Pushing the limit? Fiscal policy in the European Monetary Union

Betty C. Daniel
Department of Economics
University at Albany
Albany, NY 12222, USA
bdaniel@albany.edu

Christos Shiamptanis
Department of Economics
Wilfrid Laurier University
Waterloo, ON N2L 3C5, Canada
cshiamptanis@wlu.ca

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Abstract

Governments are confronted with the growing realization that they face fiscal limits on the size of debt and deficits relative to GDP. These fiscal limits invalidate Bohn’s criterion for fiscal sustainability, which allows explosive debt relative to GDP, eventually violating any fiscal limit. We derive restrictions on a fiscal rule, necessary for the government to eliminate explosive behavior. These restrictions require that the response of the primary surplus to debt be relatively strong, and that the primary surplus be cointegrated with both debt and output. We test these empirical implications for a panel of eleven EMU countries, and find that they are satisfied, implying that fiscal policy does not create explosive behavior.

Key Words: European Monetary Union, monetary policy, fiscal policy, fiscal limits, panel cointegration, error correction

JEL codes: C32, C33, E42, E62, F33
1 Introduction

Fiscal authorities are facing renewed scrutiny over government debt and deficits following the worldwide recession and financial crisis that began in 2007. The scrutiny has been especially intense in European Monetary Union (EMU) countries, where fiscal problems have threatened the value of the Euro, and raised the specters of both sovereign default and a breakup of the monetary union. What is "responsible" fiscal policy, and have countries in the EMU been following such a policy?

The design of responsible monetary policy has received much more attention than that of responsible fiscal policy. Monetary policy is typically specified as a rule, often a Taylor Rule, and alternative policies involve consideration of alternative specifications of the rule. Leeper (2010) argues that we need to place fiscal policy under the same scrutiny. Specifically, we need to specify fiscal policy as a rule, determine criteria for the fiscal rule to be responsible, and test whether countries meet those criteria.

A responsible fiscal rule must be sustainable. Early tests of fiscal sustainability (Hamilton and Flavin, 1986; Wilcox, 1989; Trehan and Walsh, 1991) focused either on determining whether debt was stationary or on cointegration between government debt and the primary surplus as indicative of whether current fiscal policy satisfied the intertemporal budget constraint, and hence was sustainable. However, Bohn (2007) argued that sustainability tests based on cointegration were unnecessarily restrictive. This is because a fiscal policy could...
be sustainable with a growing gap between government debt and the primary surplus, in violation of cointegration. He demonstrates that the government’s intertemporal budget constraint is expected to hold under a fiscal rule in which the primary surplus adjusts by any positive amount to lagged debt. Therefore, Bohn (1998, 2008) claims that a fiscal policy, characterized as a rule with the primary surplus responding positively to lagged debt, is sustainable.

However, Bohn’s (1998, 2008) definition of fiscal sustainability clashes with the new and growing literature on fiscal limits (Davig et al., 2010, 2011; Davig and Leeper, 2011; Cochrane, 2011; Daniel, 2010; Daniel and Shiamptanis, 2012; Sims, 1997). This literature recognizes that there is an upper bound on the value of the primary surplus that a country can raise. A fiscal limit is in part due to the Laffer Curve; since taxes are distortionary, there is an upper bound on the level of taxes a country can raise (Trabandt and Uhlig, 2011). It also arises due to political will. There is an upper bound on a country’s willingness to tax itself and a lower bound on expenditures on public goods. In the presence of these fiscal limits, explosive debt and primary surpluses, which satisfy the intertemporal budget constraint, do not represent equilibrium paths because they will eventually violate any upper bound. Therefore, a responsible fiscal rule must do more than satisfy the government’s intertemporal budget constraint. It must also rule out explosive behavior of debt and the primary surplus relative to GDP. Bohn (2007) acknowledges that his analysis abstracts from fiscal limits. Our paper can be viewed as an extension of Bohn, necessary when countries face fiscal limits.

We define a responsible fiscal rule as satisfying the government’s intertemporal budget...
constraint and ruling out explosive behavior. We specify a simple fiscal policy rule, analogous to the Taylor Rule for monetary policy, and derive the restrictions on parameters necessary for the fiscal rule to be responsible. Satisfaction of the intertemporal budget constraint requires that the primary surplus respond positively to lagged debt, as in Bohn (1998, 2008). Ruling out explosive behavior adds the requirement that the debt and primary surplus eventually stabilize relative to output. Together these restrictions require that a responsible fiscal rule yield a globally stable system.

