# Money, capital, and real liquidity effects with habit formation

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*Abstract*. The money in utility model is reconsidered in the presence of endogenous labour and habits. With standard assumptions about preferences and a policy rule that sets the nominal interest rate by adjusting the growth rate of money, the model exhibits superneutrality in the steady state. Nevertheless, habits give rise to real liquidity effects in the short run. After an increase in the nominal interest rate, employment falls, resulting in a fall in capital accumulation and in the short- and long-term real interest rates. The adjustment of the capital stock is non-monotonic. Employment and the short- and long-term real interest rates may also adjust non-monotonically. JEL classification: E22, E52, E58

*Monnaie, capital, et effets réels de liquidité quand il y a formation d'habitudes*. On réexamine le rôle de la monnaie dans un modèle d'utilité quand travail et formation d'habitudes sont endogènes. Dans le cadre des postulats usuels à propos des préférences, et d'une règle de politique qui définit le taux d'intérêt en ajustant le rythme de croissance de la monnaie, le modèle fait preuve de super-neutralité en régime permanent. Néanmoins, les habitudes entraînent des effets réels de liquidité à court terme. Après un accroissement dans le taux d'intérêt nominal, l'emploi chute, ce qui entraîne un ralentissement dans l'accumulation du capital et une chute des taux d'intérêt réels à court et à long terme. L'ajustement dans le stock de capital n'est pas monotone. L'emploi et les taux d'intérêt réels à court et à long terme peuvent aussi ne pas s'ajuster de façon monotone.

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## 1. Introduction

The relationship between money, interest rates and the real economy has been one of the classic topics in monetary economics. In the earlier literature, the important contributions by Tobin (1965) and Sidrauski (1967), among others, focused primarily on the extent to which an anticipated permanent increase in the growth rate of money, and hence long-run inflation, can affect the level of output or its growth rate and other real variables such as capital accumulation or consumption. Tobin's aggregative model predicted that an increase in money growth by reducing the rate of return on money leads households to substitute capital for money in their portfolios, which then increases steady-state capital and output. Sidrauski considered the issue in an optimizing model with money in the utility function and neoclassical growth. In that setting, he showed that an increase in the rate of growth of money will have no effect on the steady-state capital stock or the steady-state level of consumption; that is, money is superneutral in the long run.

Following the rational expectations revolution of the 1970s and its important distinction between expected and unexpected changes in monetary policy, the more recent literature has examined not only the long-run effects of a given change in money growth but, more important, it has also provided explicit theories for potential short run effects. A key feature in this literature is the behaviour of nominal interest rates following an increase in the growth rate of money. Two main views have emerged in the recent literature. First, the long-standing 'Fisherian view' that an increase in money growth leads to a proportional increase in the nominal interest rate and, second, the 'liquidity effects view' that an increase in money growth decreases the nominal interest rate.

The Fisherian view is explained through the effect of money growth on expected inflation and is consistent with most frictionless general equilibrium business-cycle models with money. In such models, if the money growth process displays positive persistence, an unexpected increase in money growth increases expected inflation and hence the nominal interest rate (see, e.g., Christiano 1991 and Walsh 1998, chap. 3).

General equilibrium models with liquidity effects have also been advanced in the literature. These models produce a negative correlation between money growth and interest rates in the short run either by assuming sticky prices or by introducing frictions in the economy through limited participation of economic agents in financial markets; see Ohanian and Stockman (1995) for a comprehensive review. For example, in Christiano and Eichenbaum (1992) money is introduced through cash-in-advance (CIA) constraints, and agents are constrained to spend a fixed amount of the money stock every period for the purchase of consumption goods, lending the difference to financial intermediaries from which the firms in the economy borrow. In this setting, a new cash injection by the monetary authorities creates excess liquidity in the economy and requires a fall in the interest rate in order to induce the firms to hold the new money. Similarly, Alvarez, Lucas, and Weber (2001) generate a short-run liquidity effect in an exchange economy by assuming that only a fraction of the households participate in financial markets. Further, they show that the liquidity effect exists under various monetary policy rules, including ‘Taylor rules’ that set the interest rate by targeting inflation.

An inevitable consequence of general equilibrium models with consumption-leisure choice and CIA constraints is money non-superneutrality.[1] This is due to the fact that the mere existence of money through CIA constraints distorts the consumption-leisure choice, as long as the interest rate is positive. In that case, an increase in the interest rate reduces consumption and increases leisure in the steady state, thereby reducing steady-state employment and output. However, this is not consistent with most empirical evidence from a cross-section of countries that points to money superneutrality in the long run. For instance, Bullard and Keating (1995) and Stock and Watson (1997) found that, apart from a small number of countries, a permanent increase in inflation has no permanent effect on either the long-run level or the rate of growth of real output.

In this paper we present a general equilibrium model that has the property of long-run money superneutrality and at the same time generates real liquidity effects through the short-term and long-term real interest rates; that is, an increase in the money growth rate reduces the short- and long-term real interest rates.[2] We achieve this in the context of a money-in-utility model with habits and endogenous labour. The money-in-utility model makes money superneutral in the steady state, while the existence of habits generates the real liquidity effects in the short run.

We assume that monetary policy is directed at setting the nominal interest rate, not the rate of growth of money or the inflation rate. Recently, the nominal interest rate has been used as an instrument of monetary policy by central banks in many developed countries, and it may, in fact, be a more relevant way to study the design and implementation of monetary policies in these countries (see, e.g., Taylor 1993; Svensson 1997; and Friedman 2000). Interest rate policies by central banks have also been blamed for policy-induced recessions in some countries; see, for example, Fortin (1996) and DeLong (2002, 448–50).

There is a more important reason we assume that monetary policy involves fixing the nominal interest rate, rather than the rate of growth of money. When the monetary authorities set the money growth rate, the inflation rate and the nominal interest rate are endogenous, since they adjust continuously in order to clear the money market. The adjustments in the nominal interest rate lead to changes in the marginal rate of substitution between consumption and real

1 Even in a simple model with only a consumption decision, Stockman (1981) showed that if investment, along with consumption, is subject to a CIA constraint, then steady-state capital will fall when the growth rate of money rises.

2 In contrast, nominal liquidity effects arise when an increase in the rate of growth of money reduces the nominal interest rates.

money. This affects consumption and savings decisions at any instant in time, which in turn affects capital accumulation along the adjustment path; see Fischer (1979) and Cohen (1985). By assuming that monetary policy involves fixing the nominal interest rate, we are able to abstract completely from these off steady-state effects and isolate completely the 'habits effect,' which is the central feature of our model.

Habit formation over consumption has a long history in economics. Ryder and Heal (1973) first formalized the idea in an optimizing framework. Within our framework habit formation is modelled by assuming that instantaneous utility depends not only on current 'full consumption,' defined as instantaneous utility from current consumption, labour supply, and real money holdings, but also on the habitual standards of living. Habits are modelled as a weighted sum of past levels of full consumption.[3]

Given this setting and the assumption that instantaneous utility is multiplicatively separable in real money balances, we show that the model exhibits long-run superneutrality; that is, an increase in the nominal interest rate has no long-run effect on capital, consumption, employment, or real interest rates. Hence, eventually these variables return to their original levels in the new steady state. Nevertheless, habits break superneutrality along the adjustment path to the new long-run equilibrium and this generates the real liquidity effects.

