mukherjee and Balasubramanian, 2001; Caballero-Sanz et al., 2002; Faulí-Oller and Sandonís, 2002; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2005), risk aversion (Bousquet et al., 1998), incumbent innovators (Shapiro, 1985; Marjit, 1990; Wang 1998, 2002; Kamien and Tauman, 2002; Sen, 2002), leadership structure (Kabiraj, 2004, 2005; Filippini, 2005) and strategic delegation (Mukherjee, 2001; Saracho, 2002).

[3] Combinations of upfront fees and royalties are commonly observed in practice. For example, in their study of British industry, Taylor and Silberstone (1973, pp. 120–121) report: "Combinations of lump sum and running royalty are fairly common ... [O]f the 28 firms that responded ... four stated that a combination of lump-sums and running royalties accounted for the bulk of their licence income and nine said that 'royalties only' and 'combination' agreements each accounted for a substantial proportion ... of income. Generally speaking ... 'combination' income tends to be found in basic and bulk chemicals ... and also in the (non-electrical) plant and machinery field." In his survey of corporate licensing in the United States, Rostoker (1984, p. 64) report: "A down payment with running royalty was used 46% of the time, while straight royalties and paid-up licenses accounted for 39% and 13%, respectively." Combinations are also common in international licensing agreements. For instance, in their study of patent licensing contracts from US firms to Japanese firms (predominantly in electronics and machinery technologies) during 1988–1991, Yanagawa and Wada (2000) find that about 48% of the contracts (195 out of 407) involve combinations of fixed fees and royalties.

[4] For example, Taylor and Silberstone (1973, pp. 112–113) point out, "One fact which was brought out fairly clearly by the inquiry was that the great majority of licences granted or taken by industrial firms are with other industrial firms. Thus, of a total of some 600 ... licences involving only UK patents reported by 26 responding firms ... 94 per cent were with other firms ... Moreover, of licences between responding firms and other firms, over three-quarters were with firms whose main operations were in the same industrial field."

We carry out our analysis in a Cournot oligopoly where each firm operates under the same constant per unit cost prior to the innovation. The innovator has a nondrastic cost-reducing innovation[5] that it can license to some or all firms by means of combinations of upfront fees and royalties. The main questions that the paper addresses are:

(a) what is the optimal configuration of licensing policies?
(b) what is the nature of diffusion of the innovation?
(c) what are the welfare implications of licensing?
(d) who has higher incentive to innovate: an outsider or an incumbent firm, and
(e) which industry structure provides the highest incentive to innovate?

We show that for both outside and incumbent innovators, the following general conclusions are obtained when there are more than two firms other than the innovator:

(a) the licensing policy involves positive royalty for relatively significant innovations, while the royalty is zero for relatively insignificant innovations,
(b) the license is practically sold to all firms, and
(c) as the result of licensing, consumers are better off, firms are worse off and the social welfare is improved.

In particular, two important social objectives are achieved under the optimal combination of upfront fee and royalty: (i) the post-innovation price falls below its pre-innovation level and (ii) there is full diffusion of the innovation. It should be mentioned that licensing by means of upfront fee results in lower price, but relatively significant innovations are licensed only to a few firms; on the other hand, there is full diffusion of the innovation under royalty licensing, but the price stays the same at its pre-innovation level in the case of an outside innovator (Kamien and Tauman, 1984, 1986; Kamien et al., 1992).

Comparing the incentives of innovation, we show that when a potential innovator chooses the optimal level of R&D investment taking into account its future post-innovation payoff, an outsider always invests more than an incumbent firm and also has higher incentives to innovate. This is a result of several effects. First, an outsider has a lower opportunity cost since without the innovation it earns zero while an incumbent firm obtains its pre-innovation oligopoly profit. Moreover an outsider sets a lower rate of royalty that results in higher willingness to pay for a license. Although for an incumbent innovator, a high level of royalty raises its oligopoly profit and royalty payments, this is outweighed by the first two effects and consequently it ends up with a smaller incremental payoff from the innovation.

Finally, the paper sheds some light on one of the classic issues in economics: the relation between industry structure and incentives to innovate. Under royalty licensing, a perfectly competitive industry provides the highest incentive for innovation (Arrow, 1962). Under licensing by means of upfront fee, the conclusion is not as sharp, but for relatively insignificant innovations, the industry size that provides the highest incentive is decreasing in the magnitude of the innova-

[5] An innovation is *drastic* if it is significant enough to create a monopoly if only one firm has the new technology; otherwise, it is *nondrastic*. Obviously, an incumbent innovator of a drastic innovation earns the entire monopoly profit with the reduced cost. In an oligopoly, an outside innovator of a drastic innovation can also earn the monopoly profit by selling the license to only one firm through an upfront fee. So the analysis of licensing is nontrivial only for nondrastic innovations. See Section 2.4.

tion (Kamien and Tauman, 1986). We show that for the optimal combination of upfront fee and royalty, a monopoly or a duopoly never provides the highest incentive for either an outsider or an incumbent innovator. In both cases, as a function of the magnitude of the innovation, the industry size that provides the highest incentive is U-shaped. It is decreasing for relatively small innovations and increasing for larger innovations. Moreover a perfectly competitive industry provides the highest incentive if either (i) the innovation becomes very small or (ii) it approaches a drastic innovation. For relatively insignificant innovations, our conclusion is the same as in licensing by means of upfront fee, while for significant innovations, it is more in conformity with royalty licensing. One reason behind this is the nature of the optimal policy: it involves positive royalty for significant innovations and no royalty for insignificant innovations.

The paper is organized as follows. In Section 2 we present the model. In Section 3 we derive the optimal licensing schemes. In Section 4 we study the incentives to innovate. We conclude in Section 5. Some proofs are relegated to Appendix A.

2. The model

We first describe the model with an outside innovator. Consider a Cournot oligopoly with n firms producing the same product where $N = \{1, \dots, n\}$ is the set of firms. For $i \in N$, let q_i be the quantity produced by firm i and let $Q = \sum_{i \in N} q_i$. Let p denote the industry price. The inverse demand function of the industry is $p = \max\{0, a - Q\}$. With the old technology, all n firms produce with the identical constant marginal cost c, where $0 < c < a$. An innovator has been granted a patent for a new technology that reduces the marginal cost from c to $c - \varepsilon$, where $0 < \varepsilon \leqslant c$. The innovator decides to license the new technology to some or all firms of the industry. In the case of an incumbent innovator, there are $n + 1$ firms where $N_I = \{I\} \cup N$ is the set of firms and firm I is the innovator.

2.1. The licensing schemes

The set of licensing schemes available to the innovator is the set of all combinations of a non-negative upfront fee and a non-negative per-unit linear royalty.[6] For any royalty, the innovator determines the upfront fee to extract the maximum possible surplus from the licensees. The best way to do this is through *auction plus royalty* (AR) policy where the innovator first announces the level of royalty and then auctions off one or more licenses (possibly with a minimum bid) so that the upfront fee that a licensee pays is its winning bid.[7] The minimum bid is required when the innovation is licensed to all firms as without that, no firm will place a positive bid since each one is guaranteed to have a license. So a typical AR policy is given by (k, r) for $1 \leqslant k \leqslant n - 1$ and $(n, r, \underline{b})$ for $k = n$, where k is the number of licenses auctioned off, r is the per-unit uniform royalty and $\underline{b}$ is the minimum bid.

When the innovator licenses an innovation of magnitude ε with rate of royalty r, the effective unit cost of a licensee is $c - (\varepsilon - r)$. Thus if $r > \varepsilon$, a licensee is less efficient than a non-licensee. So a firm will accept a policy involving $r > \varepsilon$ only if it is compensated upfront by a negative bid. Since this is ruled out, it is sufficient to consider $r \in [0, \varepsilon]$. For any r, let us define the new

[6] See Liao and Sen (2005) for the implications of allowing negative fees and royalties.

[7] Compared to a flat upfront fee, an auction generates more competition that increases the willingness to pay for the license. Katz and Shapiro (1985) showed the superiority of auction over upfront fee without royalty and the same intuition can be carried over in the presence of royalty.

variable $\delta := \varepsilon - r$. Clearly, $\delta \in [0, \varepsilon]$. The variable δ is the *effective magnitude* of the innovation. From now onwards, the licensing policies will be expressed in terms of δ rather than r and will be denoted by (k, δ) and $(n, \delta, \underline{b})$.

2.2. The games G_0 and G_1

The licensing game with an outside innovator, denoted by G_0, has three stages. In the first stage, the innovator announces a policy (k, δ) or $(n, \delta, \underline{b})$. In the second stage, firms in N bid simultaneously for the license and the k highest bidders win the license (ties are resolved at random). The set of licensees becomes commonly known at the end of the second stage. In the third stage, all firms compete in quantities. If a firm wins the license with bid b and produces q units, it pays the innovator $b + rq = b + (\varepsilon - \delta)q$. The licensing game G_1 with an incumbent innovator is defined similarly. Note that for the game G_1, in the third stage, the innovator produces with marginal cost $c - \varepsilon$ and competes with all other n firms. For both games G_0 and G_1, we confine to subgame-perfect equilibrium outcomes with the proviso that whenever the innovator is indifferent between two licensing policies, it chooses the one where the number of licensees is higher.

2.3. The willingness to pay for a license

Let us denote by $\Phi_{\mathrm{L}}(k, \delta)$ and $\Phi_{\mathrm{N}}(k, \delta)$ the Cournot profit of a licensee and a non-licensee respectively when the number of licensees is k and the effective magnitude of the innovation is δ. For $1 \leqslant k \leqslant n - 1$, the willingness to pay for a license is

$$b(k, \delta) = \Phi_{\mathrm{L}}(k, \delta) - \Phi_{\mathrm{N}}(k, \delta). \tag{1}$$

This is because a firm knows that irrespective of whether it becomes a licensee or not, there will always be k licensees. When the policy (k, δ) is announced, in equilibrium, the highest bid is $b(k, \delta)$ and at least $k + 1$ firms place this bid. Out of the highest bidders, k firms are chosen at random to be the licensees.[8] For $k = n$, the willingness to pay for a license is

$$b(n, \delta) = \Phi_{\mathrm{L}}(n, \delta) - \Phi_{\mathrm{N}}(n - 1, \delta). \tag{2}$$

Here we subtract $\Phi_{\mathrm{N}}(n - 1, \delta)$ because for $k = n$, a firm can reduce the total number of licensees by choosing to not have a license. For the policy $(n, \delta, \underline{b})$, the innovator sets $\underline{b} = b(n, \delta)$, as this is the maximum upfront fee that a firm will pay when the license is sold to all firms.

