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Money, habits and growth

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Abstract

The money-in-utility model is re-considered with habits and endogenous growth. An increase in the inflation rate requires a fall in the steady state habits relative to capital, if initially the nominal interest rate is positive. If habits exhibit adjacent complementarity, immediately after the increase in the inflation rate savings and investment fall, reducing the growth rate. However, the long-run growth rate is not affected by the policy change. The long-run level of capital would be lower than it would have been had there been no increase in the inflation rate. These predictions are supported by our empirical evidence, and also reconcile some recent empirical evidence on inflation and growth.

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1. Introduction

Following Tobin's (1965) pioneering paper, the relationship between money and growth has become a classic topic in monetary economics. A main question in this literature has been the extent to which a permanent increase in the growth rate of money, and hence long-run inflation, can affect the level of output or its growth rate

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and other real variables such as capital. Tobin's aggregative model predicted that an increase in money growth by reducing the rate of return on money leads households to substitute capital for money in their portfolios, which then increases steady state capital and output.

Sidrauski (1967) was the first to consider the issue in an optimizing model with neoclassical growth, money and time separable preferences. In his setting, an increase in the rate of growth of money-in-utility will have no effect on the steady state capital stock or the steady state level of consumption; that is, money is superneutral. The reason for this result is that in the neoclassical growth model with time separable preferences the steady state is characterized by the equality of the rate of time preference to the marginal productivity of capital. This condition dictates the level of capital that must be maintained in the steady state, regardless of the rate of growth of money.[1]

Stockman (1981) introduced money into the model through cash in advance (CIA) constraints. He showed that there would be long-run superneutrality if only consumption expenditures are subject to a CIA constraint. If investment is also subject to a CIA constraint then steady state capital will fall when the growth rate of money rises. Building on the work of Stockman, De Gregorio (1993) and Jones and Manuelli (1995) developed endogenous growth models[2] with CIA constraints in order to justify theoretically a negative correlation between inflation and growth in the *long run*. In De Gregorio there are CIA constraints on investment as well as on consumption. Inflation reduces long-run growth because it acts as a tax on investment. In Jones and Manuelli, there are CIA constraints on consumption alone, but there is labor/leisure choice. Inflation reduces long-run growth by leading to a substitution of leisure for consumption.

The empirical literature concerned with the effects of inflation on growth and the capital stock has been as ambiguous as the theoretical literature. In the 1990s there was an upsurge of interest in empirical studies of growth, starting with the works of Barro (1991), and Mankiw et al. (1992). Most of the empirical studies concerned with the effects of inflation on growth used cross-section or panel (pooled time series and cross-section) data for several countries and found a significant negative effect of inflation on growth; see, for example, De Gregorio (1992, 1993), Fischer (1993) and Barro (1995).

Fisher and Seater (1993) studied the long-run relationship between growth and inflation by analyzing the time series properties of the two variables in a log-linear bivariate ARIMA framework. They identified a permanent change in each variable with the statistical concept of a unit root in non-stationary time series analysis, and were able to define precisely the long-run effect of inflation on growth. Applying this methodology to post-war annual data for 58 countries, Bullard and Keating (1995)

[1] Fischer (1979) worked out the full dynamics of Sidrauski's model. He showed that after a change in the rate of growth of money there will be some dynamics before a new steady state is reached. The reason was that for the money markets to clear the nominal interest rate would have to adjust continuously in order to maintain equality of the marginal rate of substitution between consumption and real balances to the return on money.

[2] See, e.g., Romer (1986), Lucas (1988) and Rebelo (1991).

found that, apart from a single country, a permanent increase in inflation had no permanent effect on real output growth. Their estimated impulse response functions also indicate that for high inflation countries an increase in the inflation rate leads to a fall in the growth rate in the short-run but not in the long run. Using the same methodology, King and Watson (1997) analyzed post-war US data and reported similar results in their tests for long-run money neutrality.

Clearly, the empirical evidence shows that the 'time-span' is important in estimating the quantitative effects of inflation on growth. In the present paper we account for time-span effects by calculating the cross-correlation functions between inflation and growth at various time lags and leads for a sample of 12 developed and developing countries over the period 1960–2001. Our findings show that the contemporaneous (zero lag) correlations between inflation and growth are negative for all countries in our sample, whereas the average correlations over all the time lags and leads are small in absolute value, and close to zero. In this sense, our empirical evidence encompass the apparently disparate findings in the cross-section and time series studies mentioned above. Our findings are, furthermore, consistent with Easterly (1996) and Bruno and Easterly (1996, 1998), who find a negative short- to medium-run relationship between inflation and growth, but no lasting damage to growth from discrete high inflation episodes.

The main purpose of this paper is to provide a theoretical underpinning to the main empirical results surveyed above. We re-examine the relationship between inflation and growth in Rebelo's (1991) endogenous growth framework using the money-in-utility model with the additional assumption that preferences exhibit habit persistence (Ryder and Heal, 1973). With habit persistence, instantaneous utility depends not only on current consumption and real money holdings, but also on the habitual standard of living. Habits are modelled as a weighted sum of past levels of utility from consumption and real money holdings.[3]

We show that an increase in the inflation rate, by increasing the cost of holding real balances, requires a fall in the steady state habitual standard of living relative to capital.[4,5] If preferences exhibit adjacent complementarity, the representative individual would want to maintain the habitual standard of living inherited from

[3]The habit persistence model has been supported empirically, and it has been used by several authors to resolve asset market puzzles. For example, Constantinides (1990) uses the model to solve the Mehra and Prescott (1985) equity premium puzzle. Backus et al. (1993) show that habit persistence helps to account for the high variation in the expected returns on the forward relative to spot markets for currencies. Heaton (1993), Ferson and Constantinides (1991), Naik and Moore (1996), and Fuhrer and Klein (1998) among other authors, provide empirical evidence in favour of habit persistence.

