Web Appendix: Outsourcing Induced by Strategic Competition

Yutian Chen[*] Pradeep Dubey[†] Debapriya Sen[‡]

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[*]Department of Economics, California State University, Long Beach, CA 90840, USA. Email: ychen7@csulb.edu

[†]Center for Game Theory, Department of Economics, SUNY at Stony Brook, NY 11794, USA & Cowles Foundation, Yale University, New Haven, CT 06520, USA. Email: pradeepkdubey@yahoo.com

[‡]Department of Economics, Ryerson University, 380 Victoria Street, Toronto, ON M5B 2K3, Canada. Email: dsen@economics.ryerson.ca

1 Proof of Lemma 1

Lemma 1 *In any Subgame Perfect Nash Equilibrium* (*SPNE*) *of* Γ *or* $\widetilde{\Gamma}$, *we must have* (i) $p_0 \geq c_1$ *and* (ii) $q_1^0 = 0$ (*firm* 1 *does not outsource to firm* 0).

Proof (i) For $\widetilde{\Gamma}$, we have $p_0 \equiv c_0 > c_1$, so consider Γ and suppose $p_0 < c_1$. Since $c_1 < c_0$, firm 0 makes $(p_1 - c_0) < 0$ dollars per unit of the total outsourced order $q_1^0 + q_2^0$ that it receives. If it could be shown that $q_1^0 + q_2^0 > 0$, there would be an immediate contradiction, because firm 0 can in fact ensure zero payoff by deviating from p_0 to some sufficiently high p_0' (e.g., $p_0' > a$), at which price neither firm will outsource anything to it.

To complete the proof, we now show that $q_1^0 + q_2^0 > 0$.

Let $q_2^0 = 0$ (otherwise we are done). If $x_2 > 0$, we must have $q_2^1 > 0$. Then, since $p_0 < c_1$, 1 will pass on this order to 0, i.e., $q_1^0 > 0$.

If $x_2 = 0$, then, as is easily verified, $x_1 > 0$, i.e., $q_1^0 + q_1^1 > 0$. But the cost of producing $q_1^0 + q_1^1$ is $p_0 q_1^0 + c_1 q_1^1$. Since $p_0 < c_1$, optimality requires that $q_1^1 = 0$, so $q_1^0 > 0$.

(ii) If $p_0 > c_1$, then firm 1 will choose $q_1^0 = 0$. If $p_0 = c_1 < c_0$ and $q_1^0 > 0$, then firm 0 obtains a negative payoff. As it can ensure a zero payoff by quoting a sufficiently high price, we must have $q_1^0 = 0$. ■

2 SPNE of Γ and $\widetilde{\Gamma}$

2.1 Preliminary observations

Let x_1, x_2 be the quantities of α produced by firms $1, 2$ and $P(.)$ be the price of α. Recall that the inverse market demand for good α is

$$P(x_1 + x_2) = a - x_1 - x_2 \text{ if } x_1 + x_2 < a \text{ and } P(x_1 + x_2) = 0 \text{ otherwise} \tag{1}$$

Also recall that any terminal node of Γ or $\widetilde{\Gamma}$ is specified by $p \equiv (p_0, p_1)$, $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$ and $x \equiv (x_1, x_2)$ ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$). For $i = 0, 1, 2$, the payoff π_i of firm i is given by

$$\pi_0(p, q) = p_0(q_1^0 + q_2^0) - c_0(q_1^0 + q_2^0) \tag{2}$$

$$\pi_1(p, q, x) = P(x_1 + x_2)x_1 + p_1 q_2^1 - (p_0 q_1^0 + c_1 q_1^1) \text{ and} \tag{3}$$

$$\pi_2(p, q, x) = P(x_1 + x_2)x_2 - (p_0 q_2^0 + p_1 q_2^1) \tag{4}$$

Fix the demand at (1). Let $\mathbb{K}(p_0)$ be the Cournot duopoly game between firms 1 and 2 where 1 has (constant unit) cost c_1 and 2 has cost p_0. We know that $\mathbb{K}(p_0)$ has a unique Nash Equilibrium (NE). For $i = 1, 2$, denote by $\phi_i(p_0)$ the NE profit of firm i in $\mathbb{K}(p_0)$.

Lemma 2 *In any SPNE of* Γ *or* $\widetilde{\Gamma}$, (i) *firm* 0 *obtains at least zero and* (ii) *firm* 1 *obtains at least* $\phi_1(c_0)$.

Proof (i) Follows by noting that firm 0 can always ensure zero payoff by setting a sufficiently high input price (e.g., $p_0 > a$) so that no firm places an order of η with it.

(ii) Observe that firm 1 always has the option of setting a sufficiently high input price (e.g., $p_1 > a$) to ensure that 2 does not order η from 1. For any such p_1, 2 orders η exclusively from 0 and the game $\mathbb{K}(p_0)$ is played in the market α. If $x_2 = 0$ (i.e. 2 supplies nothing in the market α) in the NE of $\mathbb{K}(p_0)$, then firm 1 obtains the monopoly profit which is higher than $\phi_1(c_0)$. If $x_2 > 0$, we must have $p_0 \geq c_0$ (otherwise firm 0 will obtain a negative payoff, contradicting (i)) and 1 obtains $\phi_1(p_0) \geq \phi_1(c_0)$. ■

We apply backward induction to determine SPNE of Γ and $\widetilde{\Gamma}$. We therefore begin from stage II(2) of these games.

2.2 Stage II(2) of Γ and $\widetilde{\Gamma}$

In light of Lemma 1, let $p_0 \geq c_1$. In stage II(2), p_0, p_1, q_2^1 are given ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$) and firms $1, 2$ play the simultaneous-move game $G(p_0, p_1, q_2^1)$. In this game, firm 1 chooses (q_1^0, q_1^1, x_1) subject to (a) $q_1^0 + q_1^1 \geq q_2^1$ and (b) $x_1 \leq q_1^0 + q_1^1 - q_2^1$. Firm 2 chooses (q_2^0, x_2) subject to $x_2 \leq q_2^0 + q_2^1$. Since $q_1^0 = 0$ (Lemma 1), firm 1 produces η entirely by itself at unit cost $c_1 > 0$. Since $p_0 \geq c_1 > 0$, firm 2's unit cost of ordering η from firm 0 is positive. Then, optimality requires that

(i) For firm 1, $q_1^1 = x_1 + q_2^1$ (every unit of η produced by firm 1 is utilized completely either to supply α or to fulfill the order of η for firm 2).

(ii) For firm 2, $q_2^0 = \max\{x_2 - q_2^1, 0\}$. If $x_2 \leq q_2^1$ then $q_2^0 = 0$ (if firm 2's supply of α does not exceed the amount q_2^1 of η that it has ordered from 1, then it does not order η from 0) and if $x_2 > q_2^1$ then $q_2^0 = x_2 - q_2^1$ (if firm 2's supply of α exceeds q_2^1, its order of η from firm 0 equals exactly the additional amount it needs to meet its supply).

By (i) and (ii) above, $G(p_0, p_1, q_2^1)$ reduces to the game where firms 1 and 2 simultaneously choose $x_1, x_2 \geq 0$. By (i) and (3), the payoff of firm 1 is

$$\pi_1(x_1, x_2) = P(x_1 + x_2)x_1 - c_1 x_1 + (p_1 - c_1)q_2^1 \tag{5}$$

By (ii) and (4), the payoff of firm 2 is

$$\pi_2(x_1, x_2) = \begin{cases} P(x_1 + x_2)x_2 - p_1 q_2^1 \text{ if } \ x_2 \leq q_2^1 \\ P(x_1 + x_2)x_2 - p_0 x_2 - (p_1 - p_0)q_2^1 \text{ if } \ x_2 > q_2^1 \end{cases} \tag{6}$$

Observe that the last term in the payoff of both (5) and (6) is a lump-sum upfront transfer between firms 1 and 2 obtained before the game $G(p_0, p_1, q_2^1)$. Ignoring these transfers, $G(p_0, p_1, q_2^1)$ can be viewed as a Cournot duopoly game in the market α where firm 1 has unit cost c_1 and firm 2 has built a commonly known "capacity" q_2^1 prior to the game (paying the sunk cost $p_1 q_2^1$), so that 2's unit cost is 0 if it chooses to supply $x_2 \leq q_2^1$, while it is p_0 if $x_2 > q_2^1$.