We derive two empirically testable criteria which a responsible fiscal rule must satisfy. The first is that the magnitude of the responsiveness of the primary surplus to lagged debt must be large enough to yield global stability. The second is cointegration. Since the primary surplus and debt are expected to reach a long-run equilibrium in a globally stable model, consideration of non-explosive behavior, required by fiscal limits, restores cointegration between the primary surplus and debt as a necessary requirement for a responsible fiscal rule. When the fiscal rule must satisfy the intertemporal budget constraint in addition to ruling out explosive behavior, the requirement for cointegration is not unnecessarily strong.

We conduct tests to assess our two empirically testable criteria, using annual data on real debt, real primary surpluses, and real GDP for a panel of eleven EMU countries over the period 1970-2011. We find that fiscal policy was adequately responsive to increases in debt, and that the data exhibits the required cointegration to imply that the fiscal rule has

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1 Davig (2005) also argues that global stability is necessary for a sustainable fiscal policy in the context of an empirical Markov switching model. His model has two regimes, one with expanding debt and one with shrinking debt. The government’s intertemporal budget constraint is satisfied as long as periods of expanding debt are offset by periods of shrinking debt so that the system is globally stable.
been responsible in our panel of eleven EMU countries.

A fiscal rule which satisfies our criteria for responsibility is necessary, but not sufficient, for fiscal solvency for two primary reasons. First, a responsible fiscal rule eliminates explosive behavior of debt as long as the government can achieve the primary surplus mandated by the rule. Stochastic shocks could send debt so high that the surpluses, mandated by fiscal rule which brings debt back down, would violate fiscal limits and are therefore infeasible. (Daniel and Shiamptanis, 2012). At the fiscal limit, the government cannot raise surpluses further to continue its responsible fiscal rule. Agents refuse to lend, creating a solvency crisis.²

Second, any test based on historical data is necessarily backward-looking. Politicians can make future promises, unrelated to current values of the primary surplus and debt, which are insolvent, and find themselves unable to borrow to carry out those plans. Davig et al. (2010, 2011) are concerned about unfunded future liabilities due to age-related spending. Historical data does not capture this kind of future plan. Current projections for future debts levels are so high that if agents truly believed that no adjustments would be make to deal with the unfunded liabilities, then governments would be insolvent in spite of having followed a responsible fiscal rule in the past.

Therefore, a county following a responsible fiscal rule could still encounter solvency problems due to negative shocks or due to future plans which are insolvent. However, a country following a fiscal rule which is not responsible will encounter solvency problems with cer-

² Polito and Wickens (2012) have similar objectives. They evaluate the future fiscal stance of current policy by determining whether the forecast debt/GDP ratio over a particular horizon, using VAR, is consistent with a target debt/GDP ratio. They use a VAR to forecast future debt, rather than a fiscal rule. They are interested in the measure of fiscal stance whereas Daniel and Shiamptanis (2012) are interested in fiscal solvency.
tainty. This is what we seek to determine with our tests. We are testing for responsibility which requires non-explosive behavior of government debt and the primary surplus relative to GDP.

The paper is organized as follows. Section 2 derives the restrictions on the parameters of the fiscal rule necessary for a responsible fiscal policy. Section 3 contains the empirical analysis, and Section 4 provides conclusions.

2 Derivation of Model Restrictions

In this section, we derive criteria for the fiscal rule such that the government’s intertemporal budget constraint holds, and debt and the primary surplus are not expected to explode relative to output. We label this fiscal rule responsible.

2.1 Fiscal Rule

We use a simple specification for the fiscal rule, analogous to the Taylor Rule for monetary policy. Our fiscal rule generalizes those originally introduced by Leeper (1991) by allowing the primary surplus \( s_t \) to respond to its own lag and to lagged output \( y_{t-1} \), in addition to lagged debt \( b_{t-1} \). The fiscal rule is given by

\[
    s_t = \beta_1 s_{t-1} + \beta_2 y_{t-1} + \beta_3 b_{t-1} + \nu_t, \tag{1}
\]

where all variables are expressed in real terms, and \( \nu_t \) is a mean-zero disturbance representing fiscal shocks. Fiscal shocks reflect both politically-determined shocks to taxes or government

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\(^3\) If we found irresponsible fiscal rules in countries which were borrowing, then creditors must have been expecting future policy change, something we cannot detect in historical data. This would be analogous to a finding of nonsustainability in the earlier literature.
spending, and the responses of the fiscal authority to other shocks that affect the economy.