[4]As the levels of all variables are growing over time, we consider the adjustments of the variables when they are deflated by a state variable (e.g., capital or habits).

[5]We assume that monetary policy is directed at maintaining the inflation rate at a constant level; that is, the inflation rate is exogenously determined by the government. With Rebelo's production function (constant marginal productivity of capital), this is equivalent to the central bank maintaining the nominal interest rate at a constant level. With nominal interest rates constant, we abstract from the off steady state effects discussed by Fischer (1979). The off steady state effects in our model will be due to habits. As pointed out by Friedman (2000), for example, the monetary policy followed by the U.S. Federal Reserve, and most other central banks of industrialized countries, involves interest rate targeting.

the past.[6] Hence, in the short-run there is a fall in savings, which leads to a sharp decline in investment. As a result, the growth rate of capital falls on impact. But, along the adjustment path the growth rate increases until it reaches its original level. The model, therefore, emphasizes that there must be a clear distinction between the short-run and long-run effects of inflation on growth. By contrast, De Gregorio (1993) and Jones and Manuelli (1995) predict a negative long-run effect, and do no deal with the short-run effects of inflation on growth.

Since along the adjustment path capital grows at a rate that is smaller than it would have been had the inflation rate not changed, the level of the capital stock at any point in time in the steady state will be lower than it would have been otherwise. Therefore, there is non-superneutrality in the long-run as well as along the adjustment path.

The paper is organized as follows. The model is presented in Section 2. The perfect foresight path is derived in Section 3. The effects of an increase in the inflation rate, along with some simulation results, and the relation of the predictions of the model to the empirical evidence are discussed in Section 4. Some concluding remarks are made in Section 5.

2. The model

There are three agents in the model, a representative household, a representative firm and a government. The preferences of the representative household are given by

$$\int_0^\infty \mathrm{e}^{-\theta t} U(\omega(C_t, m_t), S_t)\,\mathrm{d}t, \tag{1}$$

where C_t is current consumption at time t, m_t is current real money holdings and S_t is the current habitual standard of living. Current full consumption is given by $\omega(\cdot)$, which is a homothetic sub-utility function, measuring utility from current consumption and real money holdings.[7] Following Ryder and Heal (pp. 2–3), we assume that instantaneous utility $U(\cdot)$ is increasing in full consumption ($U_1 > 0$), is decreasing in habits ($U_2 < 0$), and is quasi-concave.

Habits are a weighted sum of past level of ω, with exponentially declining weights given to more distant values of ω. Hence

$$S_t = \rho\,\mathrm{e}^{-\rho t} \int_{-\infty}^{t} \mathrm{e}^{\rho\tau} \omega(C_\tau, m_\tau)\,\mathrm{d}\tau,$$

where ρ determines the relative weight given to ω at different dates. A larger value for ρ would involve lower weights given to more distant values of ω. The evolution

[6]Throughout this paper we will assume that preferences exhibit adjacent complementarity. With adjacent complementarity an increase in current consumption (or real money holdings) will increase the marginal utility of consumption (or real money holdings) in the near future relative to the distant future. Ryder and Heal (pp. 3–5) provide a precise definition. It is adjacent complementarity that has made the habit persistence model attractive in the asset pricing literature.

[7]Feenstra (1986) provides a theoretical rationale for the money-in-utility approach by demonstrating its functional equivalence to the liquidity-costs approach.

of S_t is thus given by

$$\dot{S}_t = \rho\,(\omega_t - S_t). \tag{2}$$

Hence, ρ also determines the speed with which habits adjust to a change in ω; with larger values implying higher speeds of adjustment.

It is natural that habits should develop over past levels of full consumption ω, and not over past levels of consumption C alone. The reason for this is that the well being of the representative agent depends not only on his past consumption C alone, but also on how smoothly this consumption was obtained through transactions. The latter is measured by the utility he derives from his past real money holdings. It then follows that the habitual standard of living of the representative agent, S, depends on past levels of ω and not on the past levels of C alone.

Output Y_t is produced by the representative firm using the AK model

$$Y_t = AK_t, \tag{3}$$

where K_t is the capital stock at time t.

The representative household owns the representative firm and holds all the capital stock in the economy. He also receives real monetary transfers τ_t from the government. There are two kinds of assets in the model, real money balances and titles to capital. Hence, the real assets of the representative agent (the household-firm) are

$$a_t = K_t + m_t. \tag{4}$$

Assuming that capital depreciates at the fixed rate δ, the representative agent's flow constraint is[8]

$$\dot{a}_t = (A - \delta)\,a_t + \tau_t - C_t - (A - \delta + \pi_t)\,m_t. \tag{5}$$

Finally, the agent should also satisfy the intertemporal solvency condition

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\,a_t \geqslant 0, \tag{6}$$

which prevents borrowing without bound.