Fix the inverse demand at (1) and firm 1's constant unit cost at c_1. For $0 \leq c_2 < a$, let $\mathbb{K}(c_2)$ be the Cournot duopoly game where firm 2 has constant unit cost c_2. In $\mathbb{K}(c_2)$, firm i's unique best response to its rival firm j's quantity x_j is

$$b^{c_i}(x_j) = \begin{cases} (a - c_i - x_j)/2 \text{ if } \ x_j \leq a - c_i \\ 0 \text{ if } \ x_j > a - c_i \end{cases} \tag{7}$$

Let $(k_1(c_2), k_2(c_2))$ be the quantities of firms 1 and 2 in the unique NE of $\mathbb{K}(c_2)$. We know that

$$(k_1(p_0), k_2(p_0)) = \begin{cases} ((a - 2c_1 + p_0)/3, (a + c_1 - 2p_0)/3) \text{ if } c_1 \leq p_0 < (a + c_1)/2 \\ ((a - c_1)/2, 0) \text{ if } p_0 \geq (a + c_1)/2 \end{cases} \tag{8}$$

$$(k_1(0), k_2(0)) = \begin{cases} ((a - 2c_1)/3, (a + c_1)/3) \text{ if } c_1 < a/2 \\ (0, a/2) \text{ if } c_1 \geq a/2 \end{cases} \tag{9}$$

For $i = 1, 2$, denote by $\phi_i(c_2)$ the NE profit of firm i in $\mathbb{K}(c_2)$.

Lemma 3 *The following hold for $G(p_0, p_1, q_2^1)$ where $b^{c_i}(x_j)$ is given by* (7):

(i) *The unique best response of firm 1 to $x_2 \geq 0$ is $b^{c_1}(x_2)$. The unique best response of firm 2 to $x_1 \geq 0$ is* (a) $b^{p_0}(x_1)$ *if $q_2^1 < b^{p_0}(x_1)$,* (b) $b^0(x_1)$ *if $q_2^1 > b^0(x_1)$ and* (c) q_2^1 *if $b^{p_0}(x_1) \leq q_2^1 \leq b^0(x_1)$.*

(ii) *If (x_1, x_2) is an NE, then* (a) $x_1 = b^{c_1}(x_2)$, (b) $x_2 = b^{p_0}(x_1)$ *if $x_2 > q_2^1$,* (c) $x_2 = b^0(x_1)$ *if $x_2 < q_2^1$ and* (d) $b^{p_0}(x_1) \leq x_2 \leq b^0(x_1)$ *if $x_2 = q_2^1$.*

(iii) (a) *If $q_2^1 \leq k_2(p_0)$, there is no NE where $x_2 < q_2^1$ and* (b) *if $q_2^1 \geq k_2(0)$, there is no NE where $x_2 > q_2^1$.*

Proof (i) The first part is direct by (5). To determine firm 2's best response(s), denote

$$m(x_1) := \min\{b^0(x_1), q_2^1\} \text{ and } M(x_1) := \max\{b^{p_0}(x_1), q_2^1\} \tag{10}$$

By (6), for $x_1 \geq 0$, the unique optimal strategy of firm 2 over $x_2 \in [0, q_2^1]$ is $m(x_1)$ while over $x_2 \in [q_2^1, \infty)$, it is $M(x_1)$. As $b^{p_0}(x_1) \leq b^0(x_1)$ and $x_2 = q_2^1$ is feasible for both $[0, q_2^1]$ and $[q_2^1, \infty)$, (a)-(c) follow by (10).

(ii) Follows by (i).

(iii)(a) If (x_1, x_2) is an NE where $x_2 < q_2^1$, then by (ii)(a) and (c), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^0(x_1)$. The unique solution to this system is $x_1 = k_1(0)$ and $x_2 = k_2(0) > k_2(p_0) \geq q_2^1$, contradicting $x_2 < q_2^1$.

(iii)(b) If (x_1, x_2) is an NE where $x_2 > q_2^1$, then by (ii)(a) and (b), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^{p_0}(x_1)$. The unique solution to this system is $x_1 = k_1(p_0)$ and $x_2 = k_2(p_0) < k_2(0) \leq q_2^1$, contradicting $x_2 > q_2^1$. ■

Lemma SII(2) (Stage II(2)) (i) *$G(p_0, p_1, q_2^1)$ has a unique NE where $q_1^0 = 0$, $q_1^1 = x_1 + q_2^1$ and which is given as follows*:

(a) (*Small capacity*) *If $q_2^1 < k_2(p_0)$, then $x_1 = k_1(p_0)$, $x_2 = k_2(p_0)$ and $q_2^0 = k_2(p_0) - q_2^1$;*

(b) (*Intermediate capacity*) *If $k_2(p_0) \leq q_2^1 \leq k_2(0)$, then $x_1 = b^{c_1}(q_2^1)$, $x_2 = q_2^1$ and $q_2^0 = 0$;*

(c) (*Large capacity*) *If $q_2^1 > k_2(0)$, then $x_1 = k_1(0)$, $x_2 = k_2(0)$ and $q_2^0 = 0$.*

(ii) *Suppose* $p_0 \geq (a+c_1)/2$. *Then the NE of* $G(p_0, p_1, q_2^1)$ *is invariant of* p_0. *Hence w.l.o.g. we may restrict* $p_0 \leq (a+c_1)/2$.

Proof (i)(a) Let $0 \leq q_2^1 < k_2(p_0)$. First we show that $(k_1(p_0), k_2(p_0))$ is an NE. Clearly $k_1(p_0)$ is (the unique) best response of firm 1 to $k_2(p_0)$. Since $b^{p_0}(k_1(p_0)) = k_2(p_0) > q_2^1$, $k_2(p_0)$ is (the unique) best response of firm 2 to $k_1(p_0)$.

To prove the uniqueness, note by Lemma 3(iii)(a) that if (x_1, x_2) is an NE, we must have $x_2 \geq q_2^1$.

If (x_1, q_2^1) is an NE, then by Lemma 3(ii)(a) and (d), $x_1 = b^{c_1}(q_2^1)$ and $q_2^1 \geq b^{p_0}(x_1) = b^{p_0}(b^{c_1}(q_2^1))$. Since $x_2 \lesseqqgtr k_2(p_0) \Leftrightarrow x_2 \lesseqqgtr b^{p_0}(b^{c_1}(x_2))$, we have $q_2^1 \geq k_2(p_0)$, a contradiction.

Hence if (x_1, x_2) is an NE, then $x_2 > q_2^1$ and by Lemma 3(ii)(a)-(b), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^{p_0}(x_1)$. The unique solution of this system has $x_1 = k_1(p_0)$ and $x_2 = k_2(p_0)$, completing the proof.

(i)(b) Let $k_2(p_0) \leq q_2^1 \leq k_2(0)$. Since for $c_2 \in \{0, p_0\}$, $x_2 \lesseqqgtr k_2(c_2) \Leftrightarrow x_2 \lesseqqgtr b^{c_2}(b^{c_1}(x_2))$, we have $b^{p_0}(b^{c_1}(q_2^1)) \leq q_2^1 \leq b^0(b^{c_1}(q_2^1))$ and by Lemma 3(i) it follows that $(b^{c_1}(q_2^1), q_2^1)$ is an NE. The uniqueness follows from Lemma 3(ii)(a)-(d) by noting that for this case there is no NE where $x_2 \neq q_2^1$.

(i)(c) Let $q_2^1 > k_2(0)$. First we show that $(k_1(0), k_2(0))$ is an NE. Clearly $k_1(0)$ is the unique best response of firm 1 to $k_2(0)$. Since $b^0(k_1(0)) = k_2(0) < q_2^1$, by (i)(b), $k_2(0)$ is the unique best response of firm 2 to $k_1(0)$.

To prove the uniqueness, note by Lemma 3(iii)(b) that if (x_1, x_2) is an NE, we must have $x_2 \leq q_2^1$.

If (x_1, q_2^1) is an NE, then by Lemma 3(ii)(a) and (d), $x_1 = b^{c_1}(q_2^1)$ and $q_2^1 \leq b^0(x_1) = b^0(b^{c_1}(q_2^1))$. Since $x_2 \lesseqqgtr k_2(0) \Leftrightarrow x_2 \lesseqqgtr b^0(b^{c_1}(x_2))$, we have $q_2^1 \leq k_2(0)$, a contradiction.

Hence if (x_1, x_2) is an NE, then $x_2 < q_2^1$ and by Lemma 3(ii)(a) and (c), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^0(x_1)$. The unique solution of this system has $x_1 = k_1(0)$ and $x_2 = k_2(0)$, completing the proof.