Remark 1. It is sufficient to consider the policy (k, δ) for $1 \leqslant k \leqslant n - 1$ and the policy $(n, \delta, b(n, \delta))$ for $k = n$. From now onwards, we shall denote a policy by simply (k, δ) where it will be implicit that for $k = n$, there is a minimum bid $b(n, \delta)$.

[8] Clearly, in equilibrium, the highest bid cannot exceed $b(k, \delta)$. If at most k firms place the highest bid, each one has an incentive to slightly reduce its bid. If at least $k + 1$ firms place the highest bid and it falls below $b(k, \delta)$, each one has an incentive to slightly increase its bid to ensure having a license. Consequently, in equilibrium, at least $k + 1$ firms bid $b(k, \delta)$.

3. Optimal licensing schemes

3.1. Drastic innovations

We begin with the case of *drastic innovations* (Arrow, 1962). A cost-reducing innovation of magnitude ε is *drastic* if the monopoly price under the new technology $(a+c-\varepsilon)/2$ does not exceed the competitive price under the old technology c (i.e. $\varepsilon \geqslant a-c$); otherwise, it is *nondrastic*. If only one firm in an oligopoly has a drastic innovation, it becomes a monopolist with the reduced cost and all other firms drop out of the market. The optimal licensing policies are quite straightforward for a drastic innovation. For the sake of completeness, they are summarized in the following proposition.

Proposition 1. *Consider a drastic cost-reducing innovation. Both games G_0 and G_1 have unique subgame-perfect equilibrium outcomes where the following hold when there are n firms other than the innovator*:

(a) *For $n=1$, the outside innovator licenses the innovation to the monopolist through an upfront fee that equals the difference between post and pre innovation monopoly profits.*
(b) *For $n \geqslant 2$, the outside innovator licenses the innovation to only one firm through an upfront fee that equals the post-innovation monopoly profit.*
(c) *The incumbent innovator becomes a monopolist by using the innovation exclusively.*
(d) *Even though the market is monopolized, the consumers are better off and the social welfare is higher as the result of the innovation. The firms are worse off except in the case of a monopoly with an outside innovator where there is no change in the payoff of the monopolist.*

3.2. Nondrastic innovations

Now consider a nondrastic innovation of magnitude ε, i.e., $\varepsilon < a-c$. Let us define the indicator variable λ that takes the value 0 for an outside innovator and 1 for an incumbent innovator. Consider the policy (k,δ). Let $p(k,\delta)$ and $Q(k,\delta)$ denote the Cournot price and industry output.[9] Denote by $q_{\mathrm{N}}(k,\delta)$, $q_{\mathrm{L}}(k,\delta)$ and $q_{\mathrm{I}}(k,\delta)$ the respective Cournot outputs of a non-licensee, licensee and the incumbent innovator. For the policy (k,δ), the royalty rate is $r=\varepsilon-\delta$ and the innovator gets $k(\varepsilon-\delta)q_{\mathrm{L}}(k,\delta)$ as royalty payments. In addition, from (1) and (2), the total fee that the innovator obtains upfront is (i) $k[\Phi_{\mathrm{L}}(k,\delta)-\Phi_{\mathrm{N}}(k,\delta)]$ for $1 \leqslant k \leqslant n-1$ and (ii) $n[\Phi_{\mathrm{L}}(n,\delta)-\Phi_{\mathrm{N}}(n-1,\delta)]$ for $k=n$. An incumbent innovator also earns its Cournot profit $\Phi_{\mathrm{I}}(k,\delta)$. Hence the payoff of the innovator is

$$\begin{aligned}\Pi(k,\delta) &= \lambda\Phi_{\mathrm{I}}(k,\delta)+k(\varepsilon-\delta)q_{\mathrm{L}}(k,\delta)+k\big[\Phi_{\mathrm{L}}(k,\delta)-\Phi_{\mathrm{N}}(k,\delta)\big]\\ &= \big[p(k,\delta)-c+\varepsilon\big]Q(k,\delta)-n\Phi_{\mathrm{N}}(k,\delta)-\varepsilon(n-k)q_{\mathrm{N}}(k,\delta) \quad \text{for } 1 \leqslant k \leqslant n-1,\end{aligned} \tag{3}$$

$$\begin{aligned}\Pi(n,\delta) &= \lambda\Phi_{\mathrm{I}}(n,\delta)+n(\varepsilon-\delta)q_{\mathrm{L}}(n,\delta)+n\big[\Phi_{\mathrm{L}}(n,\delta)-\Phi_{\mathrm{N}}(n-1,\delta)\big]\\ &= \big[p(n,\delta)-c+\varepsilon\big]Q(n,\delta)-n\Phi_{\mathrm{N}}(n-1,\delta).\end{aligned} \tag{4}$$

[9] Observe that these expressions depend on λ. So more precise notations would be $p^{\lambda}(k,\delta)$ and $Q^{\lambda}(k,\delta)$. As the value of λ will be clear from the context, we suppress the superscript λ.

The payoff function of the innovator has a simple economic interpretation. The policy (k,δ) results in Cournot price $p(k,\delta)$. At this price, the maximum that the innovator can earn is $[p(k,\delta)-c+\varepsilon]Q(k,\delta)$ which is the total industry profit when all firms have the innovation. However, the innovator cannot extract this entire surplus as it has to leave with every firm the firm's opportunity cost which is the profit of a non-licensee. So we subtract $n\Phi_{\mathrm{N}}(k,\delta)$ for $1\leqslant k\leqslant n-1$ and $n\Phi_{\mathrm{N}}(n-1,\delta)$ for $k=n$. For $k\leqslant n-1$, the innovator also incurs the *loss due to exclusion*, which is the additional cost that each non-licensee has to pay for not having the innovation, so the total loss is $\varepsilon(n-k)q_{\mathrm{N}}(k,\delta)$. To determine the optimal licensing policies, it will be useful to extend the notion of drastic innovation as follows.

***k*-drastic innovation.** For $k\geqslant 1$, a cost-reducing innovation is *k-drastic* if k is the minimum number such that if k firms have the innovation, all other firms drop out of the market and a k-firm natural oligopoly is created.

Observe that a drastic innovation is 1-drastic and any nondrastic innovation is k-drastic for some integer $k\geqslant 2$. Since $\delta=\varepsilon-r\leqslant\varepsilon$, for a nondrastic innovation ε, the effective magnitude of the innovation δ is also nondrastic for any policy (k,δ). The following lemma characterizes the resulting industry structure for any policy. The proof follows from the equilibrium conditions of the Cournot oligopoly under consideration.

Lemma 1. *For $\lambda\in\{0,1\}$, let $\delta_\lambda(0):=a-c$ and $\delta_\lambda(k):=(a-c-\lambda\varepsilon)/k$ for $k\geqslant 1$. Consider the policy (k,δ).*

(a) *For $k\geqslant 2-\lambda$, an innovation of effective magnitude δ is $(k+\lambda)$-drastic if $\delta_\lambda(k)\leqslant\delta<\delta_\lambda(k-1)$.*
(b) *$p(k,\delta)\geqslant c$ iff $\delta\leqslant\delta_\lambda(k)$ with equality iff $\delta=\delta_\lambda(k)$. For $\delta\leqslant\delta_\lambda(k)$, $p(k,\delta)$ and $q_{\mathrm{N}}(k,\delta)$ are functions of $k\delta$ only.*
(c) *If $\delta<\delta_\lambda(k)$, all firms produce positive output. If $\delta\geqslant\delta_\lambda(k)$, all non-licensee firms drop out of the market and a $(k+\lambda)$-firm natural oligopoly is created.*

Proof. See Appendix A. □

3.2.1. Industries with more than two firms other than the innovator

Let us first analyze the subgame–perfect equilibrium outcomes of the games G_0 and G_1 when there are at least three firms other than the innovator ($n\geqslant 3$). In Proposition 2 below, we obtain certain general conclusions for both G_0 and G_1.

Proposition 2. *Consider a nondrastic cost-reducing innovation of magnitude ε. Both games G_0 and G_1 have unique subgame–perfect equilibrium outcomes. The following hold for both games when there are at least three firms other than the innovator ($n\geqslant 3$).*

(a) *The innovator sells the license to at least $n-1$ firms.*
(b) *Consumers are better off, firms are worse off and the social welfare is higher as the result of the innovation.*
(c) *The post-innovation Cournot price falls below its pre-innovation level, but it does not fall below the pre-innovation marginal cost c.*

(d) *The innovator charges positive royalty for relatively significant innovations and zero royalty for relatively insignificant innovations. A pure royalty policy* (*i.e., $r > 0$ and zero upfront fee*) *is never offered.*
(e) *The innovator obtains at least $\varepsilon(a-c)$, which is the total industry profit when the price equals the pre-innovation marginal cost c and all firms operate with the new technology.*

Proof. See the discussion below for the proof of (a), (b), (c), (e), and the first and last statements of (d). See Appendix A for the proof of the second statement of (d). □

Let us discuss the results of Proposition 2. We begin with (e). It is easy to verify that an incumbent innovator obtains more than $\varepsilon(a-c)$ by licensing the innovation to all other firms using the rate of royalty $r = \varepsilon$ and an outside innovator obtains at least $\varepsilon(a-c)$ through the policy $(n-1, \delta^*)$, where $\delta^* = \min\{\varepsilon, (a-c)/(n-1)\}$.

Next consider (c). The post-innovation price is lower than its pre-innovation level as no firm is less efficient than before and some firms are more efficient than before. To see why the price does not fall below c, denote by $F(p)$ the industry profit at price p when all firms use the new technology, i.e.,

$$F(p) := (p - c + \varepsilon)Q = (p - c + \varepsilon)(a - p). \tag{5}$$

For any price p, the maximum that the innovator can obtain is $F(p)$. Note that $F(p)$ is increasing for $p < p_M \equiv (a + c - \varepsilon)/2$, which is the post-innovation monopoly price. Since the innovation is nondrastic, $p_M > c$. So $F(p)$ is increasing for $p \leqslant c$ and for $p < c$, $F(p) < F(c) = \varepsilon(a-c)$, which is the lower bound of the payoff of the innovator. This proves that the post-innovation Cournot price must be at least c.