The intuition for these results is as follows. An increase in the nominal interest rate increases the cost of holding real balances as long as the initial nominal interest rate is positive. This reduces real money holdings and, thereby, the steady-state level of habits. If preferences exhibit adjacent complementarity, the representative individual would want to maintain the habitual standards of living inherited from the past.[4] Hence, in the short run real money holdings will not fall by as much as in the long run. For the same reason, in the short run there will be a fall in savings and the labour supply, resulting in a reduction in capital accumulation. As time passes and the capital stock falls, its marginal productivity increases, which makes savings more attractive. This 'marginal productivity effect' on savings will become stronger over time; eventually it will dominate the 'habits effect' on savings described above, at which point savings will start to rise, increasing investment and growth.

3 The habit formation model has been supported empirically, and it has been used by several authors to resolve asset market puzzles. For example, Constantinides (1990) uses the model to solve the Mehra-Prescott (1985) equity premium puzzle. Backus, Gregory, and Telmer (1993) show that habit persistence helps to account for the high variation in the expected returns on the forward relative to spot markets for currencies. Heaton (1993), Ferson and Constantinides (1991), Naik and Moore (1996), and Fuhrer and Klein (1998), among other authors, provide empirical evidence in favour of habit persistence.

4 Throughout this paper we will assume that preferences exhibit adjacent complementarity. With adjacent complementarity, an increase in current full consumption will increase the marginal utility of full consumption in the near future relative to the distant future. Ryder and Heal (1973, 3–5) provide a precise definition. It is adjacent complementarity that has made the habit persistence model attractive in the asset pricing literature.

Further, following the increase in the nominal interest rate and the resulting impact fall in employment, both short- and long-term real interest rates fall immediately, thereby creating the real liquidity effects. Then, as capital and employment adjust over time, both real rates rise above their long-run levels before converging to their initial steady-state values.

The rest of the paper is organized as follows. In section 2 we present the model. In section 3 we analyse the effects of an increase in the nominal interest rate. In section 4 we provide a numerical evaluation of the model with standard preferences, production function, and parameter values. Section 5 concludes.

## 2. The model

### *2.1. The representative agent and the government*

There are two agents in the model, a representative household and a government. The preferences of the representative household are given by

$$\int_0^\infty e^{-\theta t} U(\omega\,(c_t, n_t, m_t), h_t)\,dt, \tag{1}$$

where $c_t$ is current consumption, $n_t$ is the current amount of leisure, $m_t$ is current real money holdings and $h_t$ is the current habitual standards of living.

Current full consumption is given by $\omega(\cdot)$, which is a homothetic subutility function, measuring utility from current consumption, leisure, and real money holdings, with $\omega_c > 0$, $\omega_n > 0$, and $\omega_m > 0$. Following Ryder and Heal (1973, 2–3), we assume that instantaneous utility $U(\cdot)$ is increasing in full consumption ($U_\omega > 0$), is decreasing in habits ($U_h < 0$), and is quasi-concave.

Habits depend on the standards of living the representative household derives from consumption, leisure, and real money holdings and are a weighted sum of past levels of full consumption, $\omega$, with exponentially declining weights given to more distant values of $\omega$. Hence,

$$h_t = \rho e^{-\rho t} \int_{-\infty}^{t} e^{\rho\tau} \omega(c_\tau, n_\tau, m_\tau)\,d\tau, \tag{2}$$

where $\rho$ determines the relative weight given to $\omega$ at different dates. A larger value for $\rho$ would involve lower weights given to more distant values of $\omega$. The evolution of $h_t$ is thus given by

$$\dot{h}_t = \rho(\omega_t - h_t). \tag{3}$$

Hence, $\rho$ also determines the speed with which habits adjust to a change in $\omega$, with larger values implying higher speeds of adjustment.

Output $y_t$ is produced with the production function,

$$y_t = f(k_t,\ l_t), \tag{4}$$

where $k_t$ is the capital stock at time $t$, $l_t$ is the amount of labour input at time $t$, and $f(\cdot)$ is a neoclassical production function.

If we normalize the total available time for work and leisure to unity, at any instant the representative household faces the constraint,

$$n_t + l_t = 1. \tag{5}$$

The representative agent holds all the capital stock in the economy. He also receives real monetary transfers of magnitude $\tau_t$ from the government. There are two kinds of assets in the model: titles to capital and real money balances. Hence, the real assets, $a_t$, of the representative agent are,

$$a_t = k_t + m_t. \tag{6}$$

Assuming that capital depreciates at the fixed rate $\delta$ and noting that $k_t = a_t - m_t$, the representative agent's flow constraint is

$$\dot{a}_t = f(a_t - m_t,\ l_t) - \delta(a_t - m_t) + \tau_t - c_t - \pi_t m_t, \tag{7}$$

where, in addition, $\pi_t$ is the inflation rate.[5]

Finally, the agent should also satisfy the intertemporal solvency condition,

$$\lim_{t\to\infty}\ e^{-rt}\, a_t \geq 0. \tag{8}$$

The problem of the representative agent is to maximize lifetime utility (1), subject to conditions (3), (5)–(8), and the initial conditions $h_0$ and $a_0$, taking the time paths of the inflation rate $\pi$ and the transfers $\tau$ as given. The Hamiltonian for this problem is

$$\begin{aligned} H = U(\omega(c_t,1-l_t,m_t),h_t) + \lambda_t[\rho(\omega(c_t,1-l_t,m_t) - h_t)] \\ + \mu_t[f(a_t - m_t,l_t) - \delta(a_t - m_t) + \tau_t - c_t - \pi_t m_t], \end{aligned}$$

where $\lambda_t$ and $\mu_t$ are the co-state variables associated with habits and assets. The optimality conditions are

$$H_c = U_\omega \omega_c + \lambda\rho\omega_c - \mu = 0 \tag{9}$$

$$H_m = U_\omega \omega_m + \lambda\rho\omega_m - [f_k - \delta + \pi_t]\mu = 0 \tag{10}$$

$$H_l = -U_\omega \omega_n - \lambda\rho\omega_n + f_l\mu = 0 \tag{11}$$

$$-H_h + \theta\lambda = -U_h + \lambda\rho + \theta\lambda = \dot{\lambda} \tag{12}$$

5 According to this constraint, the amount of assets the representative agent accumulates should be equal to his net income ($f(k_t,\ l_t) - \delta k_t + \tau_t$) minus his total 'expenditures' ($c_t + \pi_t m_t$).

$$-H_a + \theta\mu \equiv [-f_k + \delta + \theta]\mu = \dot{\mu}, \tag{13}$$

along with the standard transversality conditions.

The government side is kept as simple as possible. We abstract completely from government expenditures on goods and services and from public debt. Monetary policy is directed at keeping the nominal interest rate, $i$, at a constant level by the appropriate choice of the transfers, $\tau$, at any time.

Notice that the nominal interest rate is equal to the marginal productivity of capital net of depreciation plus the inflation rate (i.e., $i_t = f_k(k_t, l_t) - \delta + \pi_t$). Hence, in order to keep $i$ at a constant level the government adjusts $\pi_t$ continuously as $l_t$ and $k_t$ change. That is, when the monetary authorities set the nominal interest rate, the rate of growth of money and the corresponding inflation rate are endogenous, since they are adjusted continuously by the authorities in order to keep the nominal interest rate fixed. With this policy, the marginal rate of substitution between consumption and real money balances is constant throughout the adjustment process, which allows us to abstract completely from the off steady-state effects emphasized by Fischer (1979), who derived the detailed dynamics of the money-in-utility model assuming that the authorities control the growth rate of money.[6] The policy of fixing the nominal interest rate thus allows us to isolate completely the effects of habits.