(ii) If $p_0 \geq (a+c_1)/2 > c_1$, then $q_1^0 = 0$ and in the NE of $G(p_0, p_1, q_2^1)$, $q_2^0 > 0$ only if $q_2^1 \in [0, k_2(p_0))$ [part(i)]. Since $k_2(p_0) = 0$ for $p_0 \geq (a+c_1)/2$ [by (8)], we have $q_1^0 + q_2^0 = 0$ for $p_0 \geq (a+c_1)/2$, yielding zero payoff for firm 0. This proves (ii). ■

Lemma SII(2) shows that for firm 2, building a capacity that is too large ($q_2^1 > k_2(0)$) leads to some part of it being unutilized while a capacity that is too small ($q_2^1 < k_2(p_0)$) provides it no strategic advantage. Intermediate capacities ($k_2(p_0) \leq q_2^1 \leq k_2(0)$) are fully utilized and under such capacities, firm 2 does not order η from firm 0. For these capacities, firm 2's supply in the final good market α exactly equals its capacity (i.e. $x_2 = q_2^1$). Such intermediate capacities constitute a credible commitment from 2 to 1 that establishes firm 2 as the Stackelberg leader in the NE of $G(p_0, p_1, q_2^1)$.

2.3 Stage II(1) of Γ and $\widetilde{\Gamma}$: The leadership premium

Any node in stage II(1) corresponds to a specific price pair $(p_0, p_1) \equiv p$ (for $\widetilde{\Gamma}$, $p_0 \equiv c_0$). This is stage 1 of the game $G(p_0, p_1)$ where firm 2 chooses $q_2^1 \geq 0$. Any such q_2^1 results in the game $G(p_0, p_1, q_2^1)$, whose unique NE is characterized in Lemma SII(2). By (6) and Lemma

SII(2), the payoff of firm 2 at the unique NE of $G(p_0, p_1, q_2^1)$ is[1]

$$\pi_2^p(q_2^1) = \begin{cases} \phi_2(p_0) + (p_0 - p_1)q_2^1 \text{ if } q_2^1 < k_2(p_0) \\ P\left(b^{c_1}(q_2^1) + q_2^1\right) q_2^1 - p_1 q_2^1 \text{ if } k_2(p_0) \leq q_2^1 \leq k_2(0) \\ \phi_2(0) - p_1 q_2^1 \text{ if } q_2^1 > k_2(0) \end{cases} \tag{11}$$

Therefore in stage II(1), firm 2 solves the single-person decision problem of choosing $q_2^1 \geq 0$ to maximize $\pi_2^p(q_2^1)$.

Fix the inverse demand at (1) and the constant unit cost of firm 1 at c_1. Let $\mathbb{S}(p_1)$ be the Stackelberg duopoly with firm 2 as the leader and firm 1 the follower, where 2 has constant unit cost p_1. We know that $\mathbb{S}(p_1)$ has a unique SPNE. Let $(\widetilde{s}_1(p_1), \widetilde{s}_2(p_1))$ be the quantities of firms 1 and 2 in the SPNE of $\mathbb{S}(p_1)$. At the SPNE, let $\ell(p_1)$ be the profit of the leader (firm 2) and $f(p_1)$ the profit of the follower (firm 1).

Note from (11) that for $q_2^1 \in [k_2(p_0), k_2(0)]$, firm 2 solves the *constrained* problem of the Stackelberg leader in $\mathbb{S}(p_1)$, where 2 is restricted to choose its output in the interval $[k_2(p_0), k_2(0)]$. It will be useful to define

$$s_2(p_1) := \min\{\widetilde{s}_2(p_1), k_2(0)\} \text{ and } s_1(p_1) := b^{c_1}\left(s_2(p_1)\right) = \max\{\widetilde{s}_1(p_1), k_1(0)\} \tag{12}$$

Recall that $(k_1(p_0), k_2(p_0))$ (given in (8)) is the unique NE of $\mathbb{K}(p_0)$.

Definition In the game $G(p_0, p_1)$, the *Cournot outcome* is played if $(x_1, x_2) = (k_1(p_0), k_2(p_0))$ and the *Stackelberg outcome* is played if $(x_1, x_2) = (s_1(p_1), s_2(p_1))$.

Lemma 4 *In any SPNE of* Γ *or* $\widetilde{\Gamma}$:

(i) *If* $0 < p_1 \leq p_0$, *then firm 2 chooses* $q_2^1 = s_2(p_1)$.

(ii) $p_1 > c_1$.

(iii) *If* $p_1 \geq (a + c_1)/2$, *then* $q_2^1 = 0$.

Proof (i) Note from (11) that if $p_1 > 0$, then it is not optimal for firm 2 to choose $q_2^1 > k_2(0)$, so let $q_2^1 \leq k_2(0)$. By lemmas 1 and SII(2)(ii), $p_0 \in [c_1, (a + c_1)/2]$. If $p_0 = (a + c_1)/2$, then by (8), $k_2(p_0) = 0$ and the result is immediate from (11).

Now let $p_0 < (a + c_1)/2$. Then $s_2(p_1) \geq \widetilde{s}_2(p_1) > k_2(p_0) > 0$ for $p_1 \leq p_0$. As the unconstrained maximum of $\pi_2^p(q_2^1)$ over $q_2^1 \in [k_2(p_0), k_2(0)]$ is attained at $q_2^1 = \widetilde{s}_2(p_1)$, using (12), its constrained maximizer over $q_2^1 \in [k_2(p_0), k_2(0)]$ is $q_2^1 = s_2(p_1)$ and $\pi_2^p(s_2(p_1)) > \pi_2^p(k_2(p_0))$. Noting by (11) that for $q_2^1 \leq k_2(p_0)$, $\pi_2^p(q_2^1)$ is either increasing (if $p_1 < p_0$) or constant (if $p_1 = p_0$), it follows that the unique global optimal choice for firm 2 in stage II(1) is $q_2^1 = s_2(p_1)$.

(ii) We know that firm 1 obtains at least $\phi_1(c_0)$ in any SPNE (Lemma 2(ii)). If $p_1 \leq c_1$, then firm 1 does not obtain any positive profit as a supplier of η. In what follows, we will show that if $p_1 \leq c_1$, firm 1's profit in the final good market α cannot exceed $\phi_1(c_1)$, which is lower than $\phi_1(c_0)$. This will prove that in any SPNE, we must have $p_1 > c_1$.

[1] Fix the inverse demand at (1) and firm 1's constant unit cost at c_1. Recall that for $c_2 \in \{p_0, 0\}$, the Cournot duopoly game where firm 2 has constant unit cost c_2 is denoted by $\mathbb{K}(c_2)$. For $i = 1, 2$, the NE profit of firm i in $\mathbb{K}(c_2)$ is denoted by $\phi_i(c_2)$.

By Lemma 1, $p_0 \geq c_1$. Hence if $p_1 \leq c_1$, we have $p_1 \leq p_0$ and $s_2(p_1) \geq \widetilde{s}_2(p_1) > k_2(p_0) > 0$, so that $s_2(p_1) \in (k_2(p_0), k_2(0)]$. We consider two possibilities: $p_1 = 0$ and $p_1 > 0$.

If $p_1 = 0$, then by (11), $\pi_2^p(q_2^1)$ is increasing for $q_2^1 < k_2(p_0)$, constant for $q_2^1 > k_2(0)$ and its unique maximum over $q_2^1 \in [k_2(p_0), k_2(0)]$ is attained at $s_2(p_1) \in (k_2(p_0), k_2(0)]$. As $s_2(p_1) = \min\{\widetilde{s}_2(p_1), k_2(0)\}$, for this case it is optimal for firm 2 to choose either $q_2^1 = \widetilde{s}_2(p_1)$ or some $q_2^1 \geq k_2(0)$. If $p_1 > 0$, then by part (i), it is optimal for firm 2 to choose $q_2^1 = s_2(p_1)$, which is either $\widetilde{s}_2(p_1)$ or $k_2(0)$. Hence we conclude that if $p_1 \leq c_1$, firm 2 chooses either $q_2^1 = \widetilde{s}_2(p_1)$ or some $q_2^1 \geq k_2(0)$.

If $q_2^1 = \widetilde{s}_2(p_1) > k_2(p_0)$, firm 2 will supply $x_2 = q_2^1 = \widetilde{s}_2(p_1)$ in the market α (Lemma SII(2)(b)) and firm 1's (Stackelberg follower) profit there would be $f(p_1) \leq f(c_1) \leq \phi_1(c_1)$.[2] If $q_2^1 \geq k_2(0)$, then 2 will supply $x_2 = k_2(0)$ (Lemma SII(2)(c)) and firm 1's profit in the market α would be $\phi_1(0) < \phi_1(c_1)$.