Now consider (a). From (c), we conclude that the relevant policies for the innovator are the ones that result in a Cournot price of at least c. So by Lemma 1, it is sufficient to consider policies (k, δ) where $\delta \leqslant \delta_\lambda(k)$, i.e., δ is $(\ell + \lambda)$-drastic for $\ell \geqslant k$. For $1 \leqslant k \leqslant n-1$, from (3), the payoff of the innovator from the policy (k, δ) is

$$\Pi(k, \delta) = \left[p(k, \delta) - c + \varepsilon\right]Q(k, \delta) - n\Phi_N(k, \delta) - (n-k)\varepsilon q_N(k, \delta). \tag{6}$$

By Lemma 1, $p(k, \delta)$ and $q_N(k, \delta)$ depend only on $k\delta$. Consider the policy $(n-1, \tilde{\delta})$ such that $(n-1)\tilde{\delta} = k\delta$. Then $p(n-1, \tilde{\delta}) = p(k, \delta) \geqslant c$ and $q_N(n-1, \tilde{\delta}) = q_N(k, \delta)$. By (6), it follows that the first two terms remain the same for both policies (k, δ) and $(n-1, \tilde{\delta})$ while the last term is clearly higher for the latter policy. So the innovator is best off selling the license to at least $n-1$ firms, as claimed in (a). The economic interpretation is that for two policies that result in the same industry profit and opportunity costs for firms, the innovator chooses the one where the loss due to exclusion is minimum (i.e. the diffusion of the innovation is maximum).

Next consider (b). From (c), it follows that the consumers are better off. The firms are worse off since the net payoff of any firm is the Cournot profit of a non-licensee, which is lower than its pre-innovation level. For computing the social welfare, we can ignore the upfront fees since they are lump-sum transfers from one agent to another. Social welfare is improved as the result of the fact that the innovator has non-negative royalty payments, consumers are better off and all firms (except the sole non-licensee, if any) have a higher Cournot profit.[10] For the precise proof, denote

[10] Of course each firm gets a lower net payoff since the innovator extracts the entire surplus through an upfront fee, but upfront fees do not effect the social welfare.

by Q_1 and Q_2 the pre and post-innovation industry outputs. The pre-innovation social welfare is $W_1 = \int_0^{Q_1}[p(q) - c]\,dq$. The post-innovation consumer surplus is $[\int_0^{Q_2} p(q)\,dq] - p(Q_2)Q_2$ and the sum of payoffs of the innovator and all firms is more than $[p(Q_2) - c]Q_2$ so that the post-innovation welfare satisfies $W_2 > \int_0^{Q_2}[p(q) - c]\,dq$. Since $Q_2 > Q_1$ and $p(Q_1) > p(Q_2) \geqslant c$, it follows that $W_2 > W_1$.

Finally, consider (d). To see that for significant innovations the licensing policy involves positive royalty, consider an innovation that is significant enough so that if all firms except one has it, the price falls below c (i.e., $\varepsilon > \delta_\lambda(n-1)$). When such an innovation is sold to $n-1$ or n firms using no royalty, the innovator earns the entire industry profit $F(p)$. But $F(p) < F(c) = \varepsilon(a-c)$ since $p < c$. So by (e), setting a zero royalty cannot be optimal for this case. In fact, using the policy $(n-1, \delta_\lambda(n-1))$ (i.e., $k = n-1$, $r = \varepsilon - \delta_\lambda(n-1) > 0$) the innovator can bring down the effective magnitude of the innovation to raise the price to c, which yields the payoff $F(c) = \varepsilon(a-c)$.

Regarding the pure royalty policy, first observe that when $r < \varepsilon$, the effective magnitude of the innovation is positive ($\delta = \varepsilon - r > 0$). Then a licensee is more efficient than a non-licensee and earns a higher Cournot profit. The innovator can extract this difference in profits through an upfront fee, so setting a zero upfront fee is not optimal for $r < \varepsilon$. Next consider $r = \varepsilon$. A pure royalty policy with $r = \varepsilon$ ($\delta = 0$) is not optimal for an outside innovator since the innovator obtains $n\varepsilon(a-c)/(n+1) < \varepsilon(a-c)$ under this policy (see Kamien and Tauman, 1984). To see why it is not optimal also for an incumbent innovator, note that when $\delta = 0$, the innovator is best off choosing $k = n$. From (4) and (5), the payoff of the innovator for the policy (n, δ) is

$$\Pi(n, \delta) = F\big(p(n, \delta)\big) - n\Phi_{\mathrm{N}}(n-1, \delta).$$

Observe that the first term (maximum industry profit) and the second term (sum of opportunity costs) are both increasing in price. Since price is decreasing in δ, it follows that both are decreasing in δ. Thus on the one hand, a low value of δ leads to a higher industry profit, but on the other, it also results in a higher profit for a non-licensee and consequently a higher opportunity cost for a firm. This trade-off is settled by the respective elasticities of these revenue functions and the fact that for linear demand, elasticity is increasing in price.[11] For small values of δ, elasticity of the second term is higher, so that the marginal loss in revenue from lower industry profit is outweighed by the marginal gain in revenue due to lower profit of a non-licensee. Indeed, for $n \geqslant 3$, $\Pi(n, \delta)$ is increasing in δ at $\delta = 0$ (see Lemma A.2 of Appendix A). This proves that a pure royalty cannot be optimal.

Since the payoff of the innovator is continuously differentiable in δ and it is increasing at $\delta = 0$, it is also increasing for small positive values of δ. This implies that when ε is sufficiently small, the payoff increases in δ throughout the interval $[0, \varepsilon]$ and consequently it is maximized at $\delta = \varepsilon$ (i.e., $r = 0$). So for sufficiently insignificant innovations, the innovator sets no royalty.

The next proposition provides properties that are specific to G_0 and G_1.

Proposition 3. *Consider a nondrastic cost-reducing innovation of magnitude ε. The following hold for the respective subgame–perfect equilibrium outcomes of G_0 and G_1 when there are at least three firms other than the innovator ($n \geqslant 3$).*

[11] This intuition will continue to hold for general demand functions considered in Kamien et al. (1992) where elasticity is nondecreasing in price.

(a) *For G_0, there is a function $1 < u(n) \leqslant 2$ such that if $\varepsilon > (a-c)/u(n)$, the innovator sells the license to $n-1$ firms creating an $(n-1)$-firm natural oligopoly where the Cournot price equals c. If $\varepsilon \leqslant (a-c)/u(n)$, the license is sold to at least $n-1$ firms and the price is higher than c. The innovator obtains $\varepsilon(a-c)$ if $\varepsilon \geqslant (a-c)/u(n)$ and more than $\varepsilon(a-c)$ if $\varepsilon < (a-c)/u(n)$.*
(b) *For G_1, there is a function $h(n) > 2$ such that if $\varepsilon \geqslant (a-c)/h(n)$, the innovator sells the license to all firms. If $\varepsilon < (a-c)/h(n)$, the license is sold to at least $n-1$ firms. In both cases, the Cournot price is higher than c and the innovator obtains more than $\varepsilon(a-c)$.*
(c) *For $\lambda \in \{0,1\}$, there is an increasing function $f_\lambda(n) > 1$ such that for G_λ, the innovator sets positive royalty if $\varepsilon > (a-c)/f_\lambda(n)$. Moreover for $n \geqslant 4+\lambda$, the innovator sets zero royalty if $\varepsilon \leqslant (a-c)/f_\lambda(n)$.*
(d) *Let $m \geqslant 2$ and suppose both G_0 and G_1 have m firms (in G_1, one of these firms is the innovator). Then the rate of royalty and the post-innovation Cournot price are higher in G_1 compared to G_0.*

Proof. See Appendix A. □

See Tables A.5 and A.6 at the end of the paper for the complete characterization of equilibrium policies and payoffs. We have already shown in Proposition 2 that the innovator charges positive royalty for relatively significant innovations and zero royalty for relatively insignificant innovations. Proposition 3(c) shows that when the industry size is not too small, there is in fact a threshold level such that a positive royalty is used if and only if the magnitude of the innovation is above that level. Moreover this threshold level decreases in industry size, so the same result can be stated alternatively: for any innovation, the rate of royalty is positive for sufficiently large sizes of industry.

Part (d) of Proposition 3 is quite intuitive. Since a royalty increases the effective cost of a licensee, an incumbent innovator has an additional incentive to set higher royalty to have competitive edge over its rivals. The higher Cournot price is an immediate effect of higher royalty.

Observe from (a) and (b) that there is a discontinuity in the optimal number of licenses. For a drastic innovation the market is monopolized (Proposition 1). However, any nondrastic innovation of magnitude ε is licensed to at least $n-1$ firms even if ε is sufficiently close to being a drastic innovation. This is the result of the relative effects of two policies: royalty and upfront fee. Under royalty licensing, the innovator sells the license to all firms using rate of royalty $r = \varepsilon$, so all firms effectively operate under the pre-innovation cost. On the other hand, if the innovator is confined to pure upfront fees, significant innovations are licensed only to a few firms, which creates a relatively small natural oligopoly. For example, under licensing by means of upfront fee the innovation is licensed to only two firms when $(a-c)/2 \leqslant \varepsilon < a-c$, resulting in a natural $(2+\lambda)$-firm oligopoly and the innovator extracts the entire oligopoly profit. When the innovator uses combinations of upfront fees and royalties, the relative effects of these two policies are reflected in the optimal combination. The innovator sells the license to at least $n-1$ firms using a positive royalty but in addition also collects a positive upfront fee by setting $r < \varepsilon$.

3.2.2. Industries with at most two firms other than the innovator

To complete the analysis, we consider smaller industries where there are at most two firms other than the innovator ($n = 1, 2$).

Proposition 4. *Consider a nondrastic cost-reducing innovation of magnitude ε. The following hold in the respective subgame–perfect equilibrium outcomes of G_0 and G_1 when there are at most two firms other than the innovator ($n = 1, 2$).*

(a) *For $n = 1$, the outside innovator sells the license to the monopolist by using only an upfront fee that equals the difference between post and pre-innovation monopoly profits. The innovator obtains less than $\varepsilon(a - c)$ and the post-innovation monopoly price is higher than c.*
(b) *For $n = 2$, there is a constant $1 < t < 3/2$ such that the outside innovator sells the license to only one firm if $\varepsilon > (a - c)/t$ and to both firms otherwise. The rate of royalty is positive if and only if $(a - c)/3 < \varepsilon \leqslant (a - c)/t$. The innovator obtains less than $\varepsilon(a - c)$. The post-innovation Cournot price falls below c if and only if $2(a - c)/3 < \varepsilon \leqslant (a - c)/t$.*
(c) *For $n = 1, 2$, the incumbent innovator sells the license to all n other firms using a pure royalty policy with $r = \varepsilon$. The innovator obtains more than $\varepsilon(a - c)$ and the post-innovation Cournot price is higher than c.*
(d) *As the result of the innovation, the consumers are better off and the social welfare is higher. The firms are worse off except in the case of a monopoly with an outside innovator, where there is no change in the payoff of the monopolist.*

Proof. See Appendix A. □

Consider the Cournot profit of a firm that has unit cost c and that competes in a duopoly with a rival whose cost is $c - \varepsilon$ and denote this profit by $\widetilde{\Pi}$. When ε is nondrastic, $\widetilde{\Pi} > 0$. In case of an incumbent innovator in a duopoly ($n = 1$), regardless of the rate of royalty r, the payoff of the other firm when it does not have the innovation is $\widetilde{\Pi}$. So the best policy for the innovator would be to pay this firm $\widetilde{\Pi}$ upfront and set a sufficiently high royalty that will induce it to stay out of the market. However, given that negative upfront fees are not allowed, the upfront fee will be increased to zero and the resulting policy will be a pure royalty policy. This argument was put forward by Shapiro (1985) and later by Wang (1998) to show the superiority of royalty over upfront fee for an incumbent innovator in a duopoly. Our analysis shows that while pure royalty is still optimal when there are two firms other than an incumbent innovator, it is no longer optimal for larger industries. The optimality of a pure royalty in a duopoly relies on the fact that the opportunity cost of a firm when it uses the old technology is the constant $\widetilde{\Pi}$ that does not depend on the rate of royalty r. This argument breaks down in a general oligopoly, since the payoff of any non-licensee depends on r as well as the number of licensees.

