



# The transition to a new inflation rate in models with habit formation

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## Abstract

The money in utility model is re-considered to allow for habit forming preferences, in which habits develop over instantaneous utility from consumption and real money holdings. An increase in the inflation rate does not affect the steady state level of capital or consumption, but reduces the steady state levels of real money holdings and habits. The model has important off steady state dynamics.

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## 1. Introduction

Following the classic paper of Tobin (1965), the relationship between money and economic growth has become an important topic in monetary economics. Sidrauski (1967) considered the issue in an optimizing model with money in the utility function and neoclassical growth. In that framework, he showed that money is superneutral in the long run: an increase in the growth rate of money has no effect on the steady state levels of capital or consumption.

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In this paper we examine the complete dynamics of the Sidrauski model in the presence of habits (Ryder and Heal, 1973),<sup>1</sup> where instantaneous utility depends not only on current consumption and real money holdings, but also on the habitual standards of living inherited from the past. Habits are a weighted sum of past levels of “full consumption” defined as a composite of current consumption and real money holdings.

In order to concentrate exclusively on habit effects, we assume that the monetary authority sets the inflation rate directly, and not the rate of growth of money, thereby abstracting from the effects, arising from endogenous inflation, emphasized by Fischer (1979).<sup>2</sup> In our framework, an increase in the inflation rate reduces real money holdings, and, hence, the steady state level of habits. Although there is superneutrality in the long run, the sluggish adjustment of habits leads to important dynamics along the adjustment path to the new steady state.

Our results complement the results of Faria (2001), who considered the steady state properties of the Sidrauski model with habits when habits develop not over full consumption, but over consumption alone. We show that if habits are modelled as in Faria then there will be no dynamics after a change in the inflation rate.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyses the effects of an increase in the nominal interest rate. Section 4 concludes.

## 2. The model

The preferences of the representative agent are given by

$$\int_0^{\infty} e^{-\theta t} U(w(c_t, m_t), h_t) dt, \quad (1)$$

where  $c_t$  is current consumption at time  $t$ ,  $m_t$  is current real money holdings and  $h_t$  is the current habitual living standards. Current full consumption is given by  $w(\cdot)$ , which is a homothetic sub-utility function, measuring utility from current consumption and real money holdings.<sup>3</sup>

Habits are a weighted sum of past levels of full consumption,  $w$ , with exponentially declining weights given to more distant values of  $w$ :  $h_t = \rho e^{-\rho t} \int_{-\infty}^t e^{\rho \tau} w(h_\tau, m_\tau) d\tau$ . A larger value for  $\rho$  would involve lower weights given to more distant values of  $w$ . The evolution of  $h_t$  is thus given by

$$\dot{h}_t = \rho(w_t - h_t). \quad (2)$$

<sup>1</sup> The habit persistence model has been supported empirically (see, e.g., Heaton, 1992; Ferson and Constantinides 1991; Naik and Moore, 1996), and it has been used by several authors to resolve asset market puzzles (see, e.g., Constantinides, 1990; Backus et al., 1993).

<sup>2</sup> Fischer showed that in the Sidrauski model with money growth rate targeting an increase in the rate of growth of money will lead to changes in the inflation rate in order to maintain the equality of the marginal rate of substitution between real balances and consumption to the nominal interest rate. This then gives rise to some off steady state effects, which are absent when the inflation rate is exogenous, as it is in our paper.

<sup>3</sup> Following Ryder and Heal (pp. 2–5), we assume  $U_1 > 0$ ,  $U_2 < 0$ ,  $U(\cdot)$  is quasi-concave and satisfies the property of adjacent complementarity.

Labour is inelastically supplied, and, for convenience, is set equal to unity. Output per worker  $y_t$  is produced with a neoclassical production function,  $y_t=f(k_t)$ ; where  $k_t$  is the capital stock per worker at time  $t$ .

The representative agent holds all the real assets,  $a_t$ , in the economy, consisting of titles to capital,  $k_t$ , and real money balances,  $m_t$ . Hence,  $a_t=k_t+m_t$ . He also receives real monetary transfers of magnitude  $\tau_t$  from the government.

Assuming that capital depreciates at the fixed rate  $\delta$ , and noting that  $k_t=a_t-m_t$ , the representative agent's flow constraint is

$$\dot{a}_t = f(a_t - m_t) - \delta(a_t - m_t) + \tau_t - c_t - \pi_t m_t, \quad (3)$$

where  $\pi_t$  is the inflation rate.<sup>4</sup>

The problem of the representative agent is to maximize lifetime utility (1), subject to conditions (2), (3), and the initial conditions  $h_0$  and  $a_0$ , taking the time paths of the inflation rate  $\pi$  and the transfers  $\tau$  as given. Writing the current value Hamiltonian for this problem, one can readily obtain the following optimality conditions:

$$H_c = U_1 w_c + \lambda \rho w_c - \mu = 0, \quad (4)$$

$$H_m = U_1 w_m + \lambda \rho w_m - [f'(k_t) - \delta + \pi_t] \mu = 0, \quad (5)$$

$$-H_h + \theta \lambda = -U_2 + \lambda \rho + \theta \lambda = \dot{\lambda}, \quad (6)$$

$$-H_a + \theta \mu \equiv [-f'(k_t) + \delta + \theta] \mu = \dot{\mu}, \quad (7)$$

along with the standard transversality conditions, where  $\lambda$  is the shadow price of habits and  $\mu$  the shadow price of assets.