(iii) By (11), it is not optimal for firm 2 to choose $q_2^1 > k_2(0)$ for any positive p_1, so let $q_2^1 \leq k_2(0)$. Note that $(a+c_1)/2$ is the monopoly price under unit cost c_1. For $p_1 \geq (a+c_1)/2$, the SPNE of $\mathbb{S}(p_1)$ is $(\widetilde{s}_1(p_1), \widetilde{s}_2(p_1)) = ((a-c_1)/2, 0)$ (i.e. firm 2 supplies zero output and firm 1 supplies the monopoly output $(a-c_1)/2$). Using this in (11), for $q_2^1 \in [k_2(p_0), k_2(0)]$, the unconstrained maximizer of $\pi_2^p(q_2^1)$ is $q_2^1 = 0 \leq k_2(p_0)$. Thus, $\pi_2^p(q_2^1)$ is decreasing for $q_2^1 \in [k_2(p_0), k_2(0)]$, so consider $q_2^1 \leq k_2(p_0)$. If $p_0 \geq (a+c_1)/2$, then by (8), $k_2(p_0) = 0$ and the optimal choice for firm 2 is $q_2^1 = 0$. If $p_0 < (a+c_1)/2 \leq p_1$, then by (11), $\pi_2^p(q_2^1)$ is decreasing for $q_2^1 \in [0, k_2(p_0)]$, so the optimal choice is again $q_2^1 = 0$. ■

In the light of Lemma 4(ii), consider $p_1 > c_1 > 0$. Then the SPNE of $\mathbb{S}(p_1)$ is

$$(\widetilde{s}_1(p_1), \widetilde{s}_2(p_1)) = \begin{cases} ((a-3c_1+2p_1)/4, (a+c_1)/2 - p_1) \text{ if } c_1 < p_1 < (a+c_1)/2 \\ ((a-c_1)/2, 0) \text{ if } p_1 \geq (a+c_1)/2 \end{cases} \tag{13}$$

Using (12) and (13), by standard computations it follows that

$$(s_1(p_1), s_2(p_1)) = \begin{cases} (k_1(0), k_2(0)) \text{ if } c_1 < a/2 \text{ and } p_1 \leq (a+c_1)/6 \\ (\widetilde{s}_1(p_1), \widetilde{s}_2(p_1)) \text{ otherwise} \end{cases} \tag{14}$$

Recall that at the SPNE of $\mathbb{S}(p_1)$, the profit of firm 2 (the leader) is denoted by $\ell(p_1)$ and the profit of firm 1 (the follower) is denoted by $f(p_1)$. If firm 2 chooses $q_2^1 = s_2(p_1)$, then it obtains the (possibly constrained) Stackelberg leader profit. By (14), this profit is

$$L(p_1) := \begin{cases} \phi_2(0) - p_1 k_2(0) \text{ if } c_1 < a/2 \text{ and } p_1 \leq (a+c_1)/6 \\ \ell(p_1) \text{ otherwise} \end{cases} \tag{15}$$

Firm 1's payoff has two components: (i) the Stackelberg follower's profit and (ii) its supplier revenue $(p_1 - c_1)s_2(p_1)$. Using (14), this payoff is

$$F(p_1) := \begin{cases} \phi_1(0) + (p_1 - c_1)k_2(0) \text{ if } c_1 < a/2 \text{ and } p_1 \leq (a+c_1)/6 \\ f(p_1) + (p_1 - c_1)s_2(p_1) \text{ otherwise} \end{cases} \tag{16}$$

As $p_1 > c_1$ and $p_0 \geq c_1$, we consider two possibilities: $c_1 < p_1 \leq p_0$ and $c_1 \leq p_0 < p_1$.

[2] As $f(c_1)$ is firm 1's Stackelberg follower profit in $\mathbb{S}(c_1)$ and $\phi_1(c_1)$ is its Cournot profit in $\mathbb{K}(c_1)$, we have $f(c_1) \leq \phi_1(c_1)$.

If $c_1 < p_1 \leq p_0$, then by Lemma 4(i), the Stackelberg outcome is played in the unique SPNE of $G(p_0, p_1)$, i.e., firm 2 chooses $q_2^1 = s_2(p_1)$, supplies $x_2 = s_2(p_1)$ in the market α and acquires the (possibly constrained) Stackelberg leadership position.

If $c_1 \leq p_0 < p_1$, then it follows by (11) that (i) it is not optimal for 2 to choose $q_2^1 > k_2(0)$ and (ii) over $q_2^1 \leq k_2(p_0)$, it is optimal to choose $q_2^1 = 0$. If 2 chooses $q_2^1 = 0$, then the Cournot duopoly game $\mathbb{K}(p_0)$ is played in the market α where firm 1 obtains $\phi_1(p_0)$ and firm 2 obtains $\phi_2(p_0)$. Firm 0 supplies $q_2^0 = k_2(p_0)$ units of η to firm 2 at price p_0, so it obtains

$$\psi(p_0) = (p_0 - c_0)k_2(p_0) \tag{17}$$

If 2 chooses $q_2^1 \in [k_2(p_0), k_2(0)]$ by paying the unit price $p_1 > p_0$ for the capacity q_2^1, it can acquire the (possibly constrained) Stackelberg leadership position in the market α.

Firm 2 determines optimal q_2^1 by comparing its Stackelberg leader profit with the Cournot profit $\phi_2(p_0)$. Lemma SII(1) shows that there is a function $\tau(p_0) \in [p_0, (a+c_1)/2]$ (representing the *leadership premium*) such that 2 prefers to be the Stackelberg leader as long as $p_1 < \tau(p_0)$.

Define $\tau_1, \tau_2 : [c_1, (a + c_1)/2] \to R_+$ as

$$\tau_1(p_0) := 4p_0(a + c_1 - p_0)/3(a + c_1) \text{ and } \tau_2(p_0) := (3 - 2\sqrt{2})(a + c_1)/6 + 2\sqrt{2}p_0/3 \tag{18}$$

Denote

$$\overline{\theta}(c_1) := (\sqrt{2} - 1)(a + c_1)/2\sqrt{2} \tag{19}$$

Define the function $\tau(p_0)$ as

$$\tau(p_0) := \begin{cases} \tau_1(p_0) \text{ if } p_0 < \overline{\theta}(c_1) \\ \tau_2(p_0) \text{ if } p_0 \geq \overline{\theta}(c_1) \end{cases} \tag{20}$$

Standard computations show that (i) $\tau(p_0)$ is continuous and increasing, (ii) $\tau(p_0) > p_0$ for $p_0 \in [c_1, (a + c_1)/2)$ and (iii) $\tau((a + c_1)/2) = (a + c_1)/2$.

Lemma SII(1) (Stage II(1)) (Leadership premium) $\exists$ *a function* $\tau : [c_1, (a+c_1)/2] \to R_+$ *(given in* (20)*), such that for* $p_1 \geq c_1$ *and* $p_0 \in [c_1, (a + c_1)/2]$:

(i) *In any SPNE of* $G(p_0, p_1)$, $q_2^0 q_2^1 = 0$ *(firm* 2 *orders* η *either exclusively from firm* 0 *or exclusively from firm* 1*).*

(ii) *If* $p_1 < \tau(p_0)$, *the Stackelberg outcome is played in the unique SPNE of* $G(p_0, p_1)$ *where* $x_2 = q_2^1 = s_2(p_1)$, $x_1 = q_1^1 - q_2^1 = s_1(p_1)$ *and* $q_1^0 = q_2^0 = 0$. *Firm* 0 *obtains zero payoff, firm* 1 *obtains* $F(p_1)$ *and firm* 2 *obtains* $L(p_1)$.

(iii) *If* $p_1 > \tau(p_0)$, *the Cournot outcome is played in the unique SPNE of* $G(p_0, p_1)$ *where* $x_2 = q_2^0 = k_2(p_0)$, $x_1 = q_1^1 = k_1(p_0)$ *and* $q_1^0 = q_2^1 = 0$. *Firm* 0 *obtains* $\psi(p_0)$, *firm* 1 *obtains* $\phi_1(p_0)$ *and firm* 2 *obtains* $\phi_2(p_0)$.

(iv) *If* $p_1 = \tau(p_0)$, $G(p_0, p_1)$ *has two SPNE: the Stackelberg outcome is played in one and the Cournot outcome is played in the other.*

Proof First we prove (ii)-(iv). Part (i) follow immediately from (ii)-(iv). Denote $A_1(p) := [0, k_2(p_0)]$, $A_2(p) := [k_2(p_0), k_2(0)]$ and $A(p) = A_1(p) \cup A_2(p)$. It follows from (11) that for

$p_1 \geq c_1 > 0$, it is not optimal for firm 2 to choose $q_2^1 > k_2(0)$. Therefore any optimal q_2^1 belongs to the set $A(p)$. Let

$$A_t^*(p) := \arg\max_{q_2^1 \in A_t(p)} \pi_2^p(q_2^1) \text{ for } t = 1,2 \text{ and } A^*(p) := \arg\max_{q_2^1 \in A(p)} \pi_2^p(q_2^1)$$

Then $A^*(p) \subseteq A_1^*(p) \cup A_2^*(p)$. We prove the result by showing that $A^*(p) = \{0\}$ if $p_1 < \tau(p_0)$, $A^*(p) = \{s_2(p_1)\}$ if $p_1 > \tau(p_0)$ and $A^*(p) = \{0, s_2(p_1)\}$ if $p_1 = \tau(p_0)$.