3.3. Extension to general demand

Most of our conclusions in Proposition 2 continue to hold for both outside and incumbent innovators for the class of general demand functions considered in Kamien et al. (1992) where the elasticity of demand is nondecreasing in price. In particular, for general demand, the license is sold to all firms, except perhaps one. Moreover for significant innovations, the post-innovation price does not fall below the pre-innovation unit cost c, the rate of royalty is positive and a lower bound similar to $\varepsilon(a - c)$ is obtained.

4. Incentives to innovate

In this section, we address two issues regarding the incentives to innovate. First, we compare the incentives of outsider and incumbent innovators and second, we determine the industry structure that provides an innovator with the highest incentives to innovate. This analysis is carried out under the implicit assumption that without the innovation an outside innovator earns zero while an incumbent innovator earns its pre-innovation Cournot profit. Thus the incremental payoff of an outside innovator due to the innovation is simply its post-innovation rents, while for an incumbent innovator it is the difference between the post-innovation payoff (profit plus rents from licensing) and pre-innovation oligopoly profit.

4.1. Comparison of incentives

First consider a monopoly. For any innovation, the payoff of an outside innovator in a monopoly is the difference between post and pre-innovation monopoly profits, which is precisely the incremental payoff of an incumbent innovator who is a monopolist (Proposition 1). So under a monopoly, both innovators have the same incentive for any innovation. Next consider any industry of size at least two. When the innovation is drastic, the payoff of an outside innovator is the post-innovation monopoly profit, while the incremental payoff of an incumbent innovator is the difference between the post-innovation monopoly profit and the pre-innovation Cournot profit. So for a drastic innovation, an outside innovator has higher incentives. Lemma 2 below shows that an outsider has higher incentives also for any nondrastic innovation. The main reason behind this is that an outside innovator has a lower opportunity cost since without the innovation it earns zero while an incumbent firm obtains its pre-innovation oligopoly profit.

Lemma 2. *Suppose the industry size is at least two. For any $0 < \varepsilon \leqslant c$, denote by $\Pi_0(\varepsilon)$ and $\Pi_1(\varepsilon)$ the respective post-innovation payoffs of outside and incumbent innovators. Denote by $\overline{\Pi}$ the pre-innovation profit of the incumbent innovator and let $\Delta(\varepsilon) = \Pi_1(\varepsilon) - \overline{\Pi}$. Then $\Pi_0(\varepsilon) > \Delta(\varepsilon)$, i.e., the outside innovator has higher incremental payoff from any innovation.*

Proof. See Appendix A. □

4.1.1. R&D investment: the games $\widehat{G}_0$ and $\widehat{G}_1$

For $\lambda \in \{0, 1\}$, consider the game $\widehat{G}_\lambda$ that is obtained from G_λ by adding an R&D stage (stage 0) in the beginning where the innovator chooses an R&D investment $x \geqslant 0$.[12] This stage ends with either (i) a cost-reducing innovation of fixed magnitude $\varepsilon > 0$ (success) or (ii) no innovation (failure). It is assumed that for level of investment x, the outcome is success with probability $p(x)$ which is twice continuously differentiable, strictly increasing and strictly concave with $p(0) = 0$ and $\lim_{x\to\infty} p(x) = 1$. To guarantee a solution to the first-order conditions, we also assume that for every $y > 0$ there exists $z(y) > 0$ such that $p'(z(y)) = y$.

If the outcome of the R&D stage is a success, the game $\widehat{G}_\lambda$ proceeds as G_λ from stage 1 onwards and the innovator obtains $\Pi_\lambda(\varepsilon) - x$, the equilibrium post-innovation payoff net of investment. If the outcome is a failure, firms compete with the pre-innovation technology. Then

[12] Recall that for $\lambda = 0$ stands for an outsider and $\lambda = 1$ for an incumbent innovator. Here we continue to use the notations introduced in Lemma 2.

the payoff of an outsider is $-x$ and for an incumbent innovator, it is $\overline{\Pi} - x$. Conditional on x, the expected equilibrium payoff of an outsider is $\Psi_0(x) = [p(x)\Pi_0(\varepsilon) - x]$, while for an incumbent innovator it is $\Psi_1(x) = [p(x)\Pi_1(\varepsilon) + (1 - p(x))\overline{\Pi} - x] = [p(x)\Delta(\varepsilon) + \overline{\Pi} - x]$. Denote by x_λ^* the (unique) maximizer of $\Psi_\lambda(x)$. Then from the first order conditions it follows that

$$p'(x_0^*)\Pi_0(\varepsilon) = 1 \quad \text{and} \quad p'(x_1^*)\Delta(\varepsilon) = 1. \tag{7}$$

Then two equalities of (7), together with the strict concavity of $p(x)$ and the inequality $\Pi_0(\varepsilon) > \Delta(\varepsilon)$ (Lemma 2), imply that $x_0^* > x_1^*$. Since x_0^* is the unique maximizer of $\Psi_0(x)$,

$$\Psi_0(x_0^*) > \Psi_0(x_1^*) = p(x_1^*)\Pi_0(\varepsilon) - x_1^* > p(x_1^*)\Delta(\varepsilon) - x_1^* = \Psi_1(x_1^*) - \overline{\Pi}. \tag{8}$$

The incremental payoff due to the innovation is $\Psi_0(x_0^*)$ for an outsider and $\Psi_1(x_1^*) - \overline{\Pi}$ for an incumbent innovator. Hence (8) implies that the incentive to innovate is higher for an outsider.

Proposition 5. *Suppose the industry size is at least two. For any $0 < \varepsilon \leqslant c$, an outside innovator invests more in R&D than an incumbent firm. Moreover an outsider realizes higher incremental payoff and consequently it has higher incentives to innovate.*

4.2. Industry size and incentives

For clarity of presentation, the analysis of the relation between industry size and incentives to innovate is carried out in two steps. First we determine this relation without taking the cost of R&D into account. Then it is shown that the same conclusions are valid when the innovator optimally invests in R&D (as in Section 4.1.1).[13]

4.2.1. The case of no R&D investment

For any innovation, the industry that provides the innovator with the highest incentives to innovate is the one where the incremental payoff of the innovator is maximum. Let us begin with the case of a drastic innovation. For such an innovation, an outside innovator earns the post-innovation monopoly profit for any industry of size at least two, while for a monopoly, it cannot extract this entire profit as it has to leave with the monopolist its pre-innovation profit (Proposition 1). So any industry of size at least two provides the highest incentive for an outside innovator. An incumbent innovator of a drastic innovation becomes a monopolist with the new technology regardless of the industry size (Proposition 1) but its pre-innovation profit is minimum under perfect competition. So the perfectly competitive industry provides the highest incentive for an incumbent innovator.

Now consider a nondrastic innovation. In what follows, we first show that for both outside and incumbent innovators, a perfectly competitive industry provides higher incentive than a monopoly or a duopoly. It can be easily verified that as $n \to \infty$ (i.e., the industry structure approaches perfect competition), the post-innovation payoffs of both outside and incumbent innovators converge to $\varepsilon(a - c)$. Since for a perfectly competitive industry the pre-innovation profit of an incumbent innovator is zero, for both innovators, the incremental payoff under perfect competition is simply the post-innovation payoff $\varepsilon(a - c)$. An outside innovator obtains less than $\varepsilon(a - c)$

[13] Dasgupta and Stiglitz (1980) provide a microeconomic analysis of the relation between industry structure and incentive to innovate. However, in their model, an innovation is used exclusively by the owner and the possibility of licensing is not considered.

under a monopoly or a duopoly (Proposition 4). When the incumbent innovator is a monopolist ($n = 0$), the incremental payoff due to the innovation is $\Delta^0(\varepsilon) = (a - c + \varepsilon)^2/4 - (a - c)^2/4 < \varepsilon(a - c)$ for $\varepsilon < a - c$. When there is only one firm in the industry other than the incumbent innovator ($n = 1$), it is $\Delta^1(\varepsilon) = [(a - c - \varepsilon)^2/9 + \varepsilon(a - c)] - (a - c)^2/9 < \varepsilon(a - c)$. This proves that for both outside and incumbent innovators, a perfectly competitive industry provides higher incentive than a monopoly or a duopoly. To determine the industry size that provides the highest incentive, it is therefore sufficient to consider industries of size at least three. Proposition 6 below shows that in general, the conclusions are qualitatively the same for both types of innovators. As a function of the magnitude of the innovation, the industry size that provides the highest incentive is a U-shaped step function. It decreases for innovations that lie below a certain threshold level, reaches its minimum at the threshold and then increases.

Definition. Consider a nondrastic innovation of magnitude ε. For $\lambda \in \{0, 1\}$, define by $n_\lambda(\varepsilon)$ the industry size that provides the highest incentive to innovate the innovation of magnitude ε.

Proposition 6.