The problem of the representative agent is to maximize lifetime utility (1), subject to conditions (2), (4)–(6), and the initial conditions S_0 and a_0, taking the time paths of the inflation rate π and the transfers τ as given. As the marginal rate of substitution between C and m at any date is independent of the activities of the representative agent at other dates, and as $\omega(\cdot)$ is homothetic, we can employ the standard two-stage procedure for performing this optimization problem. At the first stage, for a given level of 'expenditures' X_t, maximize $\omega(C_t, m_t)$, subject to $X_t = C_t + (A - \delta + \pi_t)\, m_t$. This gives the indirect utility function VX_t, where $V = V(\pi_t)$ is the utility from one unit of expenditures, with $V' < 0$ and $V'' > 0$. (Henceforth, to simplify notation, we will suppress the arguments of the function V). The second

[8] According to this constraint, the amount of assets the representative agent accumulates should be equal to his net income $(A - \delta)\,a_t + \tau_t$ minus his total 'expenditures' $C_t + (A - \delta + \pi_t)\,m_t$. Although the representative agent does not actually spend on his real money holdings, $(A - \delta + \pi_t)\,m_t$ is the opportunity cost of holding m_t rather than titles to capital.

stage of the problem will then be to choose the expenditures at different dates in order to maximize (1), subject to (2), (4)–(6), and the initial conditions S_0 and a_0. Replacing $\omega(C_t, m_t)$ with VX_t in (1) and (2), and $C_t + (A - \delta + \pi_t)m_t$ with X_t in (5), we can write the current value Hamiltonian for this problem as

$$H = U(VX_t, S_t) + \lambda_t[\rho(VX_t - S_t)] + \mu_t[(A - \delta)a_t + \tau_t - X_t].$$

The optimality conditions for this problem are

$$H_X \equiv U_1 V + \lambda\rho V - \mu = 0, \tag{7}$$

$$-H_S + \theta\lambda = -U_2 + \lambda\rho + \theta\lambda = \dot{\lambda}, \tag{8}$$

$$-H_a + \theta\mu \equiv -(A - \delta)\mu + \theta\mu = \dot{\mu}, \tag{9}$$

and the standard transversality conditions.

The government side is kept as simple as possible. We abstract completely from government expenditures on goods and services, and from public debt. Monetary policy is directed at keeping the inflation rate π at a constant level, by the appropriate choice of the transfers τ at any time.[9,10] The government faces the flow budget constraint

$$\tau_t = \dot{m}_t + \pi m_t, \tag{10}$$

which says that it should finance its expenditures by seigniorage.[11]

Note that by combining the flow budget constraints of the government (10), and the private sector (5), we obtain the resource constraint of the economy

$$\dot{K}_t = (A - \delta)K_t - C_t. \tag{11}$$

This completes the model.

3. The perfect foresight path

In this section we derive the perfect foresight path of the economy. The conventional saddlepath analysis requires that in the steady state the values of all the variables involved should be time invariant, whereas in the present model all variables will be growing even in the steady state (balanced growth). Hence, in order to derive the dynamics of the model using conventional saddlepath analysis, the conditions (2), (7)–(9), and (11) will be expressed in terms of variables that are time invariant in the steady state. Specifically, using these five equations we derive a three-dimensional

[9]Thus, in the remainder of the paper we drop the subscript t from π.

[10]As explained in the introduction, this policy abstracts completely from the off steady state effects emphasized by Fischer (1979), and it allows us to isolate completely the effects of habits.

[11]Because of Ricardian equivalence, issuing bonds instead of making money transfers through helicopter drops of money would not matter in our representative agent, perfect foresight model. Marini and van der Ploeg (1988) and van der Ploeg and Alogoskoufis (1994) present overlapping generations models where open market operations are non-superneutral. Extending our model in environments where the mode of monetary policy matters is left for future work.

system of differential equations for K/S, X/S and $\dot{X}/X$, after eliminating the shadow prices λ and μ. Henceforth, to simplify notation we refer to K/S, X/S and $\dot{X}/X$ as k, x, and g respectively. In what follows, we derive the differential equation for k, x, and g sequentially.

To obtain the differential equation for k, note that from Eq. (11)

$$\frac{\dot{K}_t}{S_t} = (A - \delta)\frac{K_t}{S_t} - \frac{C_t}{S_t}. \tag{12}$$

Also note that, by definition, $\frac{\dot{K}_t}{S_t} = k_t \frac{\dot{S}_t}{S_t} + \dot{k}_t$. Hence, replacing m_t in the first stage budget constraint with $m_t = -X_t V'/V$ (i.e., Roy's identity), we obtain[12]

$$C_t = X_t W, \quad \text{where} \quad W = 1 + (A - \delta + \pi)\frac{V'}{V}. \tag{13}$$

Substituting these results into Eq. (12), and using Eq. (2), we obtain the differential equation for k:

$$\dot{k}_t = k_t[A - \delta - \rho(Vx_t - 1)] + W x_t. \tag{14}$$

Next, to derive the differential equation for x, we use the definition, $\dot{x}_t = \frac{\dot{X}_t}{S_t} - \frac{\dot{S}_t}{S_t}\, x_t$, which, in combination with Eq. (2), yields the differential equation for x:

$$\dot{x}_t = x_t\left[g_t - \rho(Vx_t - 1)\right]. \tag{15}$$

The derivation of the differential equation for g is more cumbersome, involving several manipulations of Eqs. (2), (7)–(9) and (11). Since the ultimate purpose is to express these equations in terms of k, x and g after eliminating the shadow prices, we employ a specific utility function, which is a variant of the function originally introduced by Abel (1990), and also used by Carroll et al. (2000):[13]

$$U(VX_t, S_t) = \frac{\left(\frac{VX_t}{S_t^{\gamma}}\right)^{1-\sigma}}{1-\sigma}, \tag{16}$$

where $0 \leqslant \gamma < 1$ and $\sigma > \frac{1}{1-\gamma}$. These conditions ensure that the utility function is concave in both arguments. The parameter σ is the relative risk aversion coefficient, while the parameter γ indexes the importance of habits. Habits are less important when γ is smaller. When $\gamma = 0$ we have time separable preferences. With the utility function given by Eq. (16) preferences exhibit adjacent complementarity. Thus, an increase in current full consumption ω, through its effects on habits, will increase the marginal utility of full consumption U_1 in the near future relative to the marginal utility in the distant future. As a result, preferences exhibit 'addictive' behavior: after any shock the representative agent tries to maintain the habitual standard of living

[12] Again, W is a function of π; but to simplify notation we will suppress its argument.