Given this interest rate policy, the government faces the flow budget constraint,

$$\tau_t = \dot{m}_t + \pi_t m_t, \tag{14}$$

which says that it should finance its expenditures by seigniorage.

### *2.2. The perfect foresight path*

In order to derive the perfect foresight path for this economy, we first combine the flow budget constraints of the government (14) and the private sector (7) to obtain the resource constraint for the economy,

$$\dot{k}_t = f(k_t, l_t) - \delta k_t - c_t. \tag{15}$$

6 When the monetary authorities set the money growth rate, the inflation rate and the nominal interest rate are endogenous, since they adjust continuously in order to clear the money market. The adjustment in the nominal interest rate leads to changes in the marginal rate of substitution between consumption and real money, which is always set equal to $i$, the opportunity cost of money. This affects consumption and savings decisions at any instant of time, which in turn affects capital accumulation along the adjustment path to the new steady state; see also Cohen (1985). Furthermore, the monetary authorities have another option: setting the rate of inflation. In that case the rates of growth of money and the nominal interest rate would be endogenous, and they would adjust to clear the money market. With inflation rate targeting we would still have some off steady-state effects without habits, since $i$ would be adjusting along the adjustment path.

Next, linearizing equations (9)–(11) around the initial steady state, we obtain

$$c_t - \bar{c} = A_1(h_t - \bar{h}) + A_2(\lambda_t - \bar{\lambda}) + A_3(\mu_t - \bar{\mu}) + A_4(k_t - \bar{k}) \tag{16}$$

$$l_t - \bar{l} = B_1(h_t - \bar{h}) + B_2(\lambda_t - \bar{\lambda}) + B_3(\mu_t - \bar{\mu}) + B_4(k_t - \bar{k}) \tag{17}$$

$$m_t - \bar{m} = D_1(h_t - \bar{h}) + D_2(\lambda_t - \bar{\lambda}) + D_3(\mu_t - \bar{\mu}) + D_4(k_t - \bar{k}), \tag{18}$$

where overbars denote steady-state values and, as shown in appendix A, $A_i$, $B_i$, $D_i$ $(i = 1, 2, 3, 4)$ are coefficients that involve the first and second derivatives of the utility and production functions, shadow prices, and the other parameters of the model. Now, linearizing equations (3), (12), (13), and (15) around the steady state and using equations (16)–(18), we obtain the following system of differential equations:

$$\begin{bmatrix} \dot{k}_t \\ \dot{h}_t \\ \dot{\lambda}_t \\ \dot{\mu}_t \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} \begin{bmatrix} (k_t - \bar{k}) \\ (h_t - \bar{h}) \\ (\lambda_t - \bar{\lambda}) \\ (\mu_t - \bar{\mu}) \end{bmatrix}, \tag{19}$$

where $\Phi_{ij}$ $(i, j = 1, 2, 3, 4)$ are coefficients the expressions for which are also reported in appendix A. Henceforth, we will refer to the coefficient matrix in (19) as $\Phi$.

The perfect foresight path of the economy is given by the saddlepath corresponding to the differential equation system in (19). This differential equation system has two predetermined ($k$ and $h$) and two jump ($\lambda$ and $\mu$) variables. For the rest of the paper we restrict our attention to a parameter space in which the coefficient matrix in (19) has two positive and two negative eigenvalues, in which case the model exhibits saddlepath stability. Let the negative eigenvalues be denoted by $\zeta_1$ and $\zeta_2$ and the positive eigenvalues by $\zeta_3$ and $\zeta_4$.[7] Moreover, let the eigenvector associated with the eigenvalue $\zeta_i$ be denoted by $[\nu_{i1}\ \nu_{i2}\ \nu_{i3}\ \nu_{i4}]^T$ for $i = 1, 2, 3, 4$. Then, the saddlepath is given by

$$\begin{bmatrix} (k_t - \bar{k}) \\ (h_t - \bar{h}) \\ (\lambda_t - \bar{\lambda}) \\ (\mu_t - \bar{\mu}) \end{bmatrix} = \begin{bmatrix} \nu_{11} & \nu_{21} & \nu_{31} & \nu_{41} \\ \nu_{12} & \nu_{22} & \nu_{32} & \nu_{42} \\ \nu_{13} & \nu_{23} & \nu_{33} & \nu_{43} \\ \nu_{14} & \nu_{24} & \nu_{34} & \nu_{44} \end{bmatrix} \begin{bmatrix} Z_1 e^{\zeta_1 t} \\ Z_2 e^{\zeta_2 t} \\ 0 \\ 0 \end{bmatrix}, \tag{20}$$

7 The literature dealing with perfect foresight models has traditionally assumed that the parameters of preferences and the production function are such that saddlepath stability is attained. Of course, it is possible that there are more than two negative eigenvalues for the dynamic system (19), in which case the system exhibits indeterminacy, or that there are more than two positive eigenvalues, in which case the system is unstable. The eigenvalues of (19) and their corresponding eigenvectors are long and complicated functions of the model's parameters, which are difficult to sign for all possible combinations of the parameters. However, our numerical evaluations of the model in section 4 below reveal that with reasonable functional forms and parameter values the model exhibits saddlepath stability.

where, as shown in appendix B,

$$Z_1 = \frac{-\nu_{21}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}} \qquad \text{and} \qquad Z_2 = \frac{\nu_{11}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}}. \tag{21}$$

Hence, from equations (18) and (20), the perfect foresight paths for capital and labour, respectively, are given by

$$k_t - \bar{k} = Q_1(h_0 - \bar{h})e^{\zeta_1 t} + Q_2(h_0 - \bar{h})e^{\zeta_2 t} \tag{22}$$

$$l_t - \bar{l} = Q_3(h_0 - \bar{h})e^{\zeta_1 t} + Q_4(h_0 - \bar{h})e^{\zeta_2 t}, \tag{23}$$

where

$$Q_1 = -Q_2 = \frac{\nu_{11}\nu_{21}}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}},$$

while $Q_3$ and $Q_4$ are coefficients that involve the elements of the matrix in (20), and $B_i$ $(i = 1, 2, 3, 4)$ from equation (17).[8] Below, we use these two equations to analyse the effects of an increase in the nominal interest rate on the economy's capital stock, employment, and real interest rates.

## 3. The effects of an increase in the interest rate

### *3.1. The effects on capital and employment*

In this subsection we derive the effects of an increase in the nominal interest rate, *i*, on the economy. In order to obtain tractable analytic results, we make the assumption that the full consumption $\omega$ $(c_t, n_t, m_t)$ takes the form,

$$\omega(c_t, n_t, m_t) = V(c_t, n_t)W(m_t), \tag{24}$$

where $V(c_t, n_t)$ and $W(m_t)$ are standard subutility functions, with $V(c_t, n_t)$ homothetic.[9] With this assumption about $\omega$, conditions (9) and (11) imply that the marginal rate of substitution between *c* and *n* is independent of *m*. The same conditions also imply that at any instant of time the marginal rate of substitution between *n* and *c* must be equal to the marginal productivity of labour,

$$\frac{V_n(c_t, n_t)}{V_c(c_t, n_t)} = f_l. \tag{25}$$

We can now analyse the steady state and dynamic properties of the model. Consider first the steady-state properties. Equations (5), (13), (15), and (25) can be used to determine the steady-state levels of *c, k, n*, and *l*. Since these

8 The expressions for $Q_3$ and $Q_4$ are more complicated than those for $Q_1$ and $Q_2$. To obtain these expressions substitute for $(h_t - \bar{h})$, $(\lambda_t - \bar{\lambda})$, $(\mu_t - \bar{\mu})$, and $(k_t - \bar{k})$ from equations (20) into (17).

