Fee versus royalty reconsidered

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Abstract

For an outsider innovator in a Cournot oligopoly, royalty licensing could be superior to both fixed fee and auction. The result depends on a simple fact that has been overlooked in the existing literature, namely, the number of licenses can take only integer values.

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1. Introduction

The theoretical literature on licensing of cost-reducing innovations has mainly considered innovators who are outsiders to the industry (Arrow, 1962; Kamien and Tauman, 1984, 1986; Katz and Shapiro, 1985, 1986; Kamien et al., 1992; see Kamien, 1992 for a survey). It has been shown that in a Cournot oligopoly, royalty licensing is inferior to licensing by means of fixed fee or auction for an outsider innovator, regardless of the industry size or magnitude of the innovation (Kamien and Tauman, 1984, 1986; Kamien et al., 1992). In view of this theoretical conclusion, the wide prevalence of royalties in practice (e.g., Taylor and Silberstone, 1973; Rostoker, 1984) has remained a puzzle and several, often overlapping, approaches have been taken subsequently to justify the use of royalty licensing. In particular, it has been shown that royalty can be explained by asym-
metry of information (Gallini and Wright, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Poddar and Sinha, 2002; Sen, in press), variation in the quality of innovation (Rockett, 1990), product differentiation (Muto, 1993; Wang and Yang, 1999; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2003), moral hazard (Macho-Stadler et al., 1996; Choi, 2001), risk aversion (Bousquet et al., 1998), incumbent innovator (Shapiro, 1985; Wang, 1998, 2002; Kamien and Tauman, 2002; Sen, 2002; Sen and Tauman, 2003), leadership structure (Filippini, 2001; Kabiraj, 2002, 2004) or strategic delegation (Saracho, 2002). In the present paper, we show that royalty could dominate both fixed fee and auction in a standard model of a Cournot oligopoly with an outsider innovator. This result is obtained by first showing that for any non-drastic innovation, once an oligopoly has certain threshold level of size, the payoff of the innovator from auction stays the same for any larger oligopoly, while for the royalty policy, the payoff is always increasing in the size of the oligopoly. Then obtaining a common upper bound for the payoffs from both policies, we show that the payoff from auction stays bounded away from it for all but countably many magnitudes of innovation, while the payoff from royalty can be made arbitrarily close to this bound by increasing the size of the oligopoly. In the next section, we derive our result after formally describing the model.

2. The model

We consider a Cournot oligopoly with \( n \) firms producing the same product, where \( N = \{1, \ldots, n\} \) is the set of firms. For \( i \in N \), let \( q_i \) be the quantity produced by firm \( i \) and let \( Q = \sum_{i \in N} q_i \). The demand function of the industry is given by \( Q = a - p \), for \( p \leq a \) and \( Q = 0 \), otherwise. With the old technology, all \( n \) firms produce with the identical constant marginal cost \( c \), where \( 0 < c < a \). An outsider innovator has been granted a patent for a new technology that reduces the cost from \( c \) to \( c - \varepsilon \), where \( 0 < \varepsilon < c \). The innovator decides to license the new technology to some or all firms of the industry. The innovator can sell the license either through auction or royalty. Depending on the policy, we have the following two games that model the interaction of the innovator and the firms.

2.1. The game \( G^A \)

When the innovator uses an auction policy, the game \( G^A \) is played. This game has three stages. In the first stage, the innovator chooses the number of licenses to be sold, \( k \),
where $k$ is an integer satisfying $1 \leq k \leq n$, and announces to auction off $k$ licenses through a first-price sealed bid auction.\(^5\) In the second stage, firms simultaneously and independently decide whether to bid or not and how much to bid. The $k$ highest bidders win the license and pay their respective bids to the innovator (ties are broken at random). The set of licensees become common knowledge at the end of the second stage. In the third stage, firms compete in quantities, where any licensee firm operates with the reduced cost $c - \varepsilon$ and any non-licensee firm operates under the old cost $c$.