[13] Caroll et al. have combined habits with endogenous growth in order to re-examine the causal relationship between growth and savings.

inherited from the past.[14] This is the primary driving force for the transitional dynamics of the model.

In deriving the expression for g note first that from Eq. (7)

$$\dot{\mu} = V\dot{U}_1 + \rho V\dot{\lambda}. \tag{17}$$

Substituting for $\dot{\lambda}$ and $\dot{\mu}$ from Eqs. (8) and (9) into Eq. (17), and combining the resulting equation with (7), we obtain

$$(\delta - A - \rho)\,\mu = V\dot{U}_1 - (\rho + \theta)\,VU_1 - \rho VU_2. \tag{18}$$

Taking the time derivative of Eq. (18), and dividing the resulting equation by (18), we obtain

$$\frac{\dot{\mu}}{\mu} = \frac{\ddot{U}_1 - (\rho + \theta)\,\dot{U}_1 - \rho\dot{U}_2}{\dot{U}_1 - (\rho + \theta)\,U_1 - \rho U_2}. \tag{19}$$

Next, evaluating the right-hand side of Eq. (19) using the functional form in Eq. (16), and also substituting for $\dot{\mu}/\mu$ from Eq. (9), we obtain

$$\begin{aligned}
&(2\theta + \rho + \delta - A)\left[-\sigma\left(\frac{\dot{X}}{X}\right) + \gamma(\sigma - 1)\left(\frac{\dot{S}}{S}\right)\right] \\
&\quad - (\theta + \delta - A)\left[-\rho\gamma\left(\frac{XV}{S}\right) + (\rho + \theta)\right] \\
&\qquad = \sigma(1 + \sigma)\left(\frac{\dot{X}}{X}\right)^2 + 2\gamma\sigma(1 - \sigma)\left(\frac{\dot{X}}{X}\right)\left(\frac{\dot{S}}{S}\right) - \sigma\left(\frac{\ddot{X}}{X}\right) \\
&\qquad\quad + \gamma(\gamma(1 - \sigma) + 1)\left(\frac{\dot{S}^2}{S}\right) + \gamma(\sigma - 1)\left(\frac{\ddot{S}}{S}\right) \\
&\qquad\quad - \rho\gamma(\sigma - 1)V\left(\frac{\dot{X}}{S}\right) - \rho\gamma(\gamma(1 - \sigma) + 1)\left(\frac{XV}{S}\right)\left(\frac{\dot{S}}{S}\right).
\end{aligned} \tag{20}$$

Now note that Eq. (2) implies

$$\frac{\dot{S}_t}{S_t} = \rho\left[V\frac{X_t}{S_t} - 1\right] \tag{21}$$

and

$$\frac{\ddot{S}_t}{S_t} = \rho\left[\frac{V\dot{X}_t}{S_t} - \rho\left(V\frac{X_t}{S_t} - 1\right)\right]. \tag{22}$$

[14]With such addictive behaviour, the consumption smoothing motive on the part of the representative agent is over and above the motive with time separable preferences. This is what enables Constantinides to solve the Mehra–Prescott equity premium puzzle. That is, with the extra consumption smoothing motive, the representative agent can be induced to hold the risky equity (which yields variable consumption opportunities) only if its expected rate of return is much higher than the risk free rate. This is not the case in the Mehra–Prescott artificial economy with time separable preferences, where the agent is willing to hold equities even when the equity premium is small.

Substituting Eqs. (21) and (22) into (20), and using (15), we obtain the differential equation for g:

$$\dot{g}_t = b_0 + b_1 g_t + b_2 g_t^2 + b_3 V x_t + b_4 V^2 x_t^2 + b_5 V g_t x_t, \tag{23}$$

where the expressions for b_i ($i = 0, 1, \ldots, 5$) are given in the Appendix.

Eqs. (14), (15) and (23) give us the three differential equations for k, x and g. Linearizing these equations around the steady state, we obtain

$$\begin{bmatrix} \dot{k}_t \\ \dot{x}_t \\ \dot{g}_t \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} k_t - \bar{k} \\ x - \bar{x} \\ g - \bar{g} \end{bmatrix}, \tag{24}$$

where bars denote steady state values, and the expressions for the coefficients α_{ij} are given in the Appendix. As k is the only predetermined variable in this system, for saddlepath stability the coefficient matrix should have one negative and two positive eigenvalues. Let ξ denote the negative eigenvalue. Then the stable path of the system is given by the following equations

$$k_t - \bar{k} = (k_0 - \bar{k})\, \mathrm{e}^{\xi t}, \tag{25}$$

$$x_t - \bar{x} = -\frac{(\alpha_{11} - \xi)}{\alpha_{12}} (k_0 - \bar{k})\, \mathrm{e}^{\xi t}, \tag{26}$$

$$g_t - \bar{g} = \frac{(\alpha_{11} - \xi)(\alpha_{22} - \xi)}{\alpha_{12}\, \alpha_{23}} (k_0 - \bar{k})\, \mathrm{e}^{\xi t}. \tag{27}$$

The perfect foresight path is described by Eqs. (25)–(27).

4. The effects of inflation

In this section we derive the effects of an increase in the inflation rate. We begin by analyzing the steady state effects of inflation, followed by a discussion of the simulated transitional dynamics of these variables to their steady state values. Next we discuss the special case with time separable preferences. Finally, we examine the relationship of the model's predictions to the empirical evidence.