The lagged value of the primary surplus is necessary to fit the data. It allows persistence and reflects the desire to smooth the effect of shocks over time. Bohn (1998, 2008) shows that the primary surplus must respond positively to lagged debt for the fiscal rule to satisfy the intertemporal budget constraint. In a model with all variables growing, the response to lagged output replaces the constant in Leeper’s fiscal rule.

### 2.2 Evolution of the government debt

The evolution of the government debt is derived by combining asset market equilibrium with the government’s flow budget constraint. We assume that the countries in the monetary union are small enough that they cannot affect the world price level or world interest rate. There is a single good in the world, implying goods markets equilibrium requires the law of one price. Normalizing the world price level at unity and assuming no world inflation implies that the equilibrium price level in the monetary union is the exchange rate.

We assume that international creditors are willing to buy and sell a country’s government bonds as long as its nominal interest rate, $r_t$, satisfies interest rate parity. Interest rate parity is implied by the Euler equations for a representative world agent when the covariance of the country’s interest rate with world-agent consumption is zero, or when the world agent is risk neutral. Under the additional assumption that the world interest rate ($r$) is constant, interest rate parity can be expressed as

\[
\frac{1}{1 + r_t} = \left( \frac{1}{1 + r} \right) E_t \left[ \frac{P_t}{P_{t+1}} \delta_{t+1} \right],
\]

(2)
where $E_t$ denotes the expectation conditional on time $t$ information, $P_t$ denotes the common price level in the monetary union, and $\delta_{t+1}$ is the fraction of the value of the country’s bond that will be repaid in period $t+1$.

Consider the government flow budget constraint. Letting $B_t$, $M_t$, $G_t$, and $T_t$, denote, respectively, nominal government bonds held by the public, money supply supported by the country’s bonds, government spending, and tax revenue, the country’s nominal flow government budget constraint is given by

$$B_t + M_t = \delta_t [(1 + r_{t-1}) B_{t-1} + M_{t-1}] + G_t - T_t.$$

Dividing by $P_t$, the real values of debt and the primary surplus can be expressed respectively as

$$b_t = \frac{1}{P_t} \left( B_t + \frac{1}{1 + r_t} M_t \right),$$
$$s_t = \frac{1}{P_t} \left( T_t - G_t + \frac{r_t}{1 + r_t} M_t \right).$$

The government’s real flow budget constraint can be expressed as

$$b_t = \frac{\delta_t (1 + r_{t-1})}{1 + \pi_t} b_{t-1} - s_t, \quad (3)$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate.

To solve the model, we express current debt as a linear function of lagged debt and the current primary surplus by isolating the term containing the capital loss on government debt. Define $\gamma_t$ as the capital loss on debt due to inflation or default as

$$\gamma_t = \left( 1 - \frac{\delta_t}{1 + \pi_t} \right) (1 + r_{t-1}) b_{t-1}.$$
If there is no inflation \((\pi_t = 0)\) and no default \((\delta_t = 1)\), then there is no capital loss \((\gamma_t = 0)\).

But if \(\pi_t > 0\) or \(\delta_t < 1\), then \(\gamma_t > 0\). Unanticipated capital loss can be expressed as

\[
\gamma_t - E_{t-1}\gamma_t = -\left(\frac{\delta_t}{1 + \pi_t}\right)(1 + r_{t-1}) b_{t-1} + E_{t-1}\left(\frac{\delta_t}{1 + \pi_t}\right)(1 + r_{t-1}) b_{t-1}.
\]

(4)

Using equations (3) and (4), and imposing interest rate parity from equation (2), the evolution of debt can be expressed as

\[
b_t = (1 + r) b_{t-1} - s_t - (\gamma_t - E_{t-1}\gamma_t),
\]

(5)

where \(b_t\) includes publicly held government bonds and the money supply backed by the country’s bonds, \(r\) is the world real interest rate, and \((\gamma_t - E_{t-1}\gamma_t)\) represents the unanticipated capital loss, due to inflation or default, on the country’s debt. Expectations of capital loss \((E_{t-1}\gamma_t)\) raise the country’s interest rate above the world rate, and when the actual capital loss \((\gamma_t)\) does not occur, debt accumulates in response to the higher interest rate. Higher inflation, when fully anticipated, raises \(\gamma_t\) and \(E_{t-1}\gamma_t\) equally, thereby having no effect on the evolution of government debt. The increase in the nominal interest rate, created by the increase in expected inflation, is exactly offset by the capital loss on government debt due to the actual inflation.\(^4\)