9 See also Turnovsky (2000, 268) on the usefulness of this specification for $\omega$.

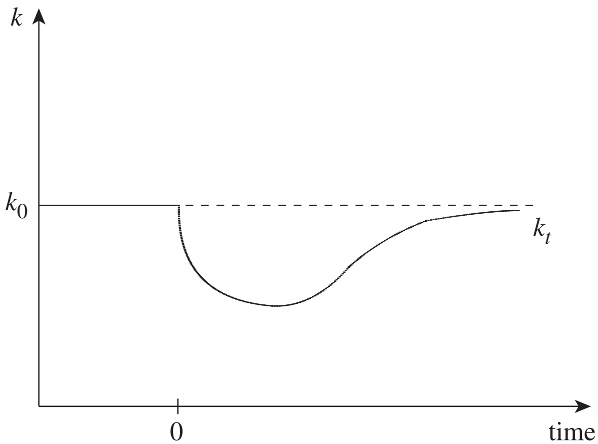


FIGURE 1 Adjustment of the capital stock

four equations are independent of the nominal interest rate, it follows immediately that money is superneutral in the steady state with respect to consumption, capital, leisure, and employment.

Nevertheless, the steady-state levels of real money holdings and habits fall when the interest rate increases. To see this, note that from equations (9) and (10) at any instant of time the marginal rate of substitution between $c$ and $m$ is equal to the nominal interest rate $i$ (i.e., $f_k - \delta + \pi$),

$$\frac{\omega_m}{\omega_c} = i. \tag{26}$$

As long as $\omega$ is not satiated in $m$ (that is, as long as $i > 0$), equation (26) implies that the steady state level of $m$ will fall when $i$ increases. Hence, from equation (3), the steady-state level of habits will also fall when the interest rate increases.

Next, we analyse the dynamic properties of the model. Consider first the dynamic adjustment of capital to the new steady state. The increase in the nominal interest rate reduces the steady-state level of habits (i.e., $(h_0 - \bar{h}) > 0$), while the steady-state level of capital is unchanged (i.e., $k_0 = \bar{k}$). From equation (22), this means that the adjustment of capital will be non-monotonic. To see this, differentiate equation (22) with respect to time and note that $Q_1 = -Q_2$ to obtain

$$\dot{k}_t = Q_1(h_0 - \bar{h})(\zeta_1 e^{\zeta_1 t} - \zeta_2 e^{\zeta_2 t}). \tag{27}$$

Hence, at time $t = 0$, we will have $\dot{k}_0 = Q_1\ (\zeta_1 - \zeta_2)(h_0 - \bar{h}) < 0$. Without loss of generality, suppose $Q_1 > 0$. With $(h_0 - \bar{h}) > 0$ for $\dot{k}_0$ to be negative we must have $(\zeta_1 - \zeta_2) < 0$, or $|\zeta_1| > |\zeta_2|$, which means that the term $(\zeta_1 e^{\zeta_1 t} - \zeta_2 e^{\zeta_2 t})$ in equation (27) is negative for small values of $t$ and positive for large values of $t$. This gives us the non-monotonic adjustment of $k$ in figure 1.

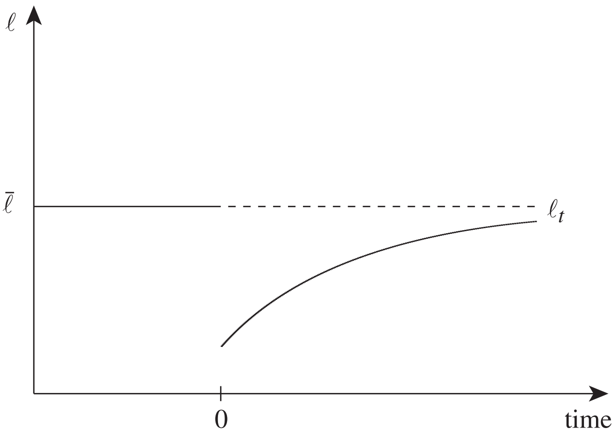


FIGURE 2 Adjustment of employment

The reason for the non-monotonic adjustment of capital is as follows. When the interest rate increases, the representative agent tries to maintain the relatively high standards of living inherited from the past. Hence, on impact consumption and leisure increase while savings fall, which reduces capital accumulation. However, as capital falls, its marginal productivity rises. This 'marginal productivity effect' will eventually dominate the 'habits effect' on savings; and there will come a time when savings will start to rise. At that point capital will also start to rise, and it will eventually return to the same steady-state level as before the increase in the nominal interest rate.

Now, consider the adjustment of employment. With adjacent complementarity in preferences, employment, $l$, falls in the short run following the increase in the nominal interest rate (i.e., $l_0 - \bar{l} < 0$). As time passes, $l$ adjusts towards its long-run level. The adjustment path of $l$ is not as clear-cut as that of capital. To see this, differentiate equation (23) with respect to time to obtain

$$\dot{l}_t = Q_3(h_0 - \bar{h})\zeta_1 e^{\zeta_1 t} + Q_4(h_0 - \bar{h})\zeta_2 e^{\zeta_2 t}. \tag{28}$$

In contrast to equation (27), equation (28) cannot be signed even at time $t = 0$. From equation (23), for $l_0 - \bar{l}$ to be negative, it must be the case that $Q_3(h_0 - \bar{h}) + Q_4(h_0 - \bar{h}) < 0$. This does not enable us to sign $\dot{l}_t$ for different values of $t$. Nevertheless, there are two dynamic paths that seem most sensible. The first case is when $\dot{l}_t > 0$ throughout the adjustment process, so that $l_t$ adjusts smoothly towards its long-run level, as shown in figure 2. The second case is when $\dot{l}_t > 0$ for small values of $t$, and $\dot{l}_t < 0$ for large values of $t$. In this second case, $l_t$ adjusts towards its long-run level in a way that overshoots its long-run value along the adjustment path. Such overshooting of employment over its steady-state level may occur because, as capital falls, its marginal productivity increases, which tends to increase the labour supply by the intertemporal substitution of leisure.

Now consider the important role of habits. To this end, consider the case without habits, which would arise when habits are fixed, with $\rho = 0$ in equations (2) and (3). In that case, we will have $h_0 = \bar{h}$ in equations (22) and (23), which would imply that there would be no dynamics after the increase in the interest rate. Hence, without habits there would be superneutrality not only in the steady state, but also along the adjustment path.

Further, notice that if $\rho > 0$ but habits do not depend on $m$ (i.e., they develop over $V(c_t, n_t)$ or over $c_t$ alone), then we would still have $h_0 = \bar{h}$, since the steady-state levels of $c$ and $n$ are unaffected by changes in $i$. With $h_0 = \bar{h}$ in equations (22) and (23), there would, again, be no dynamics after the increase in the interest rate.