4.1. Steady state effects

Evaluating Eqs. (14), (15) and (23) with $\dot{k} = \dot{x} = \dot{g} = 0$, we obtain the steady state levels of g, x and k, respectively, as

$$\overline{g} = \frac{\delta - A + \theta}{\gamma\sigma - \gamma - \sigma}, \tag{28}$$

$$\overline{x} = \frac{\theta + \gamma\sigma\rho + \delta - \gamma\rho - \sigma\rho - A}{\rho(\gamma\sigma - \gamma - \sigma)V}, \tag{29}$$

$$\overline{k} = \frac{(\theta + \gamma\sigma\rho + \delta - \gamma\rho - \sigma\rho - A)W}{\rho(\theta + \delta - A - A\sigma\gamma + A\gamma + A\sigma + \delta\sigma\gamma - \delta\gamma - \delta\sigma)V}. \tag{30}$$

From Eq. (28) it is clear that the long-run growth rate $\bar{g}$ is independent of the inflation rate; that is, monetary factors do not affect long-run growth. Nevertheless, as we shall see shortly, g_t will have important transitional dynamics which will affect the long-run *levels* of the real variables after a change in the inflation rate.

Differentiating Eqs. (29) and (30), we obtain

$$\frac{\mathrm{d}\overline{x}}{\mathrm{d}\pi} = \frac{-\overline{x}V'}{V} = \frac{\overline{m}}{\overline{S}} = \frac{E_1}{\overline{S}} > 0, \tag{31}$$

$$\frac{\mathrm{d}\overline{k}}{\mathrm{d}\pi} = \frac{\overline{k}(A - \delta + \pi)(V''/V - 2V'^2/V^2)}{W} = \frac{-(A - \delta + \pi)\overline{k}E_{11}}{W\overline{X}}, \tag{32}$$

where E_1 and E_{11} are the derivatives of the expenditure function $E = E(A - \delta + \pi, \overline{S})$ corresponding to $\omega(C, m)$ at the steady state, with $\omega = \overline{S}$.[15]

From Eq. (31), an increase in the inflation rate will require an increase in the steady state level of the expenditures/habits ratio, $\overline{x}$, because it increases the cost of holding real balances. From Eq. (32) it follows that the steady state level of the capital/habits ratio, $\overline{k}$, will increase if initially the nominal interest rate is positive. This is because with money-in-utility, steady state utility is maximized when it is satiated in real money balances, that is, when the nominal interest rate is zero (see, e.g., Blanchard and Fischer (1989, p. 191)). If initially we are away from this optimum because the nominal interest rate is positive, then an increase in the nominal interest rate will take us further away, requiring a fall in steady state utility, and in its corresponding habitual standard of living. This increases the steady state level of the capital/habits ratio $\overline{k}$.

In order to derive the responses of $\dot{K}/K$, and the growth rate of output (from Eq. (3)), first note that $X_t/K_t = x_t/k_t$. Hence from Eq. (13)

$$C_t/K_t = \frac{x_t}{k_t} W. \tag{33}$$

As V and hence W are constant along the adjustment path, Eq. (13) also implies that

$$\dot{C}_t/C_t = \dot{X}_t/X_t = g_t. \tag{34}$$

Next, divide both sides of Eq. (11) by K_t and use Eq. (33) to obtain

$$\dot{K}_t/K_t = (A - \delta) - \frac{x_t}{k_t} W. \tag{35}$$

Combining Eqs. (33) and (34) with Eq. (11), it is easy to see that the steady state level of $\dot{K}/K = \bar{g}$. Hence, the steady state rates of growth of capital and output (see Eq. (3)) are not affected by changes in the inflation rate.[16]

[15] Note that, by the properties of the expenditure function, $E_1 = \overline{m}$. Moreover, we can write $E = \overline{S}/V$. Hence, $E_1 = -\overline{S}V'/V^2$, and $E_{11} = -\overline{S}V''/V^2 + 2\overline{S}V'^2/V^3$. Setting $\overline{S} = \overline{X}V$ in the latter expression, we obtain $E_{11} = -\overline{X}V''/V + 2\overline{X}V'^2/V^2$.

[16] Clearly, as $\omega(C_t, m_t)$ is homothetic, C_t/m_t is constant for a given π. Hence, the rate of growth of m_t is also g_t; and its steady state value is unaffected by π.

Table 1
Numerical characteristics of k, x, g, and $\dot{K}_t/K_t$ following a 10% inflation shock

Variable	Initial S S	Jump to	Change (%)	New S S
k	23.3353	23.3353	0.000	24.0723
x	1.5955	1.6223	1.687	1.6523
g	0.0200	0.0179	−10.538	0.0200
$\dot{K}/K$	0.0200	0.0193	−3.639	0.0200

4.2. Transitional dynamics

For the transitional dynamics of the model, we examine the adjustment paths of the variables k, x, g and $\dot{K}_t/K_t$ to their long-run equilibrium values, assuming that the economy experiences a 10% increase in the inflation rate. We show that our model's features of endogenous growth and habit formation generate transitional dynamics for these variables that are both theoretically and empirically appealing. Subsequently, we infer the transitional dynamics for the normalized level of capital K, and demonstrate non-superneutrality in this model.

In order to compute the transitional dynamics, we first derived an explicit functional form for $V(\pi)$ by solving the *first stage* problem outlined in Section 2, assuming a Cobb–Douglas function for $\omega(C_t, m_t) = C_t^{0.8} m_t^{0.2}$. Next, we computed the time paths of k, x, g and $\dot{K}/K$ using Eqs. (25)–(27), and (35). To obtain the numerical results we assumed the following values for the remaining parameters of the model: $\gamma = 0.7$, $\rho = 0.2$, $\theta = \delta = 0.05$, $\sigma = 2$ and $A = 0.126$.[17] Finally, we set the initial value of the inflation rate at $\pi = 0.4$. This choice for π is motivated by the empirical evidence that shows a significant negative relationship between inflation and growth at inflation rates of 40% or higher (e.g., Bruno and Easterly, 1998). Given these parameter values, we traced out the behavior of these variables over time following a 10% increase in the inflation rate, and plot their adjustment paths over the period (0, 60).[18] The results are summarized in Table 1 and in Figs. 1–4.