By Lemma 4(i), $A^*(p) = \{s_2(p_1)\}$ if $p_1 \leq p_0$. So consider $p_1 > p_0$. Then it follows from (11) that $A_1^*(p) = \{0\}$. Denote

$$g(p_0) := (2/3)p_0 + (1/3)(a + c_1)/2$$

If $p_0 = (a + c_1)/2$, then $k_2(p_0) = 0$ and $A(p) = A_2(p)$, so that $A^*(p) = A_2^*(p) = \{s_2(p_1)\}$. Since $g(p_0) = \tau(p_0) = (a + c_1)/2$ for $p_0 = (a + c_1)/2$, the proof for this case is complete.

Next suppose $p_1 > p_0$ and $p_0 < (a + c_1)/2$, so that $g(p_0) > p_0$. Then there are two possibilities: $p_1 \geq g(p_0)$ and $p_1 \in (p_0, g(p_0))$.

If $p_1 \geq g(p_0)$, we have $s_2(p_1) \leq k_2(p_0)$. Hence $A_2^*(p) = \{k_2(p_0)\}$. Thus, $k_2(p_0) \in A_2^*(p) \cap A_1(p)$ but $k_2(p_0) \notin A_1^*(p) = \{0\}$. Therefore for this case, $A^*(p) = A_1^*(p) = \{0\}$.

If $p_1 \in (p_0, g(p_0))$, then $A_1^*(p) = \{0\}$ and $A_2^*(p) = \{s_2(p_1)\}$. Hence $A^*(p) \subseteq \{0, s_2(p_1)\}$. Note that $\pi_2^p(0) = \phi_2(p_0) = (a + c_1 - 2p_0)^2/9$ and $\pi_2^p(s_2(p_1)) = L(p_1)$ (given in (15)). Therefore, to determine $A^*(p)$, we have to compare $\phi_2(p_0)$ and $L(p_1)$.

Using (13) and (14) in (15), we have

$$L(p_1) = \begin{cases} \widehat{\ell}(p_1) = (a + c_1)^2/9 - p_1(a + c_1)/3 \ \text{ if } c_1 < a/2 \text{ and } p_1 \leq (a + c_1)/6 \\ \ell(p_1) = (a + c_1 - 2p_1)^2/8 \ \text{ otherwise} \end{cases} \tag{21}$$

Comparing $\phi_2(p_0) = (a + c_1 - 2p_0)^2/9$ with $\widehat{\ell}(p_1)$ and $\ell(p_1)$ we have the following where τ_1, τ_2 are given in (18).

$$\widehat{\ell}(p_1) \gtreqless \phi_2(p_0) \Leftrightarrow p_1 \lesseqgtr \tau_1(p_0) \text{ and } \ell(p_1) \gtreqless \phi_2(p_0) \Leftrightarrow p_1 \lesseqgtr \tau_2(p_0) \tag{22}$$

There are following possible cases, where $\overline{\theta}(c_1)$ is given by (19).

Case 1(a) If $c_1 < a/2$ and $p_0 \geq (a + c_1)/6 > \overline{\theta}(c_1)$, then by (20), $\tau(p_0) = \tau_2(p_0)$. Since $p_1 > p_0$, under this case we have $p_1 > (a + c_1)/6$.

Case 1(b) If $c_1 \geq a/2$, then by (19), $\overline{\theta}(c_1) < c_1 \leq p_0$ and again $\tau(p_0) = \tau_2(p_0)$.

Observe by (21) that if either 1(a) or 1(b) holds, then $L(p_1) = \ell(p_1)$. Hence by (22), $L(p_1) \gtreqless \phi_2(p_0) \Leftrightarrow p_1 \lesseqgtr \tau_2(p_0) = \tau(p_0)$. This proves the result for Case 1.

Case 2 $c_1 < a/2$ and $p_0 < (a + c_1)/6$:

Case 2(a) If $p_0 \leq \overline{\theta}(c_1)$, then $\tau(p_0) = \tau_1(p_0) \leq (a + c_1)/6$ and $\tau_2(p_0) \leq (a + c_1)/6$ [by (20)].

(i) If $p_1 \in (p_0, (a + c_1)/6]$, then by (21), $L(p_1) = \widehat{\ell}(p_1)$. Hence by (22), $L(p_1) \gtreqless \phi_2(p_0) \Leftrightarrow p_1 \lesseqgtr \tau_1(p_0) = \tau(p_0)$.

(ii) If $p_1 \in ((a + c_1)/6, g(p_0)]$, then by (21), $L(p_1) = \ell(p_1)$. Hence by (22), $L(p_1) < \phi_2(p_0)$ for $p_1 > (a + c_1)/6 \geq \tau_2(p_0)$.

The result for Case 2(a) follows by (i) and (ii) above.

Case 2(b) If $p_0 \in (\overline{\theta}(c_1), (a+c_1)/6)$, then $\tau(p_0) = \tau_2(p_0) > (a+c_1)/6$ and $\tau_1(p_0) > (a+c_1)/6$ [by (20)].

(i) If $p_1 \in (p_0, (a+c_1)/6]$, then by (21), $L(p_1) = \widehat{\ell}(p_1)$. Hence by (22), $L(p_1) > \phi_2(p_0)$ for $p_1 \leq (a+c_1)/6 < \tau_1(p_0)$.

(ii) If $p_1 \in ((a+c_1)/6, g(p_0))$, then by (21), $L(p_1) = \ell(p_1)$. Hence by (22), $L(p_1) \gtreqless \phi_2(p_0) \Leftrightarrow p_1 \lesseqgtr \tau_2(p_0) = \tau(p_0)$.

The result for Case 2(b) follows by (i) and (ii) above. ■

2.4 Stage I of Γ and $\widetilde{\Gamma}$

Now we go to the first stage of Γ and $\widetilde{\Gamma}$ where firms 0 and 1 simultaneously announce prices p_0, p_1 ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$) that result in the game $G(p_0, p_1)$, whose SPNE are characterized in Lemma S(II)(1). Lemma 5 summarizes some properties of the functions $\psi(p_0)$ (firm 0's payoff under the Cournot outcome, given by (17)) and $F(p_1)$ (firm 1's payoff under the Stackelberg outcome, given by (16)).

Define

$$\theta_0(c_1, c_0) := c_0/2 + (a+c_1)/4 \in (c_0, (a+c_1)/2) \tag{23}$$

$$\widehat{\theta}_1(c_1) := \left[a + 4c_1 - \sqrt{a^2 - 7ac_1 + c_1^2}\right]/5 \text{ and } \widehat{\theta}_2(c_1) := a/14 + 13c_1/14 \tag{24}$$

Observe that $\widehat{\theta}_2(c_1) \in (c_1, \theta_0(c_1, c_0))$ for $c_1 < a$ and

$$\widehat{\theta}_2(c_1) \gtreqless \overline{\theta}(c_1) \Leftrightarrow c_1 \gtreqless \underline{\rho}a \text{ where } \underline{\rho} \equiv 23/[121 + 84\sqrt{2}] \in (0, 1/2) \tag{25}$$

Also note that for $c_1 < \underline{\rho}a$, $\widehat{\theta}_1(c_1)$ is real and $c_1 < \widehat{\theta}_1(c_1) < \overline{\theta}(c_1) < \theta_0(c_1, c_0)$. Define

$$\widehat{\theta}(c_1) := \begin{cases} \widehat{\theta}_1(c_1) \text{ if } c_1 < \underline{\rho}a \\ \widehat{\theta}_2(c_1) \text{ if } c_1 \geq \underline{\rho}a \end{cases} \tag{26}$$

Observe that $\widehat{\theta}(c_1)$ is continuous and

$$\widehat{\theta}(c_1) \lesseqgtr \overline{\theta}(c_1) \Leftrightarrow c_1 \lesseqgtr \underline{\rho}a \tag{27}$$

Lemma 5 *There are functions $\theta_0(c_0, c_1) \in (c_0, (a+c_1)/2)$ (given in (23)) and $\widehat{\theta}(c_1) \in (c_1, \theta_0(c_0, c_1))$ (given in (26)) such that*

(i) *$\psi(p_0)$ is increasing for $p_0 \in [c_1, \theta_0)$, decreasing for $p_0 \in (\theta_0, (a+c_1)/2]$ and its unique maximum is attained at $p_0 = \theta_0$.*

(ii) *$\psi(c_0) = \psi((a+c_1)/2) = 0$, $\psi(p_0) < 0$ if $p_0 \in [c_1, c_0)$ and $\psi(p_0) > 0$ if $p_0 \in (c_0, (a+c_1)/2)$.*

(iii) *$F(\theta_0) > \phi_1(\theta_0)$.*

(iv) *$F(p_1)$ is increasing for $p_1 \in [0, (a+c_1)/2]$.*

(v) *For* $p_0 \in [c_1, \theta_0]$, $F(\tau(p_0)) \gtreqqless \phi_1(p_0) \Leftrightarrow p_0 \gtreqqless \widehat{\theta}$.