(a) *For any innovation, the perfectly competitive industry provides higher incentive to innovate than the monopoly.*
(b) *For a drastic innovation, any industry of size at least two provides the highest incentive for an outside innovator, while for an incumbent innovator the perfectly competitive industry provides the highest incentive.*
(c) *For any nondrastic innovation of magnitude ε, $n_\lambda(\varepsilon) \geqslant 3$ and it is a U-shaped step function. Specifically, there is a constant $0 < k_\lambda < (a - c)/2$ such that $n_\lambda(\varepsilon)$ is decreasing for $0 < \varepsilon < k_\lambda$ and increasing for $k_\lambda \leqslant \varepsilon < a - c$. Further, $\lim_{\varepsilon \downarrow 0} n_\lambda(\varepsilon) = \lim_{\varepsilon \uparrow a-c} n_\lambda(\varepsilon) = \infty$.*

Proof. See the discussion immediately before Proposition 6 for the proof of (a) and (b). See Appendix A for the proof of (c). □

Proposition 6 shows that the industry that provides the highest incentive to innovate approaches perfect competition if either (i) the innovation becomes very small or (ii) it is close to a drastic innovation. Let us sketch the proof of the proposition for $\varepsilon \geqslant (a - c)/2$ in case of an outside innovator. The proof for the other cases relies on similar argument. Recall from Proposition 3 that for any $n \geqslant 3$, there is a function $1 < u(n) \leqslant 2$ such that the innovator obtains exactly $\varepsilon(a - c)$ when $\varepsilon \geqslant (a - c)/u(n)$ and more than $\varepsilon(a - c)$ when $\varepsilon < (a - c)/u(n)$. Moreover, $u(n)$ is decreasing and $\lim_{n\to\infty} u(n) = 1$. So for any $\varepsilon \geqslant (a - c)/2$, $\exists m(\varepsilon) \geqslant 3$ such that $\varepsilon \geqslant (a - c)/u(n)$ for $n < m(\varepsilon)$ and $\varepsilon < (a - c)/u(n)$ for $n \geqslant m(\varepsilon)$. Hence, the payoff of the innovator is exactly $\varepsilon(a - c)$ for industries of size $n < m(\varepsilon)$ and more than $\varepsilon(a - c)$ for $n \geqslant m(\varepsilon)$. Then clearly $n_0(\varepsilon) \geqslant m(\varepsilon)$. It can be shown that $n_0(\varepsilon) < \infty$. From the properties of $u(n)$, it also follows that $m(\varepsilon)$ is increasing and $m(\varepsilon) \to \infty$ as $\varepsilon \uparrow a - c$. Hence, for $(a - c)/2 \leqslant \varepsilon_1 < \varepsilon_2$, either (i) $m(\varepsilon_2) \leqslant n_0(\varepsilon_1)$, in which case $n_0(\varepsilon_2) = n_0(\varepsilon_1)$, or (ii) $m(\varepsilon_2) > n_0(\varepsilon_1)$ so that $n_0(\varepsilon_2) \geqslant m(\varepsilon_2) > n_0(\varepsilon_1)$. Consequently, $n_0(\varepsilon_2) \geqslant n_0(\varepsilon_1)$ and $\lim_{\varepsilon\uparrow a-c} n_0(\varepsilon) = \infty$.

4.2.2. Incentives with R&D investment

Now we show that the conclusions of Proposition 6 do not change when we take into consideration the R&D investment. The pre-innovation R&D environment is assumed to be the same as in Section 4.1.1. First consider an outside innovator and denote by $\Pi_0^n(\varepsilon)$ its post-innovation payoff in the industry of size n. Then for the industry of size n, as before, the innovator chooses x to maximize its expected incremental payoff $\Psi_0^n(x) = p(x)\Pi_0^n(\varepsilon) - x$. The optimal investment for industry size n, $x^*(n)$, satisfies the first order condition

$$p'\big(x^*(n)\big)\Pi_0^n(\varepsilon) = 1 \quad \text{for all } n \quad \text{and, in particular,} \quad p'\big(x^*\big(n_0(\varepsilon)\big)\big)\Pi_0^{n_0(\varepsilon)}(\varepsilon) = 1. \tag{9}$$

Clearly, by the definition of $n_0(\varepsilon)$, $\Pi_0^{n_0(\varepsilon)}(\varepsilon) \geqslant \Pi_0^n(\varepsilon)$ for all n. Then by (9) and the strict concavity of $p(x)$, it follows that $x^*(n_0(\varepsilon)) \geqslant x^*(n)$ for all n. So the industry size $n_0(\varepsilon)$ induces the highest amount of investment. Now consider the expected incremental payoff of the innovator for an industry of size n when the amount of investment is chosen optimally (i.e., $x = x^*(n)$). This is

$$\Psi_0^n\big(x^*(n)\big) = p\big(x^*(n)\big)\Pi_0^n(\varepsilon) - x^*(n). \tag{10}$$

From (9) and (10), we have

$$\Psi_0^n\big(x^*(n)\big) = \frac{p(x^*(n))}{p'(x^*(n))} - x^*(n). \tag{11}$$

Denote $f(x) := p(x)/p'(x) - x$. Then $\Psi_0^n(x^*(n)) = f(x^*(n))$. From the strict concavity of $p(x)$, it follows that $f(x)$ is increasing since $f'(x) = -p(x)p''(x)/[p'(x)]^2 \geqslant 0$. Using the fact that $x^*(n_0(\varepsilon)) \geqslant x^*(n)$ for all n, we then conclude that $\Psi_0^{n_0(\varepsilon)}(x^*(n_0(\varepsilon))) \geqslant \Psi_0^n(x^*(n))$ for all n. So when R&D investment is chosen optimally, the industry of size $n_0(\varepsilon)$ continues to provide the highest incentive for an outside innovator. It can be shown similarly (replacing $\Pi_0^n(\varepsilon)$ by the incremental payoff of the incumbent innovator) that the industry of size $n_1(\varepsilon)$ provides the highest incentive for an incumbent innovator.

Proposition 7.

(a) *The results* (a), (b) *and* (c) *of Proposition* 6 *continue to hold when there is an R&D stage and the innovator chooses the optimal R&D investment.*
(b) *The innovator invests the highest amount in the industry that provides the highest incentive.*

5. Concluding remarks

In this paper we have considered the licensing of a cost-reducing innovation by means of combinations of upfront fees and royalties in an industry of general size. We have examined two cases, where the innovator is either an outsider, or one of the incumbent producers. For each of these cases, we have derived the optimal licensing policy and discussed its impact on the payoffs of the agents, market structure and diffusion of the innovation. We have also compared the incentives to innovate for outsider and incumbent innovators. Finally, the relation between industry structure and incentives has been analyzed.

Regarding the optimal licensing policy, our conclusion is that significant innovations involve positive royalty while no royalty is charged for insignificant innovations. This is consistent with empirical findings on actual licensing practices where all standard policies (i.e., upfront fees, royalties and combinations of both) are observed. It would be an interesting empirical test to verify if indeed licensing of significant innovations is associated with higher rate of royalty.

We have shown that licensing under combinations of upfront fees and royalties unambiguously leads to improvement of social welfare. Moreover, it results in lower industry price and the innovation is practically licensed to all firms, thus ensuring full diffusion of the innovation. It should be mentioned that under royalty licensing with an outside innovator, all firms effectively operate with the pre-innovation marginal cost and as a result the post-innovation price stays the same at its pre-innovation level. On the other hand, under licensing by means of upfront fee, significant innovations are licensed only to a few firms, so the diffusion is limited.

Regarding the relation between industry structure and incentives to innovate, we have shown that as a function of the magnitude of the innovation, the industry size that provides the highest incentive is U-shaped. Thus imperfect competition with a few firms is the most conducive industry structure for innovations of moderate magnitude. This may have an interesting policy implication: for an industry where the pace of technological progress is moderate (i.e., an industry that is not technologically stagnant, but also does not exhibit rapid breakthroughs), it might not be desirable to encourage fierce competition. It should be mentioned, however, that the perfectly competitive industry always provides higher incentive than the monopoly. This is consistent with Arrow (1962) and the underlying intuition can be stated in Arrow's own words:

> "The only ground for arguing that monopoly may create superior incentives to invent is that appropriability may be greater under monopoly than under competition. Whatever differences may exist in this direction must, of course, still be offset against the monopolist's disincentive created by his preinvention monopoly profits." (Arrow, 1962, p. 622.)

Finally, this paper has contributed to the issue of incentives of innovation by considering a potential innovator which chooses R&D investment on the basis of its future post-innovation optimal licensing policy. Doing so, the paper combines two strands of literature[14] on innovations: one that focuses on the pre-innovation environment like patent race taking the post-innovation reward as given (e.g., Loury, 1979; Lee and Wilde, 1980) and the other that determines the optimal post-innovation licensing strategy taking the innovation as given. Although our pre-innovation set-up is limited to only one potential innovator and ignores patent race, nevertheless it provides a good starting point for further research on this topic.

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Appendix A

Note that $\theta := a - c$ and $F(p) := (p - c + \varepsilon)(a - p)$. We use the notations introduced in Section 3.2. Recall that

$$\Pi(n-1,\delta) = F\big(p(n-1,\delta)\big) - n\big[p(n-1,\delta) - c\big]q_{\mathrm{N}}(n-1,\delta) - \varepsilon q_{\mathrm{N}}(k,\delta). \tag{A.1}$$

$$\Pi(n,\delta) = F\big(p(n,\delta)\big) - n\big[p(n-1,\delta) - c\big]q_{\mathrm{N}}(n-1,\delta). \tag{A.2}$$

We begin with the following lemmas, which hold for $\lambda \in \{0, 1\}$.

[14] Other papers following this approach include Gallini and Winter (1985) and Katz and Shapiro (1987). However, the licensing policies they consider are more restricted and their analysis is confined to duopolies.

Lemma A.1. *For* $1 \leqslant k \leqslant n-1$:

$$\begin{aligned}
&\text{(i)} \quad q_{\mathrm{N}}(k,\delta) = p(k,\delta) - c = \frac{k[\delta_\lambda(k) - \delta]}{n+1+\lambda}, \qquad q_{\mathrm{L}}(k,\delta) = q_{\mathrm{N}}(k,\delta) + \delta, \\
&\qquad q_{\mathrm{I}}(k,\delta) = \lambda\big[q_{\mathrm{N}}(k,\delta) + \varepsilon\big] \quad \textit{if } \delta \leqslant \delta_\lambda(k); \\
&\text{(ii)} \quad q_{\mathrm{N}}(k,\delta) = 0, \qquad q_{\mathrm{L}}(k,\delta) = p(k,\delta) - c + \delta = \frac{k\delta_\lambda(k) + (1+\lambda)\delta}{k+1+\lambda}, \\
&\qquad q_{\mathrm{I}}(k,\delta) = \lambda\big[q_{\mathrm{L}}(k,\delta) + \varepsilon - \delta\big] \quad \textit{if } \delta \geqslant \delta_\lambda(k).
\end{aligned}$$

For $k = n$ *and* $\delta \in [0, \varepsilon]$:

$$\begin{aligned}
&q_{\mathrm{L}}(n,\delta) = p(n,\delta) - c + \delta = \frac{n\delta_\lambda(n) + (1+\lambda)\delta}{n+1+\lambda}, \\
&q_{\mathrm{I}}(n,\delta) = \lambda\big[q_{\mathrm{L}}(n,\delta) + \varepsilon - \delta\big].
\end{aligned}$$

Proof. Follows from equilibrium conditions of the Cournot oligopoly under consideration. $\square$

Lemma A.2. *Suppose* $n \geqslant 3$. *For* $k \in \{n-1, n\}$ *and* $\delta \in [0, \delta_\lambda(k)]$:

(i) $\Pi(k,\delta)$ *is increasing in* δ *at* $\delta = 0$, *and*
(ii) $\Pi(k,\delta)$ *is strictly concave in* δ.