2.2. The game $G^R$

When the innovator uses a royalty policy, the game $G^R$ is played. This game also has three stages. In the first stage, the innovator announces the rate of uniform royalty $r$, where $r \geq 0$. In the second stage, firms simultaneously and independently decide whether to accept the offer or not. The set of licensees become common knowledge at the end of the second stage. In the third stage, firms compete in quantities, where any licensee firm operates with the reduced cost $c - \varepsilon$ and any non-licensee firm operates under the old cost $c$. If a licensee firm produces $q$, it pays $rq$ to the innovator.

For both games $G^A$ and $G^R$, we employ the backward induction method to find subgame-perfect equilibrium outcomes. In what follows, we classify innovations according to their relative significance. This classification plays an important role in determining the optimal licensing policy for the innovator.

2.3. Classification of innovations

To begin with, we define a drastic innovation, due to Arrow (1962).

**Drastic innovation.** A cost-reducing innovation is said to be drastic if the monopoly price under the new technology does not exceed the competitive price under the old technology; otherwise, it is non-drastic.

It is well known that for a drastic innovation, the optimal licensing policy for the innovator is to sell the license to only one firm, who becomes a monopolist with the reduced cost and the innovator collects the entire monopoly profit through a fee. The notion of drastic innovation can be extended as follows.

**$k$-drastic innovation.** For $k \geq 1$, a cost-reducing innovation is $k$-drastic if $k$ is the minimum number such that the $k$-firm oligopoly price under the new technology does not exceed the competitive price under the old technology.\(^6\)

**Exact $k$-drastic innovation.** For $k \geq 1$, a $k$-drastic cost-reducing innovation is exact $k$-drastic if the $k$-firm oligopoly price under the new technology equals the competitive price under the old technology; otherwise, it is non-exact $k$-drastic.

Observe that a drastic innovation is 1-drastic and any non-drastic innovation is $k$-drastic for some integer $k \geq 2$. For the demand $Q = a - p$ and constant cost $c$, a cost-reducing

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\(^5\) To ensure positive bids, the innovator needs to specify a minimum bid for $k = n$. We will not encounter this case in this paper.

\(^6\) Yair Tauman holds the patent for the term ‘$k$-drastic.’ He coined this term for a related paper (Sen and Tauman, 2003).
innovation of magnitude $\varepsilon$ is drastic if $\varepsilon \geq a - c$ and for $k \geq 2$, it is $k$-drastic if $(a - c)/k \leq \varepsilon < (a - c)/(k - 1)$. An innovation is exact $k$-drastic if $\varepsilon = (a - c)/k$. In an oligopoly of size $n \geq k + 1$, if $k$ firms have a $k$-drastic innovation, all other firms drop out of the market and a $k$-firm natural oligopoly is created. The Cournot price of the $k$-firm oligopoly equals $c$ if the innovation is exact $k$-drastic, while the price falls below $c$ if it is non-exact.

2.4. The main result

**Proposition 1.** Consider any non-exact $k$-drastic innovation $\varepsilon$, where $k \geq 2$. Then there exists $N(\varepsilon) \geq k + 1$ such that for any oligopoly of size $n \geq N(\varepsilon)$, royalty licensing yields higher payoff for the innovator than both fixed fee and auction.

**Proof.** Since auction dominates fixed fee, it will be sufficient to compare auction and royalty. Let us consider a $k$-drastic innovation $\varepsilon$ for $k \geq 2$, that is, $(a - c)/k \leq \varepsilon < (a - c)/(k - 1)$. For such an innovation, it is a dominated strategy for the innovator to sell more than $k$ licenses (see, e.g., Kamien and Tauman, 1986). In any oligopoly of size $n \geq k + 1$, if $k$ firms have a $k$-drastic innovation, all non-licensee firms drop out of the market and a $k$-firm natural oligopoly is created. If the innovator auctions off exactly $k$ licensees, in equilibrium, every licensee firm pays its entire Cournot profit to the innovator as winning bid. Noting that the Cournot profit of each firm in the $k$-firm oligopoly is $\left((a - c + \varepsilon)\right)^2/(k + 1)^2$, we conclude that the payoff of the innovator is given by $\Pi_n^A(k, \varepsilon) = k(a - c + \varepsilon)/(k + 1)^2$. The proof of the proposition relies on noting that $(a - c)\varepsilon$ forms the upper bound of payoffs from both auction and royalty.\(^7\) Observe that