Proof Parts (i)-(ii) follow from (17) by noting that $k_2(p_0) = (a + c_1 - 2p_0)/3$ for $p_0 \in [c_1, (a + c_1)/2]$.

(iii) Noting that $\theta_0 > (a + c_1)/6$, by (16), we have $F(p_1) = f(p_1) + (p_1 - c_1)s_1(p_1)$. As $c_1 < c_0 < \theta_0 < (a+c_1)/2$, by (13), $F(\theta_0) = (3a+2c_0-5c_1)^2/64+(a+2c_0-3c_1)(a+c_1-2c_0)/16$. As $\phi_1(\theta_0) = (5a + 2c_0 - 7c_1)^2/144$ and $a > c_0 > c_1$, we have

$$F(\theta_0) - \phi_1(\theta_0) = (17a + 62c_0 - 79c_1)(a + c_1 - 2c_0)/576 > 0$$

This proves (iii).

(iv) Follows by standard computations by using (13) and (16).

(v) First let $p_0 \geq \overline{\theta}(c_1)$. Then by (20), $\tau(p_0) = \tau_2(p_0) \geq (a + c_1)/6$. Hence by (13) and (16), $F(\tau(p_0)) = (a - 3c_1 + 2\tau(p_0))^2/16 + (\tau(p_0) - c_1)[(a + c_1)/2 - \tau(p_0)]$. Comparing it with $\phi_1(p_0) = (a - 2c_1 + p_0)^2/9$, we have the following where $\widehat{\theta}_2(c_1)$ is given by (24).

$$\text{For } p_0 \geq \overline{\theta}(c_1), \;\; \phi_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow p_0 \lesseqqgtr \widehat{\theta}_2(c_1) \tag{28}$$

Next observe that for $p_0 < \overline{\theta}(c_1)$,

$$\overline{\theta}(c_1) \gtreqqless c_1 \Leftrightarrow c_1 \lesseqqgtr \overline{\rho}a \text{ where } \overline{\rho} \equiv 1/(3 + 2\sqrt{2}) \in (0, 1/2) \text{ and } \overline{\rho} > \underline{\rho} \tag{29}$$

Case 1 $c_1 \geq \overline{\rho}a$: Then by (29), $[c_1, (a + c_1)/2) \subseteq [\overline{\theta}(c_1), (a + c_1)/2)$. As $\underline{\rho} < \overline{\rho}$, for this case we have $\widehat{\theta}(c_1) = \widehat{\theta}_2(c_1)$ [by (26)] and the result follows by (28).

Case 2 $c_1 < \overline{\rho}a$: Then by (29), $[c_1, (a + c_1)/2) = [c_1, \overline{\theta}(c_1)) \cup [\overline{\theta}(c_1), (a + c_1)/2)$.

If $p_0 \in [c_1, \overline{\theta}(c_1)]$, then by (20), $\tau(p_0) = \tau_1(p_0) < (a + c_1)/6$. Since $c_1 < \overline{\rho}a < a/2$, by (9) and (16), $F(\tau(p_0)) = (a - 2c_1)^2/9 + (\tau(p_0) - c_1)(a + c_1)/3$. Comparing it with $\phi_1(p_0) = (a - 2c_1 + p_0)^2/9$, we have

$$\phi_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow w(p_0) \gtreqqless 0 \text{ where } w(p_0) := 5p_0^2 - 2(a + 4c_1)p_0 + 3c_1(a + c_1) \tag{30}$$

Noting that (i) $w(p_0)$ is decreasing for $p_0 \in [c_1, \overline{\theta}(c_1)]$, (ii) $w(c_1) > 0$ and (iii) $w(\overline{\theta}(c_1)) \gtreqqless 0 \Leftrightarrow c_1 \gtreqqless \underline{\rho}a$, we have the following two subcases.

Subcase 2(a) $c_1 \in [\underline{\rho}a, \overline{\rho}a)$: Then for all $p_0 \in [c_1, \overline{\theta}(c_1))$, $w(p_0) > 0$ and hence by (30), $\phi_1(p_0) > F(\tau(p_0))$. Since for this case $\widehat{\theta}(c_1) = \widehat{\theta}_2(c_1) \geq \overline{\theta}(c_1)$ [(24) and (27)], the result follows by (28).

Subcase 2(b) $c_1 < \underline{\rho}a$: Then $\widehat{\theta}_2(c_1) < \bar{\theta}(c_1)$ [by (25)]. Hence by (28), $\phi_1(p_0) < F(\tau(p_0))$ for $p_0 \in [\overline{\theta}(c_1), (a + c_1)/2)$. For $p_0 \in [c_1, \overline{\theta}(c_1))$, $w(c_1) > 0 > w(\overline{\theta}(c_1))$ and $\exists\ \widehat{\theta}_1(c_1) \in (c_1, \overline{\theta}(c_1))$ [given by (24)] such that $\phi_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow p_0 \lesseqqgtr \widehat{\theta}_1(c_1)$. Noting that $\widehat{\theta}(c_1) = \widehat{\theta}_1(c_1)$ for this case [by (26)], the proof is complete. ■

Part (v) of Lemma 5 asserts that firm 1 prefers the Stackelberg outcome over the Cournot outcome for relatively large values of p_0. To see the intuition for this, observe that both $\phi_1(p_0)$ and $F(\tau(p_0))$ are increasing in p_0. While p_0 has a direct effect on $\phi_1(p_0)$, its effect on $F(\tau(p_0))$

takes place through the function $\tau(p_0)$. The latter causes a stronger effect since $\tau(p_0)$ (the leadership premium) itself increases with p_0. Firm 2 is willing pay a higher premium for larger values of p_0, which leads to higher supplier revenue for firm 1. This in turn provides a better compensation to firm 1 for its follower position in the ensuing Stackelberg game. This is the reason why the Stackelberg outcome is preferred by firm 1 for relatively large values of p_0.

Lemma SI (Stage I)

(i) *In any SPNE of* Γ: (a) $p_1 = \tau(p_0)$ *and* (b) $p_0 \in [c_1, \theta_0]$.

(ii) *In any SPNE of* $\widetilde{\Gamma}$, $p_1 \geq \tau(c_0)$.

(iii) *In any SPNE of* Γ:

(a) *the Cournot outcome is played if and only if* $p_1 = \tau(p_0)$, $p_0 \in [c_0, \theta_0]$ *and* $p_0 \leq \widehat{\theta}$.

(b) *the Stackelberg outcome is played if and only if* $p_1 = \tau(p_0)$, $p_0 \in [c_1, c_0]$ *and* $p_0 \geq \widehat{\theta}$.

(iv) *In any SPNE of* $\widetilde{\Gamma}$:

(a) *the Cournot outcome is played if and only if* $p_1 \geq \tau(c_0)$ *and* $c_0 \leq \widehat{\theta}$.

(b) *the Stackelberg outcome is played if and only if* $p_1 = \tau(c_0)$ *and* $c_0 \geq \widehat{\theta}$.

Proof (i)(a) In any SPNE of Γ, $p_0 \geq c_1$ (Lemma 1) and $p_1 > c_1$ (Lemma 4). Recall that $(a + c_1)/2$ is the monopoly price under unit cost c_1. If $p_0 \geq (a + c_1)/2$ in an SPNE of Γ, then we must have $p_1 \geq (a + c_1)/2$ as well, so that firm 1 obtains the monopoly profit and firm 2 does not order any input from either 0 or 1, resulting in zero profit for firm 0. But then 0 can deviate to $p'_0 < (a+c_1)/2 = p_1$ to ensure positive order of η from firm 2 and thus obtain positive profit. Therefore in any SPNE of Γ, we must have $p_0 < (a + c_1)/2$. Thus, $p_0 \in [c_1, (a + c_1)/2)$ and the function $\tau(p_0)$ is well defined.

If $p_1 < \tau(p_0)$, then firm 1 obtains $F(p_1)$. As F is monotonic (Lemma 5(iv)), firm 1 can deviate to $p'_1 \in (p_1, \tau(p_0))$ to obtain $F(p'_1) > F(p_1)$. So we must have $p_1 \geq \tau(p_0)$.