Proof. From Lemma A.1, we have

$$\frac{\partial \Pi(n-1,\delta)}{\partial \delta}\bigg|(\delta = 0) = \begin{cases} (n-1)(a-c+2\varepsilon)/(n+1) & \text{for } \lambda = 0, \\ (n-1)[n(a-c)+2\varepsilon]/(n+2)^2 & \text{for } \lambda = 1. \end{cases}$$

$$\frac{\partial \Pi(n,\delta)}{\partial \delta}\bigg|(\delta = 0) = \begin{cases} n[(n-1)(a-c)+(n+1)\varepsilon]/(n+1)^2 & \text{for } \lambda = 0, \\ n(n-2)(a-c-\varepsilon)/(n+2)^2 & \text{for } \lambda = 1. \end{cases}$$

Part (i) follows from the fact that all of the above expressions are positive for $n \geqslant 3$. Noting that $\partial^2 \Pi(n-1,\delta)/\partial\delta^2 = -2(n-1)^2(n+1)/(n+1+\lambda)^2 < 0$ and $\partial^2 \Pi(n,\delta)/\partial\delta^2 = -2n(n^2 - n + 1)/(n+1+\lambda)^2 < 0$, part (ii) follows. $\square$

Lemma A.3. *Suppose* $n \geqslant 3$ *and* $\varepsilon \geqslant (a-c)/2$. *Then the maximum of* $\Pi(n-1,\delta)$ *for* $\delta \in [0, \delta_\lambda(n-1)]$ *is attained at* $\delta = \delta_\lambda(n-1)$ *and* $\Pi(n-1, \delta_\lambda(n-1)) = \varepsilon(a-c)$.

Proof. Observe that

$$\frac{\partial \Pi(n-1,\delta)}{\partial \delta}\bigg|\big(\delta = \delta_\lambda(n-1)\big) = \frac{(n-1)[2\varepsilon - (a-c)]}{n+1+\lambda} \geqslant 0 \quad \text{for } \varepsilon \geqslant \frac{a-c}{2}.$$

Since $\Pi(n-1,\delta)$ is strictly concave for $\delta \in [0, \delta_\lambda(n-1)]$ (by Lemma A.2), the first part follows. The second part follows from standard computations. $\square$

Remark A.1. Under the policy $(n-1, \delta_\lambda(n-1))$, the post-innovation Cournot price equals c. Consequently the sole non-licensee does not produce and an $(n-1+\lambda)$-firm natural oligopoly is created.

Proof of Lemma 1. Follows from Lemma A.1. $\square$

Proof of Proposition 2. See the discussion after Proposition 2 for the proof of (a), (b), (c), (e) and the first and last statements of (d). The second statement of (d) follows from Lemma A.2(i). □

Proof of Proposition 3. By Proposition 2(a), it is sufficient to consider $k \in \{n-1, n\}$. By Lemma 1(b) and Proposition 2(c), it is sufficient to consider $\delta \leqslant \delta_\lambda(k)$.

Case 1. Consider $k = n-1$ and $\delta \in [0, \min\{\varepsilon, \delta_\lambda(n-1)\}]$. The unconstrained maximum of $\Pi(n-1, \delta)$ is attained at $\delta = \tilde{\delta}_\lambda(n)$, where

$$\tilde{\delta}_\lambda(n) := \frac{(n+1-\lambda)(a-c) + 2(n+1-\lambda n)\varepsilon}{2(n^2-1)}.$$

The optimal policy for $k = n-1$ is given below, where $\tilde{\rho}_\lambda(n) := \varepsilon - \tilde{\delta}_\lambda(n)$, $\rho^*_\lambda(n) := \varepsilon - \delta_\lambda(n-1)$, $f_0(n) := 2n-4$ and $f_1(n) := 2(n^2-2)/n$.

Case 2. Consider $k = n$ and $\delta \in [0, \min\{\varepsilon, \delta_\lambda(n)\}]$. The unconstrained maximum of $\Pi(n, \delta)$ is attained at $\delta = \bar{\delta}_\lambda(n)$, where

$$\bar{\delta}_\lambda(n) := \frac{(n-\lambda-1)(a-c) - (2n\lambda - n - \lambda - 1)\varepsilon}{2(n^2-n+1)}.$$

The optimal policy for $k = n$ is given below, where $\bar{\rho}_\lambda(n) := \varepsilon - \bar{\delta}_\lambda(n)$, $\rho^{**}_\lambda(n) := \varepsilon - \delta_\lambda(n)$, $w_0(n) := n(n+1)/(n^2-n+2)$ and $w_1(n) := n(2n-1)/(n-2)$.

To prove (a), we compare Tables A.1 and A.3. First observe that $w_0(n) < 2$ for all $n \geqslant 3$. When $\varepsilon \geqslant \theta/w_0(n)$, the innovator obtains $\varepsilon(a-c)$ for $k = n-1$ and less than $\varepsilon(a-c)$ for $k = n$. When $\varepsilon \in [\theta/2, \theta/w_0(n))$, (i) $\Pi^n_0(n, \bar{\rho}_0(n)) < \varepsilon(a-c)$ for $3 \leqslant n \leqslant 6$ and (ii) for $n \geqslant 7$, ∃ a function $u(n)$ (where $w_0(n) < u(n) < 2$) such that $\Pi^n_0(n, \bar{\rho}_0(n)) < \varepsilon(a-c)$ if $\varepsilon > \theta/u(n)$ and the reverse inequality holds otherwise. This fact, together with Remark A.1, proves (a). Table A.5 provides a complete characterization of equilibrium policies for the outside innovator.

To prove (b), we compare Tables A.2 and A.4. When $\varepsilon \geqslant \theta/2$, the innovator obtains $\varepsilon(a-c)$ for $k = n-1$ and more than $\varepsilon(a-c)$ for $k = n$. Observe that $w_1(n) > f_1(n) > 2$ for $n \geqslant 3$. When $\varepsilon \in [\theta/f_1(n), \theta/2)$, (i) $\Pi^n_1(n, \bar{\rho}_1(n)) > \Pi^n_1(n-1, \tilde{\rho}_1(n))$ for $n \in \{3, 4\}$ and (ii) for $n \geqslant 5$, ∃ a function $h(n)$ (where $2 < h(n) < f_1(n)$) such that $\Pi^n_1(n, \bar{\rho}_1(n)) \geqslant \Pi^n_1(n-1, \tilde{\rho}_1(n))$ if

Table A.1
Outside innovator ($\lambda = 0$)

$k = n-1$	$\varepsilon \in (0, \theta/f_0(n)]$	$\varepsilon \in (\theta/f_0(n), \theta/2)$	$\varepsilon \in [\theta/2, \theta)$
Royalty	$r = 0$	$r = \tilde{\rho}_0(n)$	$r = \rho^*_0(n)$
Payoff	$\Pi^n_0(n-1, 0)$	$\Pi^n_0(n-1, \tilde{\rho}_0(n))$	$\varepsilon(a-c)$

Table A.2
Incumbent innovator ($\lambda = 1$)

$k = n-1$	$\varepsilon \in (0, \theta/f_1(n)]$	$\varepsilon \in (\theta/f_1(n), \theta/2)$	$\varepsilon \in [\theta/2, \theta)$
Royalty	$r = 0$	$r = \tilde{\rho}_1(n)$	$r = \rho^*_1(n)$
Payoff	$\Pi^n_1(n-1, 0)$	$\Pi^n_1(n-1, \tilde{\rho}_1(n)$	$\varepsilon(a-c)$

Table 3
Outside innovator ($\lambda = 0$)

$k = n$	$\varepsilon \in (0, \theta/(2n-1)]$	$\varepsilon \in (\theta/(2n-1), \theta/w_0(n))$	$\varepsilon \in [\theta/w_0(n), \theta)$
Royalty	$r = 0$	$r = \bar{\rho}_0(n)$	$r = \rho_0^{**}(n)$
Payoff	$\Pi_0^n(n, 0)$	$\Pi_0^n(n, \bar{\rho}_0(n))$	Less than $\varepsilon(a - c)$

Table 4
Incumbent innovator ($\lambda = 1$)

$k = n$	$\varepsilon \in (0, \theta/w_1(n)]$	$\varepsilon \in (\theta/w_1(n), \theta)$
Royalty	$r = 0$	$r = \bar{\rho}_1(n)$
Payoff	$\Pi_1^n(n, 0)$	$\Pi_1^n(n, \bar{\rho}_1(n))$

$\varepsilon \geqslant \theta/h(n)$ and the reverse inequality holds otherwise. This proves (b). Table A.6 provides a complete characterization of equilibrium policies for the incumbent innovator.

Part (c) follows from Tables A.5 and A.6. Part (d) also follows from these tables by standard comparisons. □

Proof of Proposition 4. Part (a) is obvious. We prove (b), (c) and (d).

Proof of (b). Let $n = 2$ and $\delta \in [0, \varepsilon]$. For $k = 1$, it follows from (A.1) that the maximum of $\Pi(1, \delta)$ is attained at $\delta = \varepsilon$. For $k = 2$, it follows from (A.2) that the maximum of $\Pi(2, \delta)$ is attained at $\delta = \varepsilon$ if $\varepsilon \leqslant (a - c)/3$ and at $\delta = \bar{\delta}$ if $\varepsilon > (a - c)/3$, where $\bar{\delta} \equiv \varepsilon/2 + (a - c)/6$. The first two statements of (b) follow by noting that (i) for $\varepsilon \leqslant (a - c)/3$, $\Pi(2, \varepsilon) > \Pi(1, \varepsilon)$ and (ii) for $\varepsilon > (a - c)/3$, $\Pi(2, \bar{\delta}) \geqslant \Pi(1, \varepsilon)$ iff $\varepsilon \leqslant (a - c)/t$ where $t \equiv 3(\sqrt{2} - 1)$.