\[
(a - c)\varepsilon - \Pi_n^A(k, \varepsilon) = \left[k(a - c) - \varepsilon\right]\left[k\varepsilon - (a - c)\right]/(k + 1)^2 \geq 0,
\]

with equality iff the innovation is exact $k$-drastic [i.e., $\varepsilon = (a - c)/k$]. Thus, for any non-exact $k$-drastic innovation [i.e., $(a - c)/k < \varepsilon < (a - c)/(k - 1)$], we have $\Pi_n^A(k, \varepsilon) < (a - c)\varepsilon$. Let us denote

\[
(a - c)\varepsilon - \Pi_n^A(k, \varepsilon) = \delta_1(\varepsilon). \tag{1}
\]

Note that $\Pi_n^A(k, \varepsilon)$ does not vary with $n$ for any $n \geq k + 1$, rendering $\delta_1(\varepsilon)$ to be a constant with respect to $n$.

Now consider the policy where the innovator auctions off $m \leq k - 1$ licenses. When there are $m \leq k - 1$ licensees for a $k$-drastic innovation, every firm produces positive output. Let $q_0^m(m, \varepsilon)$ and $q_1^m(m, \varepsilon)$ denote the Cournot outputs of a non-licensee and licensee respectively. Then

\[
q_0^m(m, \varepsilon) = (a - c - m\varepsilon)/(n + 1) \quad \text{and} \quad q_1^m(m, \varepsilon) = q_0^m(m, \varepsilon) + \varepsilon. \tag{2}
\]

Let $\pi_n^0(m, \varepsilon)$ and $\pi_n^1(m, \varepsilon)$ denote the respective Cournot profits. Then

\[
\pi_n^0(m, \varepsilon) = \left[q_0^m(m, \varepsilon)\right]^2 \quad \text{and} \quad \pi_n^1(m, \varepsilon) = \left[q_1^m(m, \varepsilon)\right]^2. \tag{3}
\]

\(^7\) The pre-innovation competitive output is $a - c$. Thus, $(a - c)\varepsilon$ is the total reduction in cost from the innovation for the pre-innovation competitive output.
When the innovator auctions off \( m \) licenses, every licensee firm bids \( \pi_1^n(m, \varepsilon) - \pi_0^n(m, \varepsilon) \) and the innovator earns

\[
\Pi_A^n(m, \varepsilon) = m \left[ \pi_1^n(m, \varepsilon) - \pi_0^n(m, \varepsilon) \right].
\]

Observe from (2) that \( \lim_{n \to \infty} q_0^n(m, \varepsilon) = 0 \) and \( \lim_{n \to \infty} q_1^n(m, \varepsilon) = \varepsilon \). Then from (3), it follows that \( \lim_{n \to \infty} \Pi_A^n(m, \varepsilon) = m\varepsilon^2 \). Since \( \varepsilon < (a - c)/(k - 1) \) and \( m \leq k - 1 \), we have \( m\varepsilon^2 \leq (k - 1)\varepsilon^2 < (a - c)\varepsilon \). Let \( (a - c)\varepsilon - (k - 1)\varepsilon^2 = \delta_2(\varepsilon) \). Now we choose a sufficiently small positive constant \( \delta_3(\varepsilon) < \delta_2(\varepsilon) \). Then, there exists \( N_A(\varepsilon) \geq k + 1 \) such that for \( m \leq k - 1 \),

\[
\Pi_A^n(m, \varepsilon) < (k - 1)\varepsilon^2 + \delta_2(\varepsilon) - \delta_3(\varepsilon) = (a - c)\varepsilon - \delta_3(\varepsilon) \quad \text{for all } n \geq N_A(\varepsilon). \tag{4}
\]

Let \( \delta(\varepsilon) = \min\{\delta_1(\varepsilon), \delta_3(\varepsilon)\} \). Then, from (1) and (4), we have the following observation, where \( \Pi_A^n(\varepsilon) \) is the payoff of the innovator from the optimal auction policy.