If $p_1 > \tau(p_0)$, firm 0 obtains $\psi(p_0)$. If p'_0 is marginally higher or lower than p_0, we will have $p_1 > \tau(p'_0)$ and 0 would obtain $\psi(p'_0)$ by deviating to p'_0. As ψ is increasing for $p_0 \in [c_1, \theta_0)$ and decreasing for $p_0 \in (\theta_0, (a + c_1)/2)$ (Lemma 5(i)), there are gainful deviations for firm 0 if $p_0 \neq \theta_0$.

Now let $p_0 = \theta_0$ and $p_1 > \tau(\theta_0)$. Then firm 1 obtains $\phi_1(\theta_0)$. By deviating to $p'_1 = \theta_0 < \tau(\theta_0)$, firm 1 would obtain $F(\theta_0) > \phi_1(\theta_0)$ (Lemma 5(iii)). Hence we must have $p_1 = \tau(p_0)$.

(i)(b) Since $p_0 \in [c_1, (a + c_1)/2)$, if the claim is false, then $p_0 \in (\theta_0, (a + c_1)/2)$. As $p_1 = \tau(p_0)$ [from part (i)(a)], by Lemma SII(1), firm 0 obtains either zero payoff (the Cournot outcome) or $\psi(p_0)$ (the Stackelberg outcome). Let firm 0 deviate to $p'_0 = \theta_0 < p_0$ so that $\tau(\theta_0) < \tau(p_0) = p_1$. Then 0 would obtain $\psi(\theta_0)$ which is positive and more than $\psi(p_0)$ (Lemma 5(i)-(ii)), proving the result.

(ii) Since $c_0 \in (c_1, (a + c_1)/2)$, $\tau(c_0)$ is well defined and (ii) follows from the second paragraph of the proof of (i)(a) by taking $p_0 \equiv c_0$.

For the proofs of (iii) and (iv), note that $\widehat{\theta} < \theta_0 < (a + c_1)/2$.

(iii)(a) The "if" part: Let $p_1 = \tau(p_0)$, $p_0 \in [c_0, \theta_0]$ and $p_0 \leq \widehat{\theta}$. Then there is an SPNE of $G(p_0, p_1)$ where the Cournot outcome is played (Lemma SII(1)(iv)). In this SPNE, firm 0 obtains $\psi(p_0) \geq 0$ (since $p_0 \geq c_0$) and firm 1 obtains $\phi_1(p_0)$. We prove the result by showing that neither 0 nor 1 has a gainful unilateral deviation.

By deviating to $p_0' \geq (a + c_1)/2$, firm 0 would obtain zero, so such a deviation is not gainful. Now let $p_0' < (a + c_1)/2$. If firm 0 deviates to $p_0' < c_0$, it would obtain at most zero. If it deviates to $p_0' > p_0$, then $\tau(p_0') > \tau(p_0) = p_1$ and firm 0 would obtain $0 \leq \psi(p_0)$. If it deviates to $p_0' \in [c_0, p_0)$, then $\tau(p_0') < \tau(p_0) = p_1$ and it would obtain $\psi(p_0')$. Since $p_0' < p_0$ and $p_0', p_0 \in [c_0, \theta_0] \subset [c_1, \theta_0]$, by Lemma 5(i) it follows that $\psi(p_0') < \psi(p_0)$, so this deviation is also not gainful.

Now consider firm 1, who obtains $\phi_1(p_0)$. If it deviates to $p_1' \leq c_1$, it would obtain at most $\phi_1(c_1) < \phi_1(c_0) \leq \phi_1(p_0)$ (see the proof of Lemma 4(ii), pp. 6-7). So let $p_1' > c_1$. If firm 1 deviates to $p_1' > p_1 = \tau(p_0)$, it would still obtain $\phi_1(p_0)$. If it deviates to $p_1' < p_1 = \tau(p_0)$, it would obtain $F(p_1') < F(\tau(p_0))$ (by the monotonicity of F). Since $F(\tau(p_0)) \leq \phi_1(p_0)$ for $p_0 \leq \widehat{\theta}$ (Lemma 5(v)), we have $F(p_1') < \phi_1(p_0)$, so this deviation is not gainful. This completes the proof of the "if" part.

The "only if" part: By (i), $p_1 = \tau(p_0)$ and $p_0 \in [c_1, \theta_0]$ in any SPNE. Under the Cournot outcome, firm 0 obtains $\psi(p_0)$ and firm 1 obtains $\phi_1(p_0)$. If $p_0 < c_0$, then $\psi(p_0) < 0$. As firm 0 can deviate to $p_0' = (a + c_1)/2$ to obtain zero payoff, if the Cournot outcome is played in an SPNE of Γ, we must have $p_0 \geq c_0$. Since $p_0 \leq \theta_0$, we conclude that $p_0 \in [c_0, \theta_0]$.

By deviating to $p_1' > p_1 = \tau(p_0)$, firm 1 obtains $F(p_1')$ which can be made arbitrarily close to $F(\tau(p_0))$ by choosing p_1' sufficiently close to p_1. To ensure that firm 1's deviation is not gainful *for any* $p_1' \in (0, p_1)$, we need $\phi_1(p_0) \geq F(\tau(p_0))$, so by Lemma 5(v) we must have $p_0 \leq \widehat{\theta}$.

(iii)(b) The "if" part: Let $p_1 = \tau(p_0)$, $p_0 \in [c_1, c_0]$ and $p_0 \geq \widehat{\theta}$. Then there is an SPNE of $G(p_0, p_1)$ where the Stackelberg outcome is played. In this SPNE, firm 0 obtains zero payoff and firm 1 obtains $F(p_1) = F(\tau(p_0))$. We prove the result by showing that neither 0 nor 1 has a gainful unilateral deviation.

By deviating to $p_0' \geq (a + c_1)/2$, firm 0 would obtain zero, so such a deviation is not gainful. Now let $p_0' < (a + c_1)/2$. If 0 deviates to $p_0' > p_0$, then $p_1 = \tau(p_0) < \tau(p_0')$ and it would still obtain zero payoff. If it deviates to $p_0' < p_0 \leq c_0$, then it would obtain at most zero and such a deviation is also not gainful.

Now consider firm 1, who obtains $F(\tau(p_0)) = F(p_1)$. If it deviates to $p_1' > p_1 = \tau(p_0)$, it would obtain $\phi_1(p_0)$. Since $p_0 \geq \widehat{\theta}$, by Lemma 5(v) we have $\phi_1(p_0) \leq F(\tau(p_0)) = F(p_1)$, so this deviation is not gainful. Since $p_1 = \tau(p_0) > p_0 \geq c_1$, we have $p_1 > c_1$. If firm 1 deviates to $p_1' \in [0, c_1]$, it would obtain at most $\phi_1(c_1) \leq \phi_1(p_0) \leq F(p_1)$. Finally if it deviates to $p_1' \in (c_1, p_1)$, it would obtain $F(p_1')$ and by the monotonicity of F, such a deviation is also not gainful.

The "only if" part: By (i), $p_1 = \tau(p_0)$ and $p_0 \in [c_1, \theta_0]$ in any SPNE. Under the Stackelberg outcome, firm 0 obtains zero and firm 1 obtains $F(\tau(p_0))$. If $p_0 > c_0$, let firm 0 deviate to $p_0' \in (c_0, p_0)$. Then $p_1 = \tau(p_0) > \tau(p_0')$ and firm 0 would obtain $\psi(p_0') > 0$, making such a deviation gainful for 0. Therefore we must have $p_0 \leq c_0$. Since $p_0 \geq c_1$, we conclude that

$p_0 \in [c_1, c_0]$.

If firm 1 deviates to $p_1' > p_1 = \tau(p_0)$, it would obtain $\phi_1(p_0)$. To ensure that this deviation is not gainful, we need $\phi_1(p_0) \leq F(\tau(p_0))$, so by Lemma 5(v), we must have $p_0 \geq \widehat{\theta}$.

(iv) Note that for the game $\widetilde{\Gamma}$, $p_0 \equiv c_0$ and firm 0 does not play any strategic role there.

(iv)(a) The "if" part: Let $p_1 \geq \tau(c_0)$ and $c_0 \leq \widehat{\theta}$. Then there is an SPNE of $G(c_0, p_1)$ where the Cournot outcome is played (Lemma SII(1)). In this SPNE, firm 1 obtains $\phi_1(c_0)$. We prove the result by showing firm 1 does not have a gainful unilateral deviation. If firm 1 deviates to $p_1' \leq c_1$, it would obtain at most $\phi_1(c_1) < \phi_1(c_0)$ (see the proof of Lemma 4(ii), pp. 6-7), so consider $p_1' > c_1$. If firm 1 deviates to $p_1' > \tau(c_0)$, it would still obtain $\phi_1(c_0)$. If it deviates to $p_1' < \tau(c_0)$, it would obtain $F(p_1') < F(\tau(c_0))$ (by the monotonicity of F). Since $c_0 \leq \widehat{\theta}$, we have $F(\tau(c_0)) \leq \phi_1(c_0)$ (Lemma 5(v)), so this deviation is not gainful. Finally, if firm 1 deviates to $p_1' = \tau(c_0)$, then it obtains either $\phi_1(c_0)$ (the Cournot outcome) or $F(\tau(c_0)) \leq \phi_1(c_0)$ (the Stackelberg outcome), so this deviation is not gainful as well. This completes the proof of the "if" part.