Next, consider the role of multiplicative separability of preferences in equation (24). This assumption makes the marginal rate of substitution between consumption and leisure independent of real money holdings and yields superneutrality in the steady state. To see this, notice from equation (13) that the steady-state real interest rate is constrained to be equal to the rate of time preference in this (neoclassical) model. If condition (24) is satisfied, then changes in $i$ will not affect the marginal rate of substitution between consumption and leisure (see equation (25)). Hence, labour supply in the steady state will not change, and there cannot be any change in steady-state capital in order to maintain the equality of the rate of time preference to the real interest rate (i.e., equations (13), (15), and (25) can be solved in the steady state independently of $m$).[10]

In addition, if preferences are time separable (no habits) then condition (24) ensures that changes in real money holdings brought about by a change in the nominal interest rate will not affect the consumption-leisure decision of the representative agent (and there is superneutrality) even along the adjustment path.[11]

### *3.2. Liquidity effects and the term structure of interest rates*

In this subsection we discuss the real liquidity and term structure effects of an increase in the nominal interest rate brought about by a fully anticipated increase in money growth. Clearly, from the point of view of the real effects of monetary policy, the real liquidity effects are more important than the nominal liquidity effects; see Ohanian and Stockman (1995). The term structure effects are important both because most investment decisions by firms involve long term commitments for which the long-term real interest rates are

10 See also Wang and Yip (1992) regarding the role of different assumptions about instantaneous utility on the steady-state effects of an increase in anticipated inflation in the money-in-utility model without habits.

11 See also Asako (1983) for the role of different assumptions about instantaneous utility on the transition path of an increase in anticipated inflation in the money-in-utility model without habits.

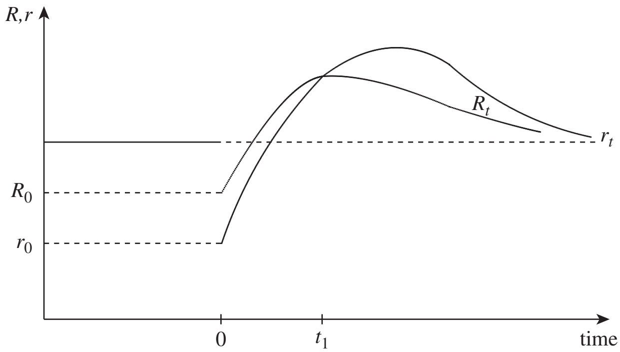


FIGURE 3 Adjustment of short and long-term real interest rates

more relevant than the short term rates and also because governments issue bonds with different maturities.

Consider first the real liquidity effects. In our model, as in most other models with flexible prices and no frictions in financial markets, the nominal interest rate is increased for Fisherian reasons, through an increase in the expected inflation rate, which in turn is brought about by an increase in the rate of growth of money. At the same time, the model generates important negative effects on both the short-term and long-term real interest rates, and thus it gives rise to a 'real liquidity effects' view of money growth that is complimentary to the 'nominal liquidity effects' view outlined in the introduction.

To understand the liquidity effects, notice first that in this model the short-term real interest rate, $r_t$, is equal to the marginal productivity of capital net of depreciation,

$$r_t = f_k(k_t, l_t) - \delta. \tag{29}$$

Because the steady-state levels of $k$ and $l$ are not affected by the increase in the nominal interest rate, there are no steady-state effects on the real interest rate. Also, we know that on impact $k$ is fixed at $k_0$, while $l$ falls to $l_0$. Thus, the fall in $l$ on impact will reduce the short-term real rate to some level $r_0$ below the initial steady-state level as shown in figure 3; this is the real short-term liquidity effect that is caused by the fall in employment following the increase in the nominal interest rate.

It is also useful to analyse the adjustment of the short-term interest rate towards its steady-state level. Differentiating equation (29) with respect to time, we obtain the dynamic adjustment of $r_t$,

$$\dot{r}_t = f_{kk}\dot{k}_t + f_{kl}\dot{l}_t. \tag{30}$$

After the impact effect, $\dot{k}_t < 0$ while $\dot{l}_t > 0$, both of which tend to increase $r_t$. With $k$ and $l$ adjusting, as in figures 1 and 2, the real interest rates will be increasing over time in the short run, but eventually, after $\dot{k}_t$ starts to become positive, $r_t$ will start to decline over time, until it reaches its steady-state level. Hence, the adjustment of $r_t$ will be as shown in figure 3. On impact, $r_t$ will fall, then it will rise above its steady-state level, and eventually it will be declining over time until it returns to its initial level.[12]

Next, we analyse the effects on the long-term real interest rate and the term structure.[13] Consider a long-term bond paying a constant real coupon of unity forever, and let the price of the bond at time $t$ be $P_t$. We assume that there are $b$ number of bonds available in the economy that have been issued by firms in the past.[14] To simplify the analysis we assume that $b$ is fixed and no other long-term bonds are issued.

With long-term bonds, the total real value of the assets of the representative agent is

$$a_t = k_t + m_t + P_t b_t, \tag{31}$$

where $b_t$ is his bond holdings, which in equilibrium will be equal to the fixed supply $b$.

Moreover, now the representative agent's flow constraint is

$$\dot{a}_t = f(k_t, l_t) - \delta k_t + \tau_t + b_t - c_t - \pi_t m_t + \dot{P}_t b_t, \tag{32}$$

according to which the improvement in the asset position of the representative agent, $\dot{a}_t$, is equal to income $f(k_t, l_t) - \delta k_t + \tau_t + b_t$, less consumption expenditures $c_t$, less the fall in the value of real money holdings due to inflation $\pi_t m_t$, plus the capital gains on bond holdings $\dot{P}_t b_t$.

Maximizing lifetime utility (1), subject to conditions (3), (5), (8), (31), (32), and the initial conditions $h_0$ and $a_0$, gives, among other conditions, the following arbitrage condition, which ensures that in equilibrium the instantaneous rate of return from long-term bonds is equal to $r_t$,

$$r_t = \frac{1 + \dot{P}_t}{P_t}. \tag{33}$$

If $R_t$ is the real rate of interest on the long-term bond, then by definition $R_t = 1/P_t$, which means that equation (33) can be rewritten as a relationship between short- and long-term real interest rates,

12 At this point, it is important to note that, because we have $\dot{l}_t > 0$ throughout the adjustment period, it is possible that the effect on dynamics of $r_t$ coming through $\dot{l}_t$ will be dominant when $\dot{k}_t$ becomes positive. In that case, the adjustment of $r_t$ will be monotonic: on impact, $r_t$ will fall, and then it will adjust monotomically towards its long-run level.

13 Our analysis of the term structure is similar to that of Fisher and Turnovsky (1992), who discuss the effects of fiscal policies in a model without money.

14 The analysis would not be different if we assume instead that the government had issued these bonds, in which case the government's budget constraint (14) would become $\tau_t + b = \dot{m}_t + \pi_t m_t$, the left-hand side of which is equal to total government expenditures in the form of transfers and interest on bonds, while the right-hand side is the government revenue from seignorage.

$$r_t = R_t - \frac{\dot{R}_t}{R_t}. \tag{34}$$

Solving equation (34) forward, we obtain the long-term interest rate as a weighted sum of expected future short-term rates,

$$R_t = \frac{1}{\int_t^\infty e^{-\int_t^\tau r_v dv} d\tau}. \tag{35}$$

First, notice from equation (34) that in the steady state $r_t = R_t = \bar{R}$. Hence, the steady-state level of the long-term real interest rate is not affected by an increase in the nominal interest rate.