Observing that $\Pi(2, \varepsilon) = 8\varepsilon(a-c)/9$, $\Pi(2, \bar{\delta}) = [(a-c)^2 + 42\varepsilon(a-c) + 9\varepsilon^2]/54 < \varepsilon(a-c)$ for $\varepsilon \in ((a-c)/3, (a-c)/t]$ and $\Pi(1, \varepsilon) = \varepsilon[2(a-c)+\varepsilon]/3 < \varepsilon(a-c)$ for $\varepsilon < a-c$, it follows that the innovator obtains less than $\varepsilon(a - c)$.

To prove the last statement, first note that for $\varepsilon > (a - c)/t$, the equilibrium policy has $k = 1$. Since the innovation is nondrastic, the Cournot price stays above c for $k = 1$. For $\varepsilon \leqslant (a - c)/3$, the equilibrium policy has $k = 2$ and zero royalty (i.e., $\delta = \varepsilon$). Since for $k = 2$ and $\delta = \varepsilon$, the price is less than c iff $\varepsilon \geqslant (a - c)/2$, we conclude that the price stays above c for $\varepsilon \leqslant (a - c)/3$. Finally, consider $\varepsilon \in ((a - c)/3, (a - c)/t]$. Then the equilibrium policy is $(2, \bar{\delta})$. The proof is complete by noting that for this policy, the price falls below c iff $\varepsilon \in (2(a - c)/3, (a - c)/t]$.

Proof of (c). Observe that for any $n \geqslant 1$, an incumbent innovator obtains more than $\varepsilon(a - c)$ by selling the license to all n other firms by using a pure royalty policy with $r = \varepsilon$. Then by the same argument as in the proof of (c) of Proposition 2, it follows that the post-innovation Cournot price stays above c.

For $n = 1$, the Cournot profit of the only other firm without the innovation is a constant that does not depend on the licensing policy. Taking $n = 1$ in (A.2), the innovator's problem then reduces to maximizing $F(p(1, \delta))$ where $F(p) := (p - c + \varepsilon)(a - p)$. Since $F(p)$ is increasing for $p < p_{\mathrm{M}} \equiv (a + c - \varepsilon)/2$ (the post-innovation monopoly price) and $p(1, \delta)$ is increasing for $\delta \in [0, \varepsilon]$, we conclude that for $F(p(1, \delta))$ is maximized at $\delta = 0$, i.e., $r = \varepsilon$.

Next consider $n = 2$. By (b) of Lemma 1, it is sufficient to consider policies (k, δ) where $\delta \in [0, \min\{\varepsilon, \delta_1(k)\}]$. For $k = 1$, it follows from (A.1) that the maximum of $\Pi(1, \delta)$ is attained at $\delta = \delta_1(1)$ if $\varepsilon \geqslant (a - c)/2$ and at $\delta = \varepsilon$ if $\varepsilon < (a - c)/2$. For $k = 2$, it follows from (A.2) that the maximum of $\Pi(2,\delta)$ is attained at $\delta = 0$ (i.e., $r = \varepsilon$). Noting that $\Pi(2,0) > \varepsilon(a-c) = \Pi(1,\delta_1(1))$ and $\Pi(2,0) - \Pi(1,\varepsilon) = [2(a-c) - 3\varepsilon]/16 > 0$ for $\varepsilon < (a-c)/2$, the result follows.

Proof of (d). In the case of an incumbent innovator, the result follows from (c) by noting that the innovator is more efficient than before and all other firms are effectively as efficient as before. The conclusion in the case of an outside innovator is obvious for $n = 1$. Finally consider an outside innovator with $n = 2$. If $\varepsilon > (a-c)/t$ or $\varepsilon \leqslant 2(a-c)/3$, the post-innovation Cournot price stays above c and the result follows by the same argument as in the proof of Proposition 2(b). So let $\varepsilon \in (2(a-c)/3, (a-c)/t]$ and denote by Q_1 and Q_2 the pre and post-innovation industry outputs respectively. Since for this case all firms have the innovation, the post-innovation sum of payoffs of the innovator and all firms equals $[p(Q_2) - (c-\varepsilon)]Q_2$. The post-innovation consumer surplus is $[\int_0^{Q_2} p(q)\,dq] - p(Q_2)Q_2$. So the post-innovation social welfare is $W_2 = \int_0^{Q_2}[p(q) - (c-\varepsilon)]\,dq$. The pre-innovation welfare is $W_1 = \int_0^{Q_1}[p(q) - c]\,dq < \int_0^{Q_1}[p(q) - (c-\varepsilon)]\,dq$. Since $p(Q_1) > p(Q_2) > c - \varepsilon$, it follows that $W_2 > W_1$. □

Proof of Lemma 2. We consider an industry of size m for every $m \geqslant 2$ and denote $m = n+1$. From Tables A.5 and A.6, we partition the interval $(0,\theta)$ into finitely many subintervals according to the licensing policies of outside and incumbent innovators. Then standard comparison shows that for every $m \geqslant 2$, in each of these subintervals, the payoff of the outside innovator is higher than the incremental payoff of the incumbent innovator. □

Proof of Proposition 6. Parts (a) and (b) have been proved in the main text. For (c), we provide the details of the proof only for significant innovations. The proof for the remaining cases is analogous and relies on certain standard comparisons based on Tables A.5 and A.6. We denote $x \equiv (a-c)/\varepsilon$. Since $0 < \varepsilon < a - c$, we have $x > 1$.

Outside innovator. Consider $\varepsilon \geqslant (a-c)/2$, i.e., $x \in (1,2]$. Note from Table A.5 that $u(n) = 2$ for $3 \leqslant n \leqslant 6$, $u(n+1) < u(n)$ for $n \geqslant 6$ and $\lim_{n\to\infty} u(n) = 1$. Thus for any $x \in (1,2]$, $\exists$ an integer $N(x) \geqslant 6$ such that $x \in (u(N(x)+1), u(N(x))]$, so that the payoff of the innovator is $\varepsilon(a-c)$ for $n \leqslant N(x)$ and it is $\Pi_0^n(n, \bar\rho_0(n)) > \varepsilon(a-c)$ for $n \geqslant N(x)+1$. Hence $n_0(\varepsilon) \geqslant N(x)+1 \geqslant 7$. For $n \geqslant 7$, $\exists$ a function $\gamma(n) > u(n)$ such that $\Pi_0^{n+1}(n+1, \bar\rho_0(n+1)) \geqslant \Pi_0^n(n, \bar\rho_0(n))$ if $x \in (1, \gamma(n)]$ (with equality iff $x = \gamma(n)$) and the reverse inequality holds if $x > \gamma(n)$. Moreover $\gamma(n+1) < \gamma(n)$ and $\lim_{n\to\infty}\gamma(n) = 1$. Thus, for every $x \in (u(N(x)+1), u(N(x))]$, $\exists$ an integer $\overline{N}(x) \geqslant N(x)$ such that $x \in (\gamma(\overline{N}(x)+1), \gamma(\overline{N}(x))]$. Hence, $\gamma(n) \geqslant x$ for $N(x) \leqslant n \leqslant \overline{N}(x)$ and $\gamma(n) < x$ for $n \geqslant \overline{N}(x)+1$. Denoting $\nu := \overline{N}(x)$, we then have

$$\Pi_0^{\nu+1}\big(\nu+1, \bar\rho_0(\nu+1)\big) \geqslant \Pi_0^{\nu}\big(\nu, \bar\rho_0(\nu)\big) > \cdots > \Pi_0^{N(x)}\big(N(x), \bar\rho_0\big(N(x)\big)\big),$$

$$\Pi_0^{\nu+1}\big(\nu+1, \bar\rho_0(\nu+1)\big) > \Pi_0^{\nu+2}\big(\nu+2, \bar\rho_0(\nu+2)\big) > \cdots$$

so that for $n \geqslant N(x)+1$, $\Pi_0^n(n, \bar\rho_0(n))$ is maximum when $n = \nu+1 = \overline{N}(x)+1$. Hence, $n_0(\varepsilon) = \overline{N}(x)+1$.

Table A.5
Equilibrium policy and payoff[15] of an outside innovator for industries of size $n \geqslant 2$

$n=2$	$\varepsilon \in (0, \theta/3]$	$\varepsilon \in (\theta/3, \theta/(3\sqrt{2}-3)]$	$\varepsilon \in (\theta/(3\sqrt{2}-3), \theta)$
Policy	$k=2, r=0$	$k=2, r=\bar{\rho}_0(2)$	$k=1, r=0$
Payoff	$8\varepsilon(a-c)/9$	$\Pi_0^2(2, \bar{\rho}_0(2))$	$\varepsilon[2(a-c)+\varepsilon]/3$

$n=3$	$\varepsilon \in (0, \theta/5]$	$\varepsilon \in (\theta/5, \theta/d_0]$	$\varepsilon \in (\theta/d_0, \theta/2]$	$\varepsilon \in (\theta/2, \theta)$
Policy	$k=3, r=0$	$k=3, r=\bar{\rho}_0(3)$	$k=2, r=0$	$k=2, r=\rho_0^*(3)$
Payoff	$\Pi_0^3(3,0)$	$\Pi_0^3(3, \bar{\rho}_0(3))$	$\Pi_0^3(2,0)$	$\varepsilon(a-c)$

$n=4,5$	$\varepsilon \in (0, \theta/\ell(n)]$	$\varepsilon \in (\theta/\ell(n), \theta/f_0(n)]$	$\varepsilon \in (\theta/f_0(n), \theta/2]$	$\varepsilon \in (\theta/2, \theta)$
Policy	$k=n, r=0$	$k=n-1, r=0$	$k=n-1, r=\tilde{\rho}_0(n)$	$k=n-1, r=\rho_0^*(n)$
Payoff	$\Pi_0^n(n,0)$	$\Pi_0^n(n-1,0)$	$\Pi_0^n(n-1, \tilde{\rho}_0(n))$	$\varepsilon(a-c)$

$n=6$	$\varepsilon \in (0, 2\theta/39]$	$\varepsilon \in (2\theta/39, \theta/8]$	$\varepsilon \in (\theta/8, \theta/d_1)$	$\varepsilon \in [\theta/d_1, \theta/d_2]$	$\varepsilon \in (\theta/d_2, \theta/2]$	$\varepsilon \in (\theta/2, \theta)$
Policy	$k=6$, $r=0$	$k=5$, $r=0$	$k=5$, $r=\tilde{\rho}_0(6)$	$k=6$, $r=\bar{\rho}_0(6)$	$k=5$, $r=\tilde{\rho}_0(6)$	$k=5$, $r=\rho_0^*(6)$
Payoff	$\Pi_0^6(6,0)$	$\Pi_0^6(5,0)$	$\Pi_0^6(5, \tilde{\rho}_0(6))$	$\Pi_0^6(6, \bar{\rho}_0(6))$	$\Pi_0^6(5, \tilde{\rho}_0(6))$	$\varepsilon(a-c)$