The "only if" part: By (ii), $p_1 \geq \tau(c_0)$ in any SPNE. Under the Cournot outcome, firm 1 obtains $\phi_1(c_0)$. Note that $\tau(c_0) > c_0 > c_1$. By deviating to $p_1' \in (c_1, \tau(c_0))$, firm 1 obtains $F(p_1')$ which can be made arbitrarily close to $F(\tau(c_0))$ by choosing p_1' sufficiently close to $\tau(c_0)$. To ensure that firm 1's deviation is not gainful *for any* $p_1' \in (c_1, \tau(c_0))$, we need $\phi_1(c_0) \geq F(\tau(c_0))$, so by Lemma 5(v) we must have $c_0 \leq \widehat{\theta}$.

(iv)(b) The "if" part: Let $p_1 = \tau(c_0)$ and $c_0 \geq \widehat{\theta}$. Then there is an SPNE of $G(c_0, p_1)$ where the Stackelberg outcome is played. In this SPNE, firm 1 obtains $F(p_1) = F(\tau(c_0))$. We prove the result by showing that firm 1 does not have a gainful unilateral deviation. If firm 1 deviates to $p_1' > p_1 = \tau(c_0)$, it would obtain $\phi_1(c_0)$. Since $c_0 \geq \widehat{\theta}$, by Lemma 5(v), $\phi_1(c_0) \leq F(\tau(c_0)) = F(p_1)$, so this deviation is not gainful. As $p_1 = \tau(c_0) > c_0 > c_1$, we have $p_1 > c_1$. If firm 1 deviates to $p_1' \in [0, c_1]$, it would obtain at most $\phi_1(c_1) < \phi_1(c_0) \leq F(p_1)$, so such a deviation is not gainful. Finally if it deviates to $p_1' \in (c_1, p_1) = (c_1, \tau(c_0))$, it would obtain $F(p_1')$ and by the monotonicity of F, such a deviation is also not gainful.

The "only if" part: By (ii), $p_1 \geq \tau(c_0)$ in any SPNE. If $p_1 > \tau(c_0)$, then there is no SPNE of $G(c_0, p_1)$ where the Stackelberg outcome is played (Lemma SII(2)), therefore if the Stackelberg outcome is played in an SPNE, we must have $p_1 = \tau(c_0)$. Under the Stackelberg outcome, firm 1 obtains $F(p_1) = F(\tau(c_0))$. If firm 1 deviates to $p_1' > p_1 = \tau(c_0)$, it would obtain $\phi_1(c_0)$. To ensure that this deviation is not gainful, we need $\phi_1(c_0) \leq F(\tau(c_0))$, so by Lemma 5(v), we must have $c_0 \geq \widehat{\theta}$. ■

3 Proofs of Theorems 1 and 2

3.1 Proof of Theorem 1

Theorem 1 (Strategic outside firm) *There is a threshold* $\widehat{\theta} \equiv \widehat{\theta}(c_1) \in (c_1, (a + c_1)/2)$ *such that, in the game* Γ *the following hold.*

(I) *If* $c_0 \in (c_1, \widehat{\theta})$, *there is a continuum of SPNE, indexed by supplier prices* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_0, \widehat{\theta}]$. *For any such* (p_0, p_1), *firm* 2 *outsources* η *to the outside firm* 0 *and* *the Cournot outcome is played in* $G(p_0, p_1)$, *i.e.*, $q_2^1 = 0$, $q_1^1 = x_1 = k_1(p_0)$ *and* $q_2^0 = x_2 = k_2(p_0)$.

(II) *If* $c_0 \in (\widehat{\theta}, (a + c_1)/2)$, *there is a continuum of SPNE, indexed by supplier prices* $(p_0, p_1) \in (Graph\ \tau)[\widehat{\theta}, c_0]$. *For any such* (p_0, p_1), *firm* 2 *outsources* η *to firm* 1 *and the Stackelberg outcome is played in* $G(p_0, p_1)$, *i.e.*, $q_2^0 = 0$, $q_1^1 = q_2^1 + x_1$, $q_2^1 = x_2 = s_2(p_1)$ *and* $x_1 = s_1(p_1)$.

(III) *Finally, if* $c_0 = \widehat{\theta}$, *there are two SPNE with the same supplier prices* $(p_0, p_1) = (c_0, \tau(c_0))$. *In the first SPNE firm* 2 *outsources* η *to* 0 *and the Cournot outcome is played in* $G(p_0, p_1)$; *in the the second SPNE firm* 2 *outsources* η *to* 1 *and the Stackelberg outcome is played.*

Proof (I) Let $c_0 \in (c_1, \widehat{\theta})$. Then there is no SPNE of Γ where the Stackelberg outcome is played (Lemma SI, part (iii)(b)) and a continuum of SPNE (indexed by (p_0, p_1) where $p_0 \in [c_0, \widehat{\theta}]$ and $p_1 = \tau(p_0)$) where the Cournot outcome is played (Lemma SI, part (iii)(a)). The outsourcing pattern under the Cournot outcome follows from Lemma SII(1).

(II) Let $c_0 \in (\widehat{\theta}, (a + c_1)/2)$. Then there is no SPNE of Γ where the Cournot outcome is played (Lemma SI, part (iii)(a)) and a continuum of SPNE (indexed by (p_0, p_1) where $p_0 \in [\widehat{\theta}, c_0]$ and $p_1 = \tau(p_0)$) where the Stackelberg outcome is played (Lemma SI, part (iii)(b)). The outsourcing pattern under the Stackelberg outcome follows from Lemma SII(1).

(III) Let $c_0 = \widehat{\theta}$. Then by part (iii) of Lemma SI, there are two SPNE of Γ, one where the Cournot outcome is played and one where the Stackelberg outcome is played, each having $p_0 = c_0 = \widehat{\theta}$ and $p_1 = \tau(c_0)$. The outsourcing pattern again follows from Lemma SII(1). ■

3.2 Proof of Theorem 2

Theorem 2 (Competitive outside fringe) *There is a threshold* $\widehat{\theta} \equiv \widehat{\theta}(c_1) \in (c_1, (a+c_1)/2)$ *(same as the threshold in Theorem* 1 *with the strategic outside firm) such that, in the game* $\widetilde{\Gamma}$ *the following hold.*

(I) *Same as Theorem* 1 *except that* $p_1 \in [\tau(c_0), \infty)$ *and the continuum of SPNE are equivalent in real terms.*

(II) *Same as Theorem* 1 *except that the continuum of SPNE collapses to a unique SPNE with* $p_1 = \tau(c_0)$.

(III) *Finally, if* $c_0 = \widehat{\theta}$, *there is a continuum of SPNE. In one of these,* $p_1 = \tau(c_0)$ *and firm* 2 *outsources* η *to firm* 1. *The rest are indexed by* $p_1 \in [\tau(c_0), \infty)$, *are equivalent in real terms and have firm* 2 *outsourcing to firm* 0.

Proof (I) Let $c_0 \in (c_1, \widehat{\theta})$. Then there is no SPNE of $\widetilde{\Gamma}$ where the Stackelberg outcome is played (Lemma SI, part (iv)(b)) and a continuum of SPNE, indexed by $p_1 \in [\tau(c_0), \infty)$, where the Cournot outcome is played (Lemma SI, part (iv)(a)). The outsourcing pattern under the Cournot outcome follows from Lemma SII(1). As firm 2 does not order any input from firm 1 in any of these SPNE, they are equivalent in real terms.

(II) Let $c_0 \in (\widehat{\theta}, (a+c_1)/2)$. Then there is a unique SPNE of $\widetilde{\Gamma}$ where $p_1 = \tau(c_0)$ and the Stackelberg outcome is played (Lemma SI, part (iv)). The outsourcing pattern under the Stackelberg outcome follows from Lemma SII(1).

(III) Let $c_0 = \widehat{\theta}$. By Lemma SI(iv), there is one SPNE where $p_1 = \tau(c_0)$ and the Stackelberg outcome is played and a continuum of SPNE, indexed by $p_1 \in [\tau(c_0), \infty)$ and equivalent in real terms, where the Cournot outcome is played. The outsourcing pattern again follows from Lemma SII(1). ■