Next, consider the impact effect on $R_t$ of an increase in the nominal interest rate. Because along the adjustment path the short-term real interest rate will be above its impact level $r_0$, and because the impact level of the long-term interest rate $R_0$ is a weighted sum of $r_0$ and all the future short-term real rates, it must be the case that on impact the long-term real interest rate is above the short-term real rate: $R_0 > r_0$. To establish a liquidity effect it must also be the case that $R_0 < \bar{R}$, where $\bar{R}$ is the initial (and final) steady-state value of the long-term real interest rate. We prove $R_0 < \bar{R}$ by contradiction, as follows. Suppose $R_0 > \bar{R}$. Then, in view of fact that $R_t$ is a weighted sum of $r_t$ and all future short-term rates, $R_t$ will never reach $\bar{R}$ as required by superneutrality in this model. Consequently, it must be the case that $R_t$ falls on impact and thus $R_0 < \bar{R}$. This result establishes the real liquidity effect with respect to the long-term real interest rate.

The dynamic adjustment of $R_t$ relative to $r_t$, is shown in figure 3. Notice from equation (34) that if $R_t > r_t$ we must have $\dot{R}_t/R_t > 0$; that is, if the long-term real rate exceeds the short-term real rate, the long-term real rate must be rising over time. Hence, after the increase in $i$ the long-term real interest rate must be rising over time. Also, because of the non-monotonic adjustment of $r_t$, there must come a time $t_1$ where the current $r_t$ is just equal to the weighted sum of all future short-term real interest rates. Once this time is reached, $R_t$ will start to fall. Hence, at the later stages of the adjustment process we will have $\dot{R}_t/R_t < 0$, and from equation (34), $R_t < r_t$.[15]

In sum, on impact, after the increase in the nominal interest rate, both $r_t$ and $R_t$ fall, thereby generating a real liquidity effect on the term structure of the real interest rates. Further, at the early stages of the adjustment process both the short-term and long-term real rates rise over time. But there comes a time when the long-term rate starts to fall, followed by a time when the short-term

15 Of course, if, as stated in fn. 12 above, the adjustment of the short-term real interest rate is monotonic, then the long-term real interest rates will also adjust monotonically. In that case, throughout the adjustment path the long-term rate will stay above the short-term rate; moreover, both $r_t$ and $R_t$ will stay below their steady-state levels.

real rates start to fall. Eventually, both real rates converge to the level they were at before the increase in the nominal interest rate.

## 4. A numerical simulation

In this section we provide a brief numerical evaluation of the model based on specific functional forms for preferences and the production function and parameter values similar to those used in the literature. Given this information, we then compute the effects of an increase in the nominal interest rate from 4% to 6%.

For preferences, we employ the following variant of the function for instantaneous utility that was originally introduced by Abel (1990):

$$U(\omega(c_t, n_t, m_t), h_t) = \frac{\left(\frac{\omega(c_t, n_t, m_t)}{h_t^{\gamma}}\right)^{1-\sigma}}{1-\sigma}. \tag{36}$$

Note that $\sigma$ is the relative risk aversion coefficient, while $\gamma$ indexes the importance of habits. Habits are less important when $\gamma$ is smaller. When $\gamma = 0$, we have time separable preferences. We assume that $0 \leq \gamma < 1$ and $\sigma > 1/(1-\gamma)$. These conditions on $\gamma$ and $\sigma$ ensure that the utility function is concave in both arguments.

The subutility function $\omega(c_t, n_t, m_t)$ is assumed to be Cobb-Douglas,

$$\omega(c_t, n_t, m_t) = c_t^{\alpha} n_t^{\beta} m_t^{\eta}, \tag{37}$$

which clearly satisfies condition (24).

Further, following the real business cycle literature, we assume that the production function is also Cobb-Douglas,

$$f(k_t, l_t) = k_t^{\upsilon} l_t^{1-\upsilon}. \tag{38}$$

We set the parameter values based, for the most part, on recent simulation studies. We set the rate of time preference $\theta$ equal to 0.02, because we wanted to have the long-run real interest rates (both short term and long term) equal to that value. Following Ohanian and Stockman (1995), we set the relative risk aversion parameter $\sigma$ equal to 2, the parameters $\alpha$ and $\beta$ in the subutility function $\omega$ at 0.37 and 0.63, respectively, and the shares of capital and labour incomes $\upsilon$ and $1-\upsilon$ at 0.36 and 0.64, respectively. We could not find a value in the literature for the third parameter in $\omega$, and we chose to set $\eta$ at 0.2. The parameter $\gamma$, which indexes the importance of habits, and the parameter $\rho$, which determines the speed with which habits adjust to a change in $\omega$, were set at 0.7 and 0.2, respectively, as in Carroll, Overland, and Weil (2000). Finally, we set the depreciation rate $\delta$ equal to 0.012 because we wanted to achieve a short-term real interest rate net of depreciation equal to 0.02 in the steady state; the same depreciation rate is also used in Christiano and Eichenbaum (1992).

TABLE 1
The steady state and impact values of $k$, $l$, $r$, and $R$ after an increase in the nominal interest rate from 4% to 6%

| Variable | S.S. | Impact | Change (%) |
|---|---|---|---|
| $k$ | 13.2915 | 13.2915 | 0.0 |
| $l$ | 0.3029 | 0.2513 | −17.0 |
| $r$ | 0.0200 | 0.0164 | −18.0 |
| $R$ | 0.0200 | 0.0197 | −1.5 |

With these functional forms and parameter values and assuming that initially the nominal interest rate is equal to 4%, we compute the coefficient matrix in the dynamic system (19) as

$$\Phi = \begin{bmatrix} 0.02619785836 & 0.072219418 & -0.3818363143 & 3.884626874 \\ -0.0006495707204 & -0.2019014050 & 0.03674520691 & -0.3738288184 \\ 0.004301049308 & -0.1005923686 & 0.1563790572 & 0.3067995480 \\ 0.0001344402194 & 0.03671807958 & -0.00005048081 & -0.00660843342 \end{bmatrix}. \quad (39)$$

This coefficient matrix has two positive and two negative eigenvalues: 0.1282603473, 0.05357800501, −0.1830692647, −0.02470201046. Hence, with these functional forms and parameter values the dynamic system (19) exhibits saddlepath stability. We also experimented with different values of the parameters in the neighbourhood of their set values, and the saddlepath stability of the model is maintained.[16]

The impact effects of an increase in the nominal interest rate from 4% to 6% are shown in table 1.

Notice, first, that capital, being a state variable, does not change on impact. However, the increase in the nominal interest rate leads to a fall in employment from its steady-state value of 0.3029 to the value of 0.2513; this is a fall of 17%. This impact fall in employment reduces the marginal productivity of capital, lowering the short-term real interest rate to 0.016 from its steady-state level of 0.02. Since the real long-term interest rate is a weighted sum of all future short-term rates, it does not fall by as much as the short-term real rate; it falls to only 0.0197 from its steady-state level of 0.02. The percentage reduction in the two interest rates is more telling. Whereas the short-term real rate falls by 18%, the long-term real rate falls by only 1.5%. This evidence, in turn, implies that the liquidity effects of a fall in real rates are likely to be modest even in the short run, since long-term real investment decisions depend more on the long-term real interest rate than the short-term real rate.