$n \geqslant 7$	$\varepsilon \in (0, \theta/\ell(n)]$	$\varepsilon \in (\theta/\ell(n), \theta/f_0(n)]$	$\varepsilon \in (\theta/f_0(n), \theta/v(n))$	$\varepsilon \in [\theta/v(n), \theta/u(n)]$	$\varepsilon \in (\theta/u(n), \theta)$
Policy	$k=n$, $r=0$	$k=n-1$, $r=0$	$k=n-1$, $r=\tilde{\rho}_0(n)$	$k=n$, $r=\bar{\rho}_0(n)$	$k=n-1$, $r=\rho_0^*(n)$
Payoff	$\Pi_0^n(n,0)$	$\Pi_0^n(n-1,0)$	$\Pi_0^n(n-1, \tilde{\rho}_0(n))$	$\Pi_0^n(n, \bar{\rho}_0(n))$	$\varepsilon(a-c)$

Royalties:

$$\tilde{\rho}_0(n) := \frac{2(n-2)\varepsilon - \theta}{2(n-1)}, \qquad \bar{\rho}_0(n) := \frac{(n-1)[(2n-1)\varepsilon - \theta]}{2(n^2-n+1)}, \qquad \rho_0^*(n) := \varepsilon - \frac{\theta}{n-1}.$$

Payoffs:

$$\Pi_0^n(n,0) = \frac{n(\theta+\varepsilon)^2 - n[\theta-(n-1)\varepsilon]^2}{(n+1)^2},$$

$$\Pi_0^n(n-1,0) = \frac{(n-1)(\theta+2\varepsilon)^2 - (n-1)[\theta-(n-1)\varepsilon]^2}{(n+1)^2},$$

$$\Pi_0^n(n-1, \tilde{\rho}(n)) = \frac{\theta^2 + 4n\theta\varepsilon + 4\varepsilon^2}{4(n+1)},$$

$$\Pi_0^n(n, \bar{\rho}(n)) = \frac{n[(n-1)^2\theta^2 + 2(2n^3+n^2+1)\theta\varepsilon + (n+1)^2\varepsilon^2]}{4(n+1)^2(n^2-n+1)}.$$

Notes for Table A.5:

$$d_0 \equiv \frac{(8+2\sqrt{7})}{3}, \qquad d_2 \equiv \frac{(210-\sqrt{5642})}{67}, \qquad d_1 \equiv \frac{(210+\sqrt{5642})}{67},$$

$$\ell(n) := \frac{n^2+n-3}{2} \qquad f_0(n) := 2n-4, \qquad v(n) := \frac{n^3-n+\sqrt{(n+1)(n^2-n+1)(n^3-6n^2+5n-4)}}{2n^2-n+1},$$

$$u(n) := \begin{cases} 2 & \text{for } 3 \leqslant n \leqslant 6 \\ \frac{(n+1)(1+\sqrt{n^2-n+1})^2}{n(n-1)^2} & \text{for } n \geqslant 7. \end{cases}$$

[15] Recall that $\theta := a-c$. Since ε is nondrastic, $0 < \varepsilon < \theta$.

Table A.6
Equilibrium policy and payoff of an incumbent innovator when there are $n \geqslant 1$ firms other than the innovator

$n=1,2$	$\varepsilon \in (0,\theta)$
Policy	$k=n,\ r=\varepsilon$
Payoff	$(a-c-\varepsilon)^2/(n+2)^2+\varepsilon(a-c)$

$n=3$	$\varepsilon \in (0,\theta/15]$	$\varepsilon \in (\theta/15,\theta)$
Policy	$k=3,\ r=0$	$k=3,\ r=\bar{\rho}_1(3)$
Payoff	$\Pi_1^3(3,0)$	$\Pi_1^3(3,\bar{\rho}_1(3))$

$n=4$	$\varepsilon \in (0,\theta/14]$	$\varepsilon \in (\theta/14,\theta/s_1]$	$\varepsilon \in (\theta/s_1,\theta/s_2)$	$\varepsilon \in [\theta/s_2,\theta)$
Policy	$k=4,\ r=0$	$k=4,\ r=\bar{\rho}_1(4)$	$k=3,\ r=0$	$k=4,\ r=\bar{\rho}_1(4)$
Payoff	$\Pi_1^4(4,0)$	$\Pi_1^4(4,\bar{\rho}_1(4))$	$\Pi_1^4(3,0)$	$\Pi_1^4(4,\bar{\rho}_1(4))$

$n\geqslant 5$	$\varepsilon \in (0,\theta/g(n)]$	$\varepsilon \in (\theta/g(n),\theta/f_1(n)]$	$\varepsilon \in (\theta/f_1(n),\theta/h(n))$	$\varepsilon \in [\theta/h(n),\theta)$
Policy	$k=n,\ r=0$	$k=n-1,\ r=0$	$k=n-1,\ r=\tilde{\rho}_1(n)$	$k=n,\ r=\bar{\rho}_1(n)$
Payoff	$\Pi_1^n(n,0)$	$\Pi_1^n(n-1,0)$	$\Pi_1^n(n-1,\tilde{\rho}_1(n))$	$\Pi_1^n(n,\bar{\rho}_1(n))$

Royalties:

$$\tilde{\rho}_1(n) := \frac{2(n^2-2)\varepsilon - n\theta}{2(n^2-1)}, \qquad \bar{\rho}_1(n) := \frac{n(2n-1)\varepsilon-(n-2)\theta}{2(n^2-n+1)}.$$

Payoffs:

$$\Pi_1^n(n,0) = \frac{(\theta+\varepsilon)^2+n(\theta+\varepsilon)^2-n(\theta-n\varepsilon)^2}{(n+2)^2},$$

$$\Pi_1^n(n-1,0) = \frac{(\theta+2\varepsilon)^2+(n-1)(\theta+2\varepsilon)^2-(n-1)(\theta-n\varepsilon)^2}{(n+2)^2},$$

$$\Pi_1^n(n-1,\tilde{\rho}_1(n)) = \frac{\theta^2+4n\theta\varepsilon+4\varepsilon^2}{4(n+1)},$$

$$\Pi_1^n(n,\bar{\rho}_1(n)) = \frac{(n^3+4)(\theta+\varepsilon)^2+4n^2(n+1)^2\theta\varepsilon}{4(n+2)^2(n^2-n+1)}.$$

Notes for Table A.6:

$$s_2 \equiv \frac{43-3\sqrt{13}}{4}, \qquad s_1 \equiv \frac{43+3\sqrt{13}}{4}, \qquad g(n) := \frac{n^2+3n-1}{2}, \qquad f_1(n) := \frac{2(n^2-2)}{n},$$

$$h(n) := \frac{n^4+5n^3+2n^2-4n+4+(n+2)\sqrt{(n+1)(n^2-n+1)(n^3+4)}}{2n^3+n^2-4n}.$$

Observation 1. *$\overline{N}(x)$ is decreasing for $x \in (1,2]$.*

Proof. Consider $1 < x_1 < x_2 \leqslant 2$. Then there are integers $\overline{N}(x_1)$ and $\overline{N}(x_2)$ such that $x_1 \in (\gamma(\overline{N}(x_1)+1), \gamma(\overline{N}(x_1))]$ and $x_2 \in (\gamma(\overline{N}(x_2)+1), \gamma(\overline{N}(x_2))]$. Suppose to the contrary that $\overline{N}(x_1) < \overline{N}(x_2)$, i.e., $\overline{N}(x_1)+1 \leqslant \overline{N}(x_2)$. Since $\gamma(n+1) < \gamma(n)$, we then have

$$\gamma\big(\overline{N}(x_2)+1\big) < \gamma\big(\overline{N}(x_2)\big) \leqslant \gamma\big(\overline{N}(x_1)+1\big) < \gamma\big(\overline{N}(x_1)\big). \tag{A.3}$$

Since $x_1 \in (\gamma(\overline{N}(x_1)+1), \gamma(\overline{N}(x_1))]$ and $x_2 \in (\gamma(\overline{N}(x_2)+1), \gamma(\overline{N}(x_2))]$, from (A.3), it follows that $x_2 \leqslant x_1$, a contradiction. □

Since $x = (a-c)/\varepsilon$, $n_0(\varepsilon) = \overline{N}(x)+1$ and $\overline{N}(x)$ is decreasing for $x \in (1,2]$, we conclude that $n_0(\varepsilon)$ is increasing for $\varepsilon \in [(a-c)/2, a-c)$. Since $x \in (\gamma(\overline{N}(x)+1), \gamma(\overline{N}(x))]$ and $\gamma(n) \to 1$ as $n \to \infty$, we conclude that $\overline{N}(x) \to \infty$ as $x \downarrow 1$, so that $n_0(\varepsilon) \to \infty$ as $\varepsilon \uparrow a-c$.

Incumbent innovator. Consider $\varepsilon \geqslant (a-c)/h(3)$ (note from Table A.6 that $h(3) > 2$). Then from the properties of $h(n)$, it follows that $\varepsilon \geqslant (a-c)/h(n)$ for all $n \geqslant 3$ and from Table A.6, the payoff of the innovator is $\Pi_1^n(n, \bar{\rho}_1(n))$ so that the incremental payoff is $\Delta^n(\varepsilon) = \Pi_1^n(n, \bar{\rho}_1(n)) - (a-c)^2/(n+2)^2$. For all $x > 1$, $\Delta^n(\varepsilon)$ is increasing in n for $2 \leqslant n \leqslant 8$. For $n \geqslant 8$, $\exists$ a function $\phi(n) > 1$ such $\Delta^{n+1}(\varepsilon) \geqslant \Delta^n(\varepsilon)$ iff $x \in (1, \phi(n)]$. Moreover $\phi(n+1) < \phi(n)$, $\phi(12) < h(3) < \phi(11)$ and $\lim_{n\to\infty} \phi(n) = 1$. All these facts imply that for any $x \in (1, h(3)]$, $\exists$ an integer $\widetilde{N}(x) \geqslant 12$ such that $x \in [\phi(\widetilde{N}(x)), \phi(\widetilde{N}(x)-1))$, so that $n_1(\varepsilon) = \widetilde{N}(x)+2$. By an argument analogous to the case of an outside innovator, we can show that $\widetilde{N}(x)$ is decreasing for $x \in (1, h(3)]$ and $\widetilde{N}(x) \to \infty$ as $x \downarrow 1$. This proves that $n_1(\varepsilon)$ is increasing for $\varepsilon \in [(a-c)/h(3), a-c)$. Further, $n^1(\varepsilon) \to \infty$ as $\varepsilon \uparrow a-c$. □

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