The government side is kept as simple as possible. We abstract completely from government expenditures on goods and services, and from public debt. Monetary policy is directed at keeping the inflation rate at a constant level  $\pi$  by the appropriate choice of the transfers  $\tau$  at any time. The government faces the flow budget constraint

$$\tau_t = \dot{m}_t + \pi m_t, \quad (8)$$

which says that it should finance its expenditures by seigniorage.

### 3. Effects of an increase in the inflation rate

To work out the effects of an increase in the inflation rate, first combine the flow budget constraints of the government (8), and the private sector (3) to obtain the economy's resource constraint

$$\dot{k}_t = f(k_t) - \delta k_t - c_t. \quad (9)$$

<sup>4</sup> In addition, the agent should also satisfy the standard no Ponzi game condition.

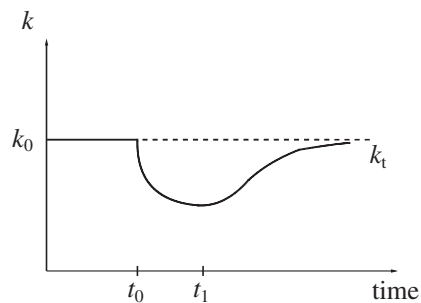
From Eq. (7) it is clear that the steady state level of the capital/labour ratio,  $k$ , is independent of the inflation rate. Thus, with the steady state level of  $k$  not affected by  $\pi$ , it must be the case that the steady state level of output and, from Eq. (9), the steady state level of consumption are also not affected by  $\pi$ . This is the Sidrauski long run superneutrality result for an economy with habits defined over full consumption.

Nevertheless, notice that the increase in the inflation rate reduces both the steady state level of real money holdings and, via Eq. (2), the steady state level of habits. This has important implications for the dynamics of the model.

Consider the adjustment of capital to the new steady state, assuming that the parameters of the model are such that the model exhibits saddlepath stability.<sup>5</sup> The increase in  $\pi$  reduces the steady state level of habits,  $\bar{h}$ , below  $h_0$  while the steady state level of capital is unchanged (that is,  $k_0 = \bar{k}$ ). However, immediately after the increase in  $\pi$  the representative agent would want to maintain the relatively high standards of living inherited from the past. He will, thus, reduce his money holdings by an amount that is smaller than its new long run level. He will also increase his consumption expenditures and reduce his savings. This will lead to capital decumulation.

As capital falls, its marginal productivity rises. This “marginal productivity effect” becomes stronger over time, and eventually dominates the “habits effect”. Thus, at some point savings pick up and capital starts to increase, eventually returning to its original level.

In terms of the figure below, immediately after the increase in  $\pi$  at time  $t_0$ ,  $k$  starts declining smoothly from its initial steady state level  $k_0$ , until there comes a time  $t_1$ , when the representative agent starts to replenish his capital stock. At that point  $k$  starts to increase and eventually returns to its original level.



Clearly, although an increase in the inflation rate has no long-run real effects, it does affect the time path of the economy in the short-run, due to the stickiness of habits. These important dynamics are absent if habits develop not over full consumption, but over consumption alone. Faria (2001) considers the steady state implications of this case.

With habits developing over  $c$  alone we have  $h_t = \rho e^{-\rho t} \int_{-\infty}^t e^{\rho \tau} c_\tau d\tau$ , and the evolution of  $h_t$  is given by  $\dot{h}_t = \rho(c_t - h_t)$ . In this case, as  $h$  does not depend on  $m$  the steady state level of habits are not affected by the increase in the inflation rate:  $\dot{h}_t = 0$  gives us a unique value for the steady state level of habits, with  $\bar{h} = \bar{c}$ : As habits do not need to adjust, there are no dynamics.

<sup>5</sup> Our numerical evaluation of the model suggests that with standards functional forms and parameter values the model does exhibit saddlepath stability. These results are available from the authors upon request.

#### **4. Conclusions**

In this paper we have explored the effects of monetary policies intended to reduce/increase the inflation rate in the money-in-utility model when preferences exhibit habit persistence. In this setting, an increase in the inflation rate has important off steady state effects when habits develop over full consumption, whereas these dynamics are absent when habits develop over consumption alone.

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