**Observation 1.** For every non-exact \( k \)-drastic innovation \( \varepsilon \), where \( k \geq 2 \), \( \exists N_A(\varepsilon) \geq k + 1 \) such that \( \Pi_A^n(\varepsilon) \leq (a - c)\varepsilon - \delta(\varepsilon) \) for any oligopoly of size \( n \geq N_A(\varepsilon) \).

Next, consider the royalty policy. The optimal royalty policy for the innovator is to charge \( r = \varepsilon \) (see, e.g., Kamien and Tauman, 1984). In equilibrium, every firm accepts the offer and the payoff of the innovator from the royalty policy in an oligopoly of size \( n \) is given by \( \Pi_R^n(\varepsilon) = n(a - c)\varepsilon/(n + 1) \). Observing that \( \lim_{n \to \infty} \Pi_R^n(\varepsilon) = (a - c)\varepsilon \), we conclude the following.

**Observation 2.** For every non-exact \( k \)-drastic innovation \( \varepsilon \), where \( k \geq 2 \), \( \exists N_R(\varepsilon) \geq k + 1 \) such that \( \Pi_A^n(\varepsilon) > (a - c)\varepsilon - \delta(\varepsilon) \) for any oligopoly of size \( n \geq N_R(\varepsilon) \).

Then by taking \( N(\varepsilon) = \max\{N_A(\varepsilon), N_R(\varepsilon)\} \), the result follows from Observations 1 and 2. \( \Box \)

The following example illustrates the superiority of royalty over auction.

**Example.** Consider a Cournot oligopoly in 20 firms where the demand function is \( Q = 18 - p \), the pre-innovation cost is 8 and the magnitude of the innovation is 6 (i.e., \( n = 20, a = 18, c = 8 \) and \( \varepsilon = 6 \)). This is a non-exact 2-drastic innovation, as \( (a - c)/2 < \varepsilon < a - c \). So, it is a dominated strategy for the innovator to auction off more than 2 licenses. When the innovator auctions off exactly two licenses, the payoff is \( \Pi_A^n(2, \varepsilon) = 2(a - c + \varepsilon)^2/9 = 3584/63 \). When only one license is auctioned, it is \( \Pi_A^n(1, \varepsilon) = (a - c + n\varepsilon)^2/(n + 1)^2 - (a - c - \varepsilon)^2/(n + 1)^2 = 2412/63 \). When the innovator sells the license to all firms charging the rate of royalty \( r = \varepsilon \), the payoff is \( \Pi_R^n(\varepsilon) = n(a - c)\varepsilon/(n + 1) = 3600/63 \), which is more than both \( \Pi_A^n(2, \varepsilon) \) and \( \Pi_A^n(1, \varepsilon) \).

2.5. **Discussion**

It is evident from the proof of Proposition 1 that for an exact \( k \)-drastic innovation \( \varepsilon \) [i.e., \( \varepsilon = (a - c)/k \)], the innovator obtains \( (a - c)\varepsilon \) by auctioning off \( k \) licenses and then
auction dominates royalty. In the existing literature, the analysis has been carried out in terms of \((a - c)/\varepsilon\). For example, Kamien and Tauman (1986, pp. 477–478) stated that when \(n \geq \lfloor 2(a - c)/\varepsilon - 1 \rfloor\), the optimal number of licenses to auction off is \((a - c)/\varepsilon\) and the innovator obtains \((a - c)\varepsilon\). However, \((a - c)/\varepsilon\) is an integer only for exact innovations, so for all but countably many magnitudes of the innovation, it will not be possible for the innovator to sell \((a - c)/\varepsilon\) licenses and hence the payoff would be less than \((a - c)\varepsilon\). The fact that the number of licenses can only be an integer thus plays a crucial role, and once this is taken into consideration, the uniform superiority of fee over royalty no longer holds.

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References