It is clear from Table 1 that following the inflation shock all the variables other than k jump on impact before they reach their new steady state values. The capital/habits ratio, k, being a state variable does not jump but attains a higher new steady state value. The expenditures/habits ratio, x, increases by 1.69% on impact, and then adjusts smoothly to its higher new steady state value. The growth rate of the economy, g, falls by 10.54% on impact before it adjusts back to its initial level of 2%. Finally, the rate of capital accumulation, $\dot{K}/K$, falls by 3.64% on impact and then it adjusts back to its initial level of 2%.

[17]This value of A guarantees that the steady state growth rate of the economy is 2%. Also the value of $\gamma = 0.7$ is intended to represent medium to strong habit effects and the value of $\rho = 0.2$ allows for protracted transition dynamics.

[18]We assume that the increase in inflation takes place at time $t = 0$. All the numerical results were obtained using the computer package Maple 7.

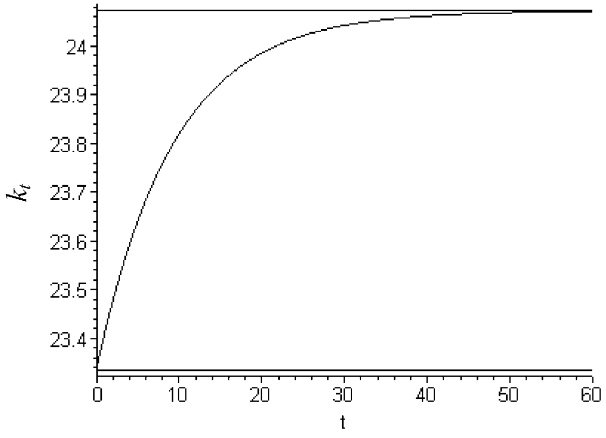

Fig. 1. The adjustment path of the capital/habits ratio (k_t).

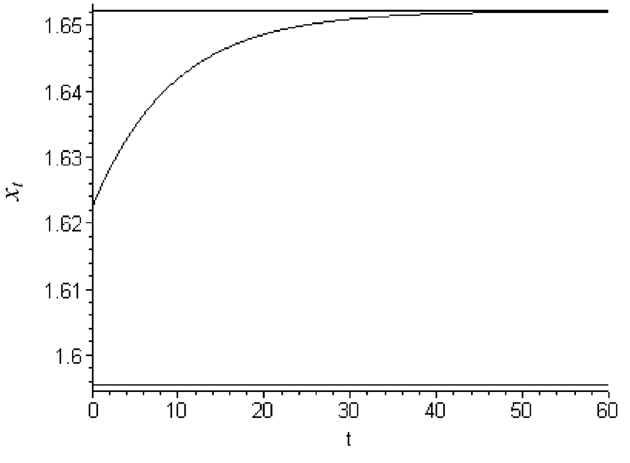

Fig. 2. The adjustment path of the expenditures/habits ratio (x_t).

Figs. 1–4 show the adjustment paths of k, x, g, and $\dot{K}/K$, respectively. The economic intuition for these adjustment paths is as follows. As explained above, the increase in the inflation rate reduces steady state welfare, requiring a fall in the steady state habitual standard of living relative to capital K and expenditures X; that is, the steady state levels of k and x increase in Figs. 1 and 2. With adjacent complementarity, the representative agent attempts to maintain the relatively high standard of living inherited from the past. Hence, on impact, savings fall in order to maintain the habitual standard of living. The fall in savings explains the initial increase in expenditures to habits ratio, x, and the initial fall in investment, and thus in $\dot{K}_t/K_t$ as shown in Fig. 4. This fall in investment also reduces the rate of growth of

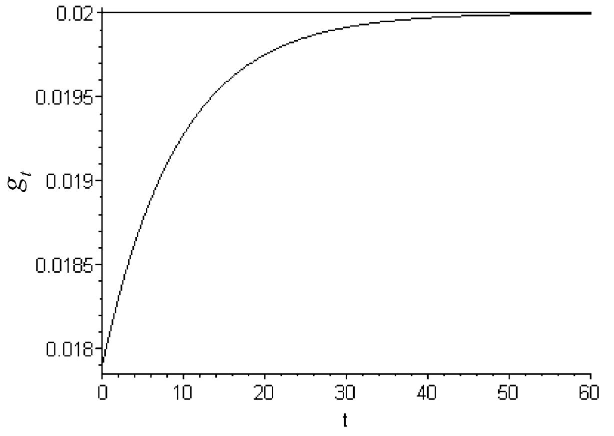

Fig. 3. The adjustment path of the growth rate (g_t).

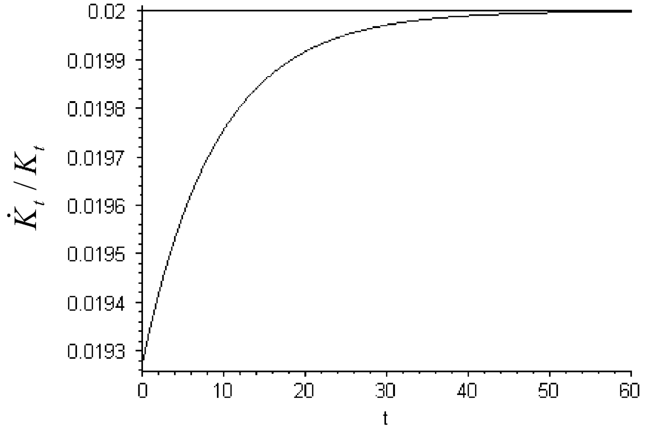

Fig. 4. The adjustment path of the growth rate of capital ($\dot{K}_t/K_t$).

expenditures; that is, g_t falls in Fig. 3. As monetary factors do not affect the long run growth rates, $\dot{K}_t/K_t$ and g_t increase over time until they return to their steady state levels in Figs. 3 and 4.