We also calculated the dynamic paths of capital, employment, and the two real interest rates, using the eigenvalues and eigenvectors of the matrix (39) and

16 All the numerical simulations were carried out using the mathematical package Maple 7.

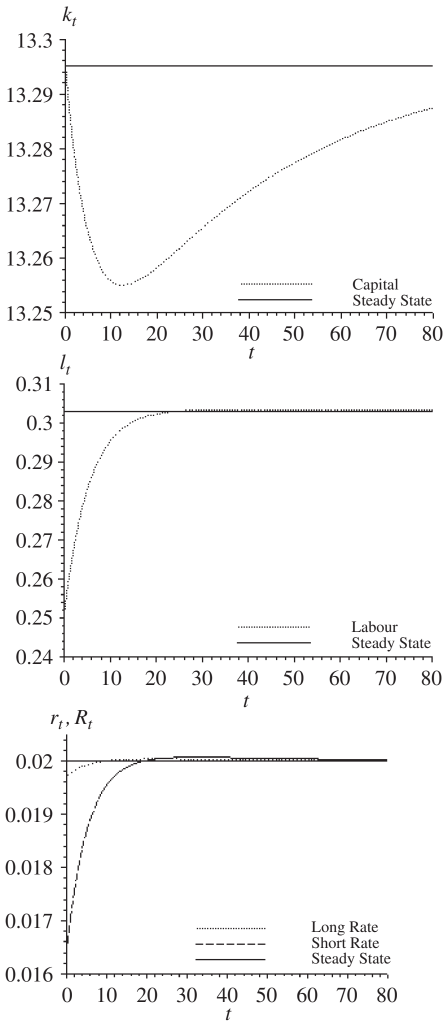


FIGURE 4 The simulated adjustment paths of $k_t$, $l_t$, $r_t$, and $R_t$, with $\rho = 0.2$

the equations (22), (23), (29), and (35). Figure 4 shows the simulated adjustment paths of $k$, $l$, $r$, and $R$ along the perfect foresight path following the increase in the nominal interest rate from 4% to 6%. It is clear from this figure that the simulated adjustment paths for these variables correspond closely to the predictions of the model and figures 1, 2, and 3. Initially, capital starts to fall, but at some point in time this trend is reversed and it returns to its steady-state value. Employment falls on impact and then increases over time until it converges to its initial steady-state value. Further, as predicted by our theory, the impact fall in the short-term real rate is much larger than the fall in the

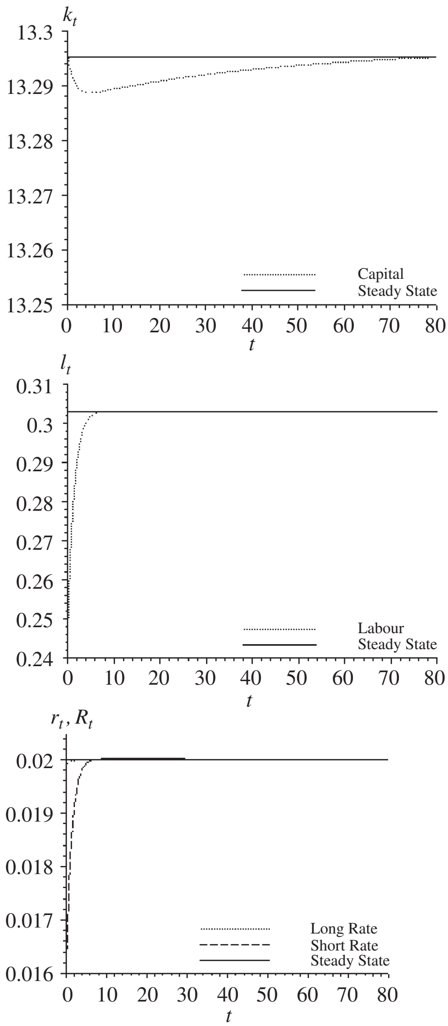


FIGURE 5 The simulated adjustment paths of $k_t$, $l_t$, $r_t$, and $R_t$, with $\rho = 0.8$

long-term real rate, both rates overshoot their long-run values during the adjustment process, and both rates eventually reach their common initial steady state asymptotically. Clearly, the existence of habits in this model makes the real liquidity effects persistent rather than transient.

One can implement an extensive study of the effects of parameter changes on the simulations. For brevity, we compare the simulated paths reported in figure 4 with the corresponding paths one would obtain if habits adjusted faster, by setting the parameter $\rho$ at a higher value. The simulated paths of $k$, $l$, $r$ and $R$ with $\rho = 0.8$ are shown in figure 5. Comparing figures 4 and 5, one can readily see that, when habits adjust faster, these variables adjust faster to their steady-state levels. For example, capital reaches its minimum level earlier in the process, before adjusting back to its steady-state level. Another indicator of the higher speed of adjustment is the substantially reduced volatility in the long-term real interest rates following the nominal interest rate shock. This is expected, since the long-term rate is a weighted sum of all future short-term rates; if the short-term rate adjusts faster to its steady-state level, then the long-term rate will stay closer to its steady-state value throughout the adjustment period.

## 5. Conclusions

In this paper we presented a general equilibrium model with money in utility and habit formation in order to analyse the effects on the economy of a nominal interest rate increase by the monetary authorities. The predictions of the model are consistent with money superneutrality in the long run and non-superneutrality along the adjustment path to the steady state. The paper also presents a new channel for real liquidity effects through reductions in both the short-term and the long-term real interest rates.

We showed that an increase in the nominal interest rate increases the cost of holding real money balances, which reduces the steady-state habitual standards of living. In the short run the representative agent would want to maintain the relatively high standards of living inherited from the past, which leads to a fall in savings and labour supply. The fall in employment reduces the marginal productivity of capital, thereby reducing both short- and long-term real interest rates. The fall in employment also leads to a fall in capital accumulation.

Nevertheless, changes in the nominal interest rate have no long-run effects on capital, employment, or the real interest rates. The fall in capital accumulation that ensures the increase in the nominal interest rate tends to increase the marginal productivity of capital. This ‘marginal productivity effect’ becomes stronger over time, and at some point savings pick up and capital starts to increase. This non-monotonic adjustment of the capital stock causes the real short-term and long-term interest rates to increase above their steady-state levels along their adjustment path to the new steady state.

In recent years, an increasing number of countries have pursued monetary policies by controlling nominal interest rates. Interest rate adjustments by central banks have become very effective tools for stabilization policies, and they have also been blamed for policy-induced recessions in the post-war era. In policy debates price rigidities are viewed as the primary reason for the effectiveness of interest rate rules, with an interest rate increase reducing aggregate demand and moving the economy along its short-run Phillips curve. In this paper we emphasize habits as another important channel for the propagation of the effects of a policy that sets the nominal interest rate in an optimizing framework without price rigidities.

