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PROBLEMS AND SOLUTIONS

PROBLEMS

95.5.1. *Iterative Estimation in Partitioned Regression Models*, proposed by Denzil G. Fiebig. Consider a classical linear regression model in partitioned form:

$$y = X_1\beta_1 + x_2\beta_2 + u, \tag{1}$$

where y and u are $T \times 1$ vectors with $E(u) = 0$; X_1 and x_2 are a $T \times k$ matrix and $T \times 1$ vector of nonstochastic regressors; and β_1 and β_2 are conformable coefficient vectors.

Consider the following strategy for estimating β_2:

Estimate β_1 from the shortened regression of y on X_1.
Regress the residuals from this regression on x_2 to yield $b_2^{(1)}$.

(a) Prove that $b_2^{(1)}$ is biased.

Now consider the following iterative strategy for reestimating β_2:

Reestimate β_1 by regressing $y - x_2 b_2^{(1)}$ on X_1 to yield $b_1^{(1)}$.
Now iterate according to the following scheme:

$$b_1^{(j)} = (X_1'X_1)^{-1}X_1'(y - x_2 b_2^{(j)}), \tag{2.1}$$

$$b_2^{(j+1)} = (x_2'x_2)^{-1}x_2'(y - X_1 b_1^{(j)}), \qquad j = 1,2,\ldots. \tag{2.2}$$

(b) Determine the behavior of the bias of $b_2^{(j+1)}$ as j increases.

(c) Show that as j increases $b_2^{(j+1)}$ converges to the estimator of β_2 obtained by running OLS on (1).

95.5.2. *The Null Distribution of Nonnested Tests with Nearly Orthogonal Regression Models*, proposed by Leo Michelis. Consider the following two nonnested linear regression models:

$$H_0 : y = W\delta_0 + X\beta + u, \qquad u \sim \text{i.i.d.}(0, \sigma^2 I_n),\ 0 < \sigma^2 < \infty,$$

$$H_1 : y = W\delta_1 + Z\gamma + v, \qquad v \sim \text{i.i.d.}(0, \omega^2 I_n),\ 0 < \omega^2 < \infty,$$

where y is an $n \times 1$ vector of observations on the dependent variable, W is an $n \times r$ matrix common to both hypotheses, X and Z are the $n \times p$ and $n \times q$ observation matrices of explanatory variables specific to models H_0 and H_1, respectively, δ_0, δ_1, β, and γ are vectors of unknown regression coefficients, and u and v are $n \times 1$ vectors representing the random errors in the two models.

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Project y, X, and Z onto the space orthogonal to the subspace defined by the columns of W and assume that the following probability limits exist and the matrices Σ_{xx} and Σ_{zz} are nonsingular:

$$\operatorname*{plim}_{n\to\infty}(n^{-1}X'M_W X) = \Sigma_{xx}, \tag{A.1}$$

$$\operatorname*{plim}_{n\to\infty}(n^{-1}Z'M_W Z) = \Sigma_{zz}, \tag{A.2}$$

where $M_W = I_n - W(W'W)^{-1}W'$.

In addition to (A.1) and (A.2), suppose that

$$\operatorname*{plim}_{n\to\infty}(n^{-1/2}X'M_W Z) = \Delta, \tag{A.3}$$

where Δ is a $p \times q$ nonnull matrix of constants such that $\Delta'\beta \neq 0$.

(a) Interpret condition (A.3).
(b) Derive the asymptotic null distributions of the J (Davidson and MacKinnon, 1981) and simplified Cox (Fisher, 1983) statistics under (A.1)–(A.3).
(c) How do the results in part (b) change if $X'M_W Z = 0$?

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Davidson, R. & J.G. MacKinnon (1981) Several tests for model specification in the presence of alternative hypotheses. *Econometrica* 49, 781–793.
Fisher, G.R. (1983) Tests of two separate regressions. *Journal of Econometrics* 21, 117–132.

95.5.3. *The Moore–Penrose Inverse of a Sum of Three Matrices*, proposed by Shuangzhe Liu and Yue Ma. Suppose Z is an $n \times r$ matrix partitioned as $Z = (X, Y)$, where X and Y are $n \times p$ and $n \times (r - p)$ matrices, respectively, $p \le r \le n$. Denote $V = aI_n + bZ(Z'Z)^+Z' + cX(X'X)^+X'$, where a, b, and c are scalars, I_n is an $n \times n$ identity matrix, and $+$ indicates the Moore–Penrose inverse. Then,

(i) prove that $V^{-1} = a^{-1}I_n + a^{-1}(a+b)^{-1}bZ(Z'Z)^+Z' + (a+b)^{-1}(a+b+c)^{-1}cX(X'X)^+X'$, for a, b, and c, such that $a > 0$, $a + b > 0$, and $a + b + c > 0$, and
(ii) give V^+, for any a, b, and c.

Remark. In part (i), when Z is of full column rank and $c = 0$, algebraic equalities and econometric applications can be found in Higgins (1994) and Ma and Liu (1995).

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Higgins, M.L. (1994) Computation of the GLS estimator of a model with anticipated and unanticipated effects. *Economics Letters* 45, 125–129.
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