Next, consider the adjustment of the *normalized* value of capital, defined as $K_t\,\mathrm{e}^{-\bar{g}t}$, where $\bar{g}$ is the steady state growth rate. The adjustment of $K_t\,\mathrm{e}^{-\bar{g}t}$ is shown in Fig. 5. As the rate of growth of capital during the adjustment period is less than in the steady state, $K_t\,\mathrm{e}^{-\bar{g}t}$ will be falling over time until it reaches its new lower level. If the increase in π is implemented at time $t = 0$, then the level of capital at time t will be $K_0\mathrm{e}^{\int_0^t \dot{K}_v/K_v\,\mathrm{d}v}$. On the other hand, if the change in π had not been implemented, then the level of capital at time t would be $K_0\mathrm{e}^{-\bar{g}t}$. Hence, $K_0\mathrm{e}^{\bar{g}t} - K_0\mathrm{e}^{\int_0^t \dot{K}_v/K_v\,\mathrm{d}v}$ is the capital loss due to the increase in the inflation rate.

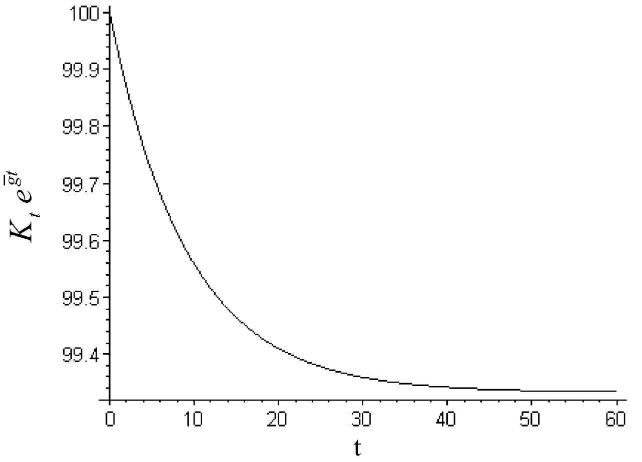

Fig. 5. The adjustment path of normalized capital ($K_t e^{\bar{g}t}$).

In terms of Fig. 4, the capital loss is given by the area above the $\dot{K}/K$ schedule that lies under the $\dot{K}/K = 0.02$ line. Setting $K_0 = 100$, $\bar{g} = 0.02$ and $t = 60$, we computed this area after evaluating $\int_0^t \dot{K}_v/K_v \, \mathrm{d}v$ using Eq. (35). It turns out that the capital loss due to the increase in inflation for this economy is in the order of 4.94% of the initial capital stock K_0. This is rather significant because it means that for every time period in the new steady state the economy will suffer a reduction in its output of about 5% of its initial level AK_0.

4.3. On alternative assumptions about preferences

In order to highlight the role played by habits, we now consider the model with time separable preferences. Also we show the implications of having habits develop over consumption C alone, and not over full consumption $\omega(C, m)$.

First consider the case with time separable preferences. In this case, we have $U_2 = 0$, and the problem of the representative agent is to maximize (1) subject to (4)–(6), and the initial condition a_0. The optimality conditions for this problem are (7) and (9) with $\lambda = 0$. Evaluating these expressions using the utility function (16), with $\gamma = 0$, and using (11), one can readily derive

$$\frac{\dot{X}_t}{X_t} = \frac{\dot{K}_t}{K_t} = \frac{A - \delta - \theta}{\sigma}. \tag{36}$$

Since Eq. (36) is independent of π, without habits changes in the inflation rate will have no effect on the growth rates of X or K. There will be no transitional dynamics. Clearly, with $\dot{K}/K$ unaffected by π, there will be no effect on the time path of capital.

Next consider the implications of having habits develop over consumption alone. In that case, we will have

$$S_t = \rho e^{-\rho t} \int_{-\infty}^{t} e^{\rho \tau} C_\tau \, d\tau, \tag{37}$$

and the evolution of S_t will be given by

$$\dot{S}_t = \rho (C_t - S_t). \tag{38}$$

The problem of the representative agent now is to maximize lifetime utility (1), subject to conditions (4)–(6), (38) and the initial conditions S_0 and a_0, taking the time paths of the inflation rate π and the transfers τ as given. In this case, as S does not depend on m it can be readily shown that the steady state level of the capital/habits ratio k is unaffected by an increase in the inflation rate. As k does not need to adjust, there will be no dynamics. Intuitively, in this model the primary driving force behind the dynamics is the requirement for habits to adjust sluggishly in response to changes in real money holdings. If habits do not depend on real money holdings they will not need to adjust in response to a change in the inflation rate, and thus there will be no dynamics.

4.4. *Theory and inflation-growth empirics*

Our model gives clear predictions about the relationship between inflation and growth. As seen in Table 1 and Fig. 3, a permanent increase in inflation leads to a sharp fall in savings and growth in order to allow the representative household to maintain his habitual standard of living, thereby inducing a negative correlation between inflation and growth in the short-run. Over time the growth rate recovers and returns to its original steady state value. Hence, the model also predicts no long-run correlation between inflation and growth.

In order to compare these theoretical predictions with the empirical evidence, we examined the time series properties of real GDP per capita growth and the rate of inflation for a cross-section of 12 developed and developing OECD countries: Australia, Canada, Ireland, Italy, Japan, Mexico, Norway, Peru, Philippines, Sweden, the UK and the US. The empirical methodology that we adopt is similar to that used by Backus et al. (1994) in studying the dynamics of the trade balance and the terms of trade, or by Stock and Watson (2000) in studying the stylized facts of the modern theories of business cycles.