## Appendix A: Coefficients in equations (16)–(19)

In this appendix we derive expressions for the values of the parameters $A_i$, $B_i$, and $C_i$ $(i=1,2,3,4)$ in equations (16)–(18) and also expressions for $\Phi_{ij}$ $(i, j=1,2,3,4)$ in the differential equation system (19). First, notice equations (9) and (10) imply equation (26). Hence, linearizing equations (9), (26), and (11), we obtain

$$\mathbf{G}\begin{bmatrix} c_t-\overline{c} \\ \ell_t-\overline{\ell} \\ \mu_t-\overline{\mu} \end{bmatrix} = \mathbf{Q}\begin{bmatrix} h_t-\overline{h} \\ \lambda_t-\overline{\lambda} \\ \mu_t-\overline{\mu} \\ k_t-\overline{k} \end{bmatrix} \tag{A1}$$

where

$$\mathbf{G}=\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}, \ \mathbf{Q}=\begin{bmatrix} q_{11} & q_{12} & q_{13} & 0 \\ 0 & 0 & 0 & 0 \\ q_{31} & q_{32} & q_{33} & q_{44} \end{bmatrix}, \tag{A2}$$

and $g_{11} = U_{\omega\omega}\omega_c^2 + [U_\omega + \lambda\rho]\,\omega_{cc}$, $g_{12}=-U_{\omega\omega}\omega_c\omega_n-[U_\omega+\lambda\rho]\omega_{cn}$, $g_{13}=U_{\omega\omega}\omega_c\omega_m+[U_\omega+\lambda\rho]\omega_{cm}$, $g_{21}=\omega_{mc}-i\omega_{cc}$, $g_{22}=-\omega_{mn}+i\omega_{cn}$, $g_{23}=\omega_{mm}-i\omega_{cm}$, $g_{31}=-U_{\omega\omega}\omega_n\omega_c-[U_\omega+\lambda\rho]\omega_{nc}$, $g_{32} = U_{\omega\omega}\omega_n^2 + [U_\omega + \lambda\rho]\omega_{nn} + \mu f_{\ell\ell}$, $g_{33}=-U_{\omega\omega}\omega_n\omega_m-[U_\omega+\lambda\rho]\ \omega_{nm}$, $q_{11}=-U_{\omega h}\omega_c$, $q_{12}=-\rho\omega_c$, $q_{13}=1$, $q_{31}=U_{\omega h}$, $q_{32}=\rho\omega_n$, $q_{33}=-f_\ell$, and $q_{34}=-\mu f_{\ell k}$.

From equation (A1) we can now obtain the expressions for $A_i$, $B_i$, and $D_i$ as

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \\ D_1 & D_2 & D_3 & D_4 \end{bmatrix} = \mathbf{G}^{-1}\,\mathbf{Q}.$$

Next, in order to obtain the differential equation system (19) we linearize equations (3), (12), (13), and (15) around the steady state and use equations (16)–(18). The coefficients in the differential equation system (19) are as follows: $\Phi_{11}=(f_k-\delta)-A_4+f_\ell B_4$, $\Phi_{12}=-A_1+f_\ell B_1$, $\Phi_{13}=-A_2+f_\ell B_2$, $\Phi_{14}=-A_3+\ \ f_\ell B_3$, $\Phi_{21}=\rho\omega_c A_4+\rho\omega_m D_4-\rho\omega_n B_4$, $\Phi_{22}=-\rho+\rho\omega_c A_1+\rho\omega_m D_1-\rho\omega_n B_1$, $\Phi_{23}=\rho\omega_c A_2+\rho\omega_m D_2-\rho\omega_n B_2$, $\Phi_{24}=\rho\omega_c A_3+\rho\omega_m D_3-\rho\omega_n B_3$, $\Phi_{31}=-U_{\omega h}\omega_c A_4-U_{\omega h}\omega_m D_4+U_{\omega h}\omega_n B_4$, $\Phi_{32}=-U_{\omega h}\omega_c A_1-U_{\omega h}\omega_m D_1+U_{\omega h}\omega_n B_1-U_{hh}$, $\Phi_{33}=\ (\rho\ +\theta)-U_{\omega h}\omega_c A_2-U_{\omega h}\omega_m D_2+U_{\omega h}\omega_n B_2$, $\Phi_{34}=-U_{\omega h}\omega_c A_3-U_{\omega h}\omega_m D_3+U_{\omega h}\ \ \omega_n B_3$, $\Phi_{41}=-\mu f_{kk}-\mu f_{k\ell}B_4$, $\Phi_{42}=-\mu f_{k\ell}B_1$, $\Phi_{43}=-\mu f_{k\ell}B_4$, and $\Phi_{44}=-f_k+\delta+\theta-\mu f_{k\ell}B_3$.

**Appendix B: Derivation of the saddlepath**

The perfect foresight path is given by the saddlepath for the differential equation system (19) in the main text:

$$\begin{bmatrix} \dot{k}_t \\ \dot{h}_t \\ \dot{\lambda}_t \\ \dot{\mu}_t \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} \begin{bmatrix} (k_t - \bar{k}) \\ (h_t - \bar{h}) \\ (\lambda_t - \bar{\lambda}) \\ (\mu_t - \bar{\mu}) \end{bmatrix}. \tag{B1}$$

As this differential equation system has two predetermined ($k$ and $h$) and two jump ($\lambda$ and $\mu$) variables, for saddlepath stability the system should have two positive and two negative eigenvalues. Let the negative eigenvalues be denoted by $\zeta_1$ and $\zeta_2$ and the positive eigenvalues by $\zeta_3$ and $\zeta_4$. Also, let the eigenvector associated with the eigenvalue $\zeta_i$ be denoted by $[\nu_{i1}\ \nu_{i2}\ \nu_{i3}\ \nu_{i4}]^T$ for $i = 1, 2, 3, 4$. Then the general solution to this differential equation system is given by

$$\begin{bmatrix} (k_t - \bar{k}) \\ (h_t - \bar{h}) \\ (\lambda_t - \bar{\lambda}) \\ (\mu_t - \bar{\mu}) \end{bmatrix} = \begin{bmatrix} \nu_{11} & \nu_{21} & \nu_{31} & \nu_{41} \\ \nu_{12} & \nu_{22} & \nu_{32} & \nu_{42} \\ \nu_{13} & \nu_{23} & \nu_{33} & \nu_{43} \\ \nu_{14} & \nu_{24} & \nu_{34} & \nu_{44} \end{bmatrix} \begin{bmatrix} Z_1 e^{\zeta_1 t} \\ Z_2 e^{\zeta_2 t} \\ Z_3 e^{\zeta_3 t} \\ Z_4 e^{\zeta_4 t} \end{bmatrix}, \tag{B2}$$

where $Z_i (i = 1, 2, 3, 4)$ are arbitrary constants corresponding to different paths.

For the *saddlepath* we should set the coefficients on the positive roots $\zeta_3$ and $\zeta_4$ equal to zero (i.e., $Z_3 = Z_4 = 0$), and then determine $Z_1$ and $Z_2$ as the solutions to the first two equations in (20) at $t = 0$. This then gives us

$$Z_1 = \frac{\nu_{22}(k_0 - \bar{k}) - \nu_{21}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}} = \frac{-\nu_{21}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}} \tag{B3}$$

$$Z_2 = \frac{-\nu_{12}(k_0 - \bar{k}) + \nu_{11}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}} = \frac{\nu_{11}(h_0 - \bar{h})}{\nu_{11}\nu_{22} - \nu_{12}\nu_{21}}, \tag{B4}$$

as $k_0 = \bar{k}$. These solution for $Z_1$ and $Z_2$ are given in the equations (21) in the main text.

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