Substituting for the primary surplus from equation (1) yields

\[
b_t = (1 + r - \beta_3) b_{t-1} - \beta_1 s_{t-1} - \beta_2 y_{t-1} - \nu_t - (\gamma_t - E_{t-1}\gamma_t).
\]

(6)

Together, equations (1) and (6) constitute a dynamic system in the primary surplus and

\(^4\) Higher money growth, which creates higher inflation, does raise the surplus by creating additional seigniorage.
debt as a function of output, the fiscal shock, and unanticipated capital loss on government
debt with all variables expressed in real terms.

2.3 Fiscal Limits

World-wide increases in government debt relative to GDP have sparked interest in a new
literature on fiscal limits, where these limits are endogenous to a country’s economic and
political system and have no relation to the limits imposed in the EMU or any other limits
imposed exogenously on a government. These internal fiscal limits recognize that there
is an upper bound to the tax revenue that can be raised because taxes are distortionary;
explicitly, there is a top to the Laffer Curve (Trabandt and Uhlig, 2011). And the political
limit on taxation could be reached prior to reaching the top of the Laffer Curve (Bi et al.,
2010). Additionally, there is a minimum below which transfers or government spending
on public goods cannot be reduced. Davig et al. (2010, 2011), Davig and Leeper (2011),
and Cochrane (2011) have explored the implications of fiscal limits for inflation, and Daniel
(2010) explores their implications for an exchange rate crisis. Sims (1997, 1999) and Daniel
and Shiamptanis (2012) consider their implications for price stability in a monetary union.
Our paper considers the implications of fiscal limits for responsible fiscal rules.

Leeper (2010) defines the fiscal limit as the point at which the government can no longer
raise taxes or reduce spending and transfers. In models without growth and with an exoge-
nous path for government spending and transfers, he models the fiscal limit as a fixed value
for taxes. We generalize Leeper and do not assume exogenous government spending and
transfers. This leads us to express the fiscal limit in terms of the primary surplus. Following
Bi (2012), we assume that there is an upper bound on the present value of the future primary surplus ($s_t$) relative to output ($y_t$) that a government can raise.

Letting $g$ denote the rate of growth of output, define $R_j = \prod_{i=1}^{j} \frac{1+r}{1+g_{t+i}}$, as the gross growth-adjusted interest rate factor. We specify the fiscal limit as requiring that the present value of primary surpluses relative to output, discounted at the growth-adjusted interest rate, be less than an upper bound according to

$$\sum_{j=1}^{\infty} (R_j^{-1}) \frac{s_{t+j}}{y_{t+j}} \leq \varphi_{\text{max}} \sum_{j=1}^{\infty} (R_j^{-1}),$$

where $\varphi_{\text{max}}$ has the interpretation as the maximum value for the surplus relative to output if this value were constant. Combining the government’s intertemporal budget constraint, derived by solving equation (5) forward, with the upper bound on the present value of primary surpluses yields an upper bound on debt according to

$$b_t = \sum_{j=1}^{\infty} \left( \frac{1}{1+r} \right)^j s_{t+j} < y_t \varphi_{\text{max}} \sum_{j=1}^{\infty} (R_j^{-1}).$$

We can illustrate that Bohn’s criterion for intertemporal budget balance does not rule out explosive behavior, required by fiscal limits. Setting $\beta_1 = \beta_2 = 0$ in equation (6) to simplify and solving forward to obtain the value of debt $N$ periods ahead yields

$$b_{t+N} = (1+r-\beta_3)^N b_t,$$

Satisfaction of the government’s intertemporal budget constraint requires that the present value of debt be zero in the limit, explicitly

$$\lim_{N \to \infty} \frac{b_{t+N}}{(1+r)^N} = \lim_{N \to \infty} \left( \frac{1+r-\beta_3}{1+r} \right)^N b_t = 0.$$

$^5$ If the growth rate were constant at $g$, the limit on debt would simplify to $b_t < y_t \left( \frac{1+g}{r-g} \right) \varphi_{\text{max}}$. 