Specifically using annual data for each country, we compute the cross-correlation function between the current real GDP per capita growth rate and the inflation rate at different time lags and leads. The data set is annual time series on real GDP per capita and inflation and covers the period 1960–2001. For all countries, except Australia, the data were obtained from the World Bank's World Development Indicators. Real GDP per capita growth was calculated from real GDP per capita in constant (1995 = 100) prices and inflation was computed from the Consumer Price Index (1995 = 100). For Australia, the same base period data were obtained from the International Financial Statistics of the IMF (IFS-CD, February 2003). Our

Table 2
Contemporaneous and average cross-correlations

Country	Contemporaneous	Average
Australia	−0.338	−0.135
Canada	−0.295	0.110
Ireland	−0.348	−0.123
Italy	−0.038	0.024
Japan	−0.523	−0.065
Mexico	−0.674	0.116
Norway	−0.031	0.132
Peru	−0.450	−0.171
Philippines	−0.579	0.001
Sweden	−0.521	0.082
United Kingdom	−0.511	−0.004
United States	−0.434	0.035

Contemporaneous and average cross correlations are computed for $j = 0$ and $-8 \leqslant j \leqslant +8$ respectively.

choice of annual frequencies was influenced by the fact that the time lags associated with the effects of monetary policy are rather long. Denoting the real GDP per capita growth rate in period t with rgdpcg_t and the rate of inflation in period $t + j$ with $\inf_{t+j}$, we computed the cross-correlation function, $\mathrm{corr}(\mathrm{rgdpcg}_t, \inf_{t+j})$, for each country for different values of j in the interval $-8 \leqslant j \leqslant +8$.

Table 2 reports the contemporaneous and average cross correlations for each country over the whole range of values of j. As shown in the second column of Table 2 the contemporaneous (i.e., $j = 0$) cross-correlations are uniformly negative for all 12 countries in the sample, as predicted by our theory. Inflation has detrimental effects on economic growth in the short run, an empirical finding that is also consistent with the results reported by most cross-section studies on economic growth and inflation. Yet, as shown in the third column of Table 2, the average cross-correlations for all the countries, over the whole range of j, are much smaller in absolute value than the contemporaneous ones, and close to zero; the maximum average cross-correlation is −0.17 (Peru), and for half of the 12 countries the average cross-correlation is less than 10% in absolute value. Like most time series studies, we interpret this evidence of negligible average correlation as evidence in support of the proposition that in the long run money has no effects on growth; see Fisher and Seater (1993). Thus, our theory throws light on and can reconcile the apparently disparate empirical results from cross-section and time series studies.

5. Conclusions

In this paper we have considered the money-in-utility model with habits and endogenous growth. We have shown that an increase in the inflation rate, by increasing the cost of holding real balances, would require a fall in the steady state habitual standard of living relative to capital, as long as initially the nominal interest rate is positive. With higher inflation and adjacent complementarity in preferences, the representative agent would want to maintain the habitual standard of living inherited from the past. Hence, on impact, there will be a fall in savings, reducing the growth rate in the economy. Over time, the growth rate will be increasing, until it reaches its original steady state level. Hence, inflation affects growth along the transition path only. It has no effect on long-run growth.

Our empirical evidence seems to validate the theoretical predictions of the model. For our sample of 12 countries, the empirical cross-correlations between real GDP per capita growth and inflation are large and negative contemporaneously, but close to zero on average over a lag-lead range of 16 years.

The existing empirical literature has given mixed evidence regarding the effects of inflation on growth. On the one hand, the cross-section/panel studies have found evidence supporting a negative effect of inflation on growth. This evidence is stronger with high frequency data, which contain strong short term effects. On the other hand, time series studies, which are concerned mainly with long-term effects, have found no significant long run effects on growth. The model presented in this paper reconciles this apparently contradictory evidence, by viewing the cross-section/panel evidence as capturing mainly the average short-run effects of inflation on growth, while the time series evidence capturing the long-term effects.

The empirical results presented in this paper are only an empirical illustration of the main predictions of the theory. For this reason, they are indicative rather than definitive. Further detailed empirical work is needed with more countries to analyze the dynamics of inflation and economic growth. We intend to pursue this project in future research.

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Appendix

In this Appendix we report the values of the coefficients in Eq. (23), and in the differential equation system (24). The coefficients in Eq. (23) are as follows:

$$b_0 = \frac{1}{\sigma}((\rho + \theta)(\theta + \delta - A) + \rho\gamma(1 - \sigma)(\rho(\gamma(1 - \sigma) + 1) - (2\theta + 2\rho + \delta - A)),$$

$$b_1 = 2\theta + \rho + \delta - A - 2\gamma\rho(1 - \sigma),$$

$$b_2 = \sigma,$$

$$b_3 = \left(\frac{\rho\gamma}{\sigma}\right)(\rho\gamma(1-\sigma)(2\sigma-1)+\theta+\rho-\sigma(2\theta+\delta-A)),$$

$$b_4 = -\rho^2\gamma(\gamma(1-\sigma)+1),$$

$$b_5 = 2\gamma\rho(1-\sigma).$$

The coefficients in the dynamic system (24) are as follows:

$$\alpha_{11} = A-\delta-\rho(V\overline{x}-1),$$

$$\alpha_{12} = W-\rho\overline{k}V,$$

$$\alpha_{22} = \overline{g}-\rho(2V\overline{x}-1),$$

$$\alpha_{23} = \overline{x},$$

$$\alpha_{32} = b_3V+b_5V\overline{g}+2b_4V^2\overline{x},$$

$$\alpha_{33} = b_1+2b_2\overline{g}+b_5V\overline{x}.$$

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