10
A positive response of the primary surplus to lagged debt, $\beta_3 > 0$, is sufficient to imply that debt grows more slowly than interest, yielding a zero limit and intertemporal budget balance. However, a small value for $\beta_3$ allows debt to grow faster than output such that debt relative to output grows forever and eventually violates any value for $\varphi^{\text{max}}$. Davig (2005) also notes that the government’s intertemporal budget constraint could be satisfied with an exploding debt relative to GDP, although he does not explicitly consider fiscal limits. Our purpose is to derive constraints on the more general fiscal rule, equation (1), to assure that debt does not grow faster than output in the limit.

The value for $\varphi^{\text{max}}$ could be stochastic as in Bi (2012), or heterogenous across countries as in Trabandt and Uhlig (2011). What matters in deriving restrictions for a responsible fiscal rule is the existence of a fiscal limit, not its magnitude or its time-varying behavior. If the value of debt relative to GDP is explosive, then the magnitude of the fiscal limit is not important because debt will eventually violate any finite limit. If we were interested in predicting a fiscal solvency crisis, as in Daniel and Shiamptanis (2012), then the magnitude of the fiscal limit, compared with the current values for debt and the primary surplus, would be important. The focus of this paper is on one necessary criterion for solvency – a fiscal rule which eliminates explosive behavior.

### 2.4 Dynamics

To complete the model, we specify output dynamics. Consistent with our empirical evidence, we specify output to be integrated of order one, giving the system a unit root. Output is

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6 We assume $r > g$. Otherwise, the government does not face a binding budget constraint.
determined by
\[ y_t - (1 + g) y_{t-1} = \rho [y_{t-1} - (1 + g) y_{t-2}] + \eta_t, \quad (7) \]
where \( 0 < \rho < 1 \) is the autocorrelation in output growth, \( 0 < g < 1 \) is the average long-run growth rate of output and \( \eta_t \) is a mean-zero output shock. Our assumption, that output is an independent integrated stochastic process, is consistent with a model in which output is driven by integrated exogenous technology shocks. Therefore, we are assuming that the primary surplus has no impact on output. In the macro literature, the sign of this effect is controversial and model specific. Our specification should be viewed as a simplification, which is standard in the literature on fiscal sustainability,\(^7\) and consistent with at least one mainstream macroeconomic model.

In the long-run, all the variables in the system grow at the average growth rate of output \((g)\). To solve for this, quasi-difference equations (1) and (6), to yield
\[ s_t - (1 + g) s_{t-1} = (\beta_1 - 1 - g) s_{t-1} + \beta_2 y_{t-1} + \beta_3 b_{t-1} + \nu_t, \quad (8) \]
\[ b_t - (1 + g) b_{t-1} = -\beta_1 s_{t-1} - \beta_2 y_{t-1} + (r - g - \beta_3) b_{t-1} - (\gamma_t - E_{t-1} \gamma_t) - \nu_t. \quad (9) \]
Setting these equations to zero and solving, with shocks taking on their expected values of zero, yields long-run relationships
\[ s_t = \varphi y_t \quad \text{and} \quad s_t = \frac{r - g}{1 + g} b_t \quad (10) \]
where
\[ \varphi = \frac{-\beta_2}{(1 + g) \left[ \frac{\beta_1}{1 + g} + \frac{\beta_3}{r - g} - 1 \right]} < \varphi_{\text{max}}, \quad (11) \]
and where \( \varphi \) has the interpretation of the long-run value of the primary surplus relative to GDP. In the long-run, the primary surplus and debt both grow at the average rate of growth of output, and the primary surplus is proportional to output and to debt, with the factor of proportionality to debt expressed as the growth-adjusted interest rate.

Using equation (10), equations (8) and (9) can be rearranged and written in error correction form as

\[
\begin{align*}
    s_t - (1 + g) s_{t-1} &= -\frac{\beta_3 (1 + g)}{r - g} \left[ s_{t-1} - \frac{r - g}{1 + g} b_{t-1} \right] - \frac{\beta_2}{\varphi} [s_{t-1} - \varphi y_{t-1}] + \nu_t, \\
    b_t - (1 + g) b_{t-1} &= -\left( 1 + g - \frac{\beta_3 (1 + g)}{r - g} \right) \left[ s_{t-1} - \frac{r - g}{1 + g} b_{t-1} \right] + \frac{\beta_2}{\varphi} [s_{t-1} - \varphi y_{t-1}] - (\gamma_t - E_{t-1} \gamma_t) - \nu_t.
\end{align*}
\] (12) (13)

The expressions in square brackets are the cointegrating vectors, equivalently the long-run relationships, given by equations (10). The error correction form implies that when the system deviates from its long-run conditions, the primary surplus and debt both adjust to these deviations. If the adjustment is in the correct direction and large enough, then the system is globally stable. A globally stable system is expected to return to long-run values. Therefore, a system which returns to these long-run relationships does not allow the primary surplus relative to output to grow forever, ruling out explosive behavior, and does not allow debt to grow forever relative to the primary surplus, implying intertemporal budget balance.

To determine the restrictions necessary for global stability, we need to solve the dynamic system, expressed in terms of the cointegrating vectors and the long-run relationship for output. Define \( Y_t \) and \( B_t \) as deviations from cointegrating vectors and \( \Gamma_t \) as the deviation of
output from its long-run growth relation according to

\[ Y_t = s_t - \varphi y_t \]

\[ B_t = s_t - \frac{r - g}{1 + g} b_t \]

\[ \Gamma_t = y_t - (1 + g) y_{t-1}. \]

Rewrite the dynamic system, comprised of equations (7), (12), and (13) in terms of \( Y_t, B_t \) and \( \Gamma_t \)

\[ Y_t = \left( 1 + g - \frac{\beta_2}{\varphi} \right) Y_{t-1} - \frac{\beta_3 (1 + g)}{r - g} B_{t-1} - \rho \varphi \Gamma_{t-1} - \varphi \eta_t + \nu_t \]

\[ B_t = -\frac{\beta_2}{\varphi} \left( \frac{1 + r}{1 + g} \right) Y_{t-1} + \left( 1 + g - \frac{\beta_3 (1 + g)}{r - g} \right) \left( \frac{1 + r}{1 + g} \right) B_{t-1} + \left( \frac{1 + r}{1 + g} \right) \nu_t + \left( \frac{r - g}{1 + g} \right) (\gamma_t - E_t \gamma_t) \]

\[ \Gamma_t = \rho \Gamma_{t-1} + \eta_t. \]

Letting \( \theta \) denote the roots of the system above, the characteristic equation is

\[ (\rho - \theta) \left\{ \left[ 1 + g - \frac{\beta_2}{\varphi} - \theta \right] \left[ \left( 1 + g - \frac{\beta_3 (1 + g)}{r - g} \right) \left( \frac{1 + r}{1 + g} \right) - \theta \right] - \frac{\beta_3 (1 + g)}{r - g} \frac{\beta_2}{\varphi} \left( \frac{1 + r}{1 + g} \right) \right\} = 0 \]

Using equation (11) to substitute for \( \varphi \), the characteristic equation simplifies to

\[ (\theta - \rho) [\theta^2 - (1 + r + \beta_1 - \beta_3) \theta + (1 + r) \beta_1] = 0. \]

Given that \( \rho \) is less than one\(^8\), the system is globally stable if the remaining roots are also less than one in absolute value. There are multiple distinct sets of restrictions on \( \beta_1 \) and \( \beta_3 \), which yield roots less than one in absolute value. We are interested in the empirically relevant set of restrictions, explicitly restrictions which admit positive values for \( \beta_1 \) and values for \( \beta_3 \) with an order of magnitude similar to an interest rate.

\(^8\) The group-mean panel estimate of \( \rho \) is 0.34
Both roots can be within the unit circle only if their product is within the unit circle. This requirement yields an upper bound on $\beta_1$ as

$$(1 + r) \beta_1 < 1.$$ (14)

Adding the requirement that the roots be real yields an upper bound on $\beta_3$ as

$$\beta_3 < 1 + r + \beta_1 - 2 ((1 + r) \beta_1)^{1/2}.$$ (15)

When $\beta_1$ and $\beta_3$ are positive and satisfy both (14) and (15), then the stability criterion is given by

$$\beta_3 > r (1 - \beta_1).$$ (16)

In a globally stable system, the values of $Y_t$, $B_t$ and $\Gamma_t$ are expected to approach zero in the limit. Equivalently, deviations from the cointegrating vectors vanish, and the system reaches the long-run relationships, given in equation (10). Equation (16) implies that the responsiveness of the primary surplus to lagged debt, given by $\beta_3$, must be larger than the real interest rate times one minus the persistence in the primary surplus. A positive,

9 If the largest root is less than unity, then both roots are inside the unit circle. The largest root is less than unity if

$$(1 + r + \beta_1 - \beta_3 + \left[ (1 + r + \beta_1 - \beta_3)^2 - 4 (1 + r) \beta_1 \right]^{1/2}) < 1$$

$$\left[ (1 + r + \beta_1 - \beta_3)^2 - 4 (1 + r) \beta_1 \right]^{1/2} < 1 - (r + \beta_1 - \beta_3).$$

Assume the right-hand side is positive and square both sides to obtain

$$\left[ (1 + r + \beta_1 - \beta_3)^2 - 4 (1 + r) \beta_1 \right] < [1 - (r + \beta_1 - \beta_3)]^2$$

Simplifying yields

$$r (1 - \beta_1) < \beta_3.$$
but weak response, implies that any initial deviations from cointegrating relationships will explode over time, violating any finite fiscal limit.

Consider the implications of the model solution for the intertemporal government budget constraint and fiscal limits. For positive values of the coefficients and for $\beta_3$ satisfying equation (15), the restrictions in equations (16) and (14) yield a system which is expected to reach long-run relationships, given by equations (10). These restrictions assure intertemporal budget balance and rule out explosive behavior. Alternatively, if both restrictions are not satisfied, then initial deviations from the cointegrating relationships are expected to grow forever. Growth of the primary surplus relative to output violates the fiscal limit, no matter how large the limit is, while growth of deviations of the primary surplus from debt service might or might not violate Bohn’s (1998, 2008) criterion for fiscal sustainability.

The introduction of fiscal limits brings the literature on fiscal sustainability full circle. Earlier work on fiscal sustainability, summarized by Trehan and Walsh (1991), defined sustainability as satisfaction of the government’s intertemporal budget constraint and argued that it required a primary surplus responsiveness to debt at least as large as the interest rate. Bohn (1998, 2008) demonstrated that any positive responsiveness would suffice because the intertemporal budget constraint did not require boundedness of the debt. However, fiscal limits require boundedness. When we add boundedness, we restore original criteria that the response of the primary surplus to lagged debt must be large enough. Our magnitudes differ only because we have added additional terms to the fiscal rule. When $\beta_1 = 0$, as assumed in earlier work, our restrictions reduce to the earlier one, where the primary surplus must
respond to lagged debt by more than the real interest rate.

A disclaimer is necessary. We are not testing whether fiscal policy is active or passive. When non-explosive equilibria exist under each regime, observable equilibrium conditions are equivalent, invalidating any tests which attempt to distinguish (Cochrane, 1998). Explicitly, government debt and the primary surplus must be cointegrated whether fiscal policy is active or passive. Additionally, a positive coefficient on lagged debt does not imply that the primary surplus responds to lagged debt. This is because debt can increase in equilibrium only if future primary surpluses are expected to increase. Therefore, the coefficient on debt could be positive even if the fiscal authority does not respond to debt. Rather, we assume that fiscal policy is passive and that the fiscal authority follows a rule under which it responds to lagged debt and other variables. Conditional on the assumption that the fiscal authority responds to debt, we ask "Does it respond enough for the equilibrium to be non-explosive?"

3 Empirical Results

The purpose of the empirical work is to test whether the fiscal rule characterizing policy in the EMU has been responsible. Our analysis yields two testable criteria: 1) the existence of cointegrating relationships between the primary surplus and debt, and between the primary surplus and output; and 2) a large enough response of the primary surplus to lagged debt to yield global stability for the dynamic system. If the fiscal rule is responsible, then both criteria should be satisfied. We test both, obtaining consistent results. Additionally, estimation of the parameters of the fiscal rule provides parameter values which researchers can use
to either calibrate the fiscal rule or forecast future values for debt and the primary surplus. We have annual data on the real primary surplus, $s_{it}$, real debt, $b_{it}$, and real GDP, $y_{it}$, for the period of forty-two years (1970-2011) for a panel of eleven EMU countries.\footnote{The countries were chosen based on data availability. They include Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. For more details see the Data Appendix.}

We use panel techniques to provide estimates of the parameters. Panels increase power, an important feature in a relatively-short time series (42 years). Mendoza and Ostry (2008) use Bohn’s (1998, 2008) criteria to assess fiscal sustainability for two large diverse panels, one of industrial countries over 1980-2005, and the other for emerging markets over 1990-2005. In contrast to Mendoza and Ostry (2008), we use panel techniques which permit full heterogeneity of both the short-run and long-run dynamics across the members in the panel, accounting for possible differences across the EMU countries.

The model is comprised of variables which can take on positive and negative values, but which also exhibit non-stationary geometric growth. This requires decisions on estimation. The standard way of dealing with geometric trends using logarithms is not available to us since the primary surplus does take on negative values. Bohn (1998, 2008) and Mendoza and Ostry (2008) transform the variables by dividing by output. We chose not to make this transformation for several reasons. First, such a transformation should eliminate the unit root in the data, invalidating the use of cointegration tests between the primary surplus and debt as a criterion for a responsible fiscal rule.\footnote{Recently, Berenguer-Rico and Carrion-I-Silvestre (2011) also use real variables instead of variables expressed as a percentage of GDP, allowing exploitation of the non-stationary characteristics of the series through multicointegration.} Second, the transformation makes the coefficients in the fiscal rule stochastic, with values depending on the realization of
the stochastic output growth rate. Third, Bohn (1998, 2008) finds that estimates of the parameters of the fiscal rule, with variables expressed as fractions of output, are sensitive to measures of transitory versus permanent spending.

We follow Hamilton and Flavin (1986), Trehan and Walsh (1991), and Berenguer-Rico and Carrion-I-Silvestre (2011) and estimate the model in levels. This allows us to exploit the time-series characteristics of the data using cointegration methods to obtain super-consistent estimates of the parameters of the cointegrating vectors. Additionally, the error correction specification makes a natural distinction between permanent and temporary changes without the need for developing a separate methodology for measuring transitory spending.

However, estimation in levels does have implications for inferences on magnitudes of parameter values. Estimation in levels carries with it the implicit assumption that the data has a linear trend instead of a geometric trend. Replacing the geometric trend with a linear trend changes the coefficients on the cointegrating relations and introduces constants into the cointegrating relationships and the error correction specification. We derive the cointegrating relationships and the error correction specification with a linear trend in Appendix A, and denote the modified coefficients in the cointegrating vectors with tildes ($\sim$). We show that stability criteria are identical in the two specifications. We also show that, at mean values, the cointegrating relationships with the linear trend are identical to those where growth has a geometric trend.
3.1 Time Series Characteristics of Data

Before we proceed with the estimation of cointegration and error correction, we establish that the variables behave as unit root processes, $I(1)$. We use panel unit root tests, which have more power than the time series unit root tests. Following Im et al. (2003), we test the null hypothesis that all series in the panel contain a unit root, against the alternative hypothesis that some of the series in the heterogeneous panel are stationary. The test is based on the average of $N$ individual augmented Dickey-Fuller (1979) (ADF) $t$-statistics. The tests, reported in Table 1, are computed using one lag and they include an individual specific constant and trend. All the tests fail to reject the null hypothesis of a unit root in $s_{it}$, $b_{it}$, and $y_{it}$ at the 5 percent level, implying that real primary surplus, real debt and real output are $I(1)$.

3.2 Cointegration

Given that the variables in the fiscal rule appear to behave as unit root processes, we test for the existence of long-run relationships using panel cointegration techniques. We estimate the following cointegrating model

$$s_{it} = a_i + \zeta_i x_{it} + e_{it}$$

where

$$e_{it} = \varrho_i e_{it-1} + \mu_{it}$$  \hspace{1cm} (17)

We also fail to reject the null of a unit root when the tests are computed alternately using two lags, and heterogeneous numbers of lags across countries, with the lag order estimated using consistent information criteria such as the Akaike and Schwarz criteria. Moreover, we do not reject the null of a unit root when we use Breitung (2000), Maddala and Wu (1999), Choi (2001) tests, and we reject the null of stationarity when we use Hadri (2000) test. Additionally, we confirm the unit root behavior of the series using panel unit root tests suggested by Breitung and Das (2005), Moon and Perron (2004), and Pesaran (2007), which account for various forms of cross-sectional dependence.
and test for the existence of cointegrating relationships between the real primary surplus and real debt, and between the real primary surplus and real GDP. In equation (17), \( a_i \) denotes the country specific fixed effects, \( s_{it} \) is the real primary surplus, \( x_{it} \) is the regressor, \( b_{it} \) or \( y_{it} \), and \( \zeta_i \) is the cointegrating parameter, \( r_i \) or \( \tilde{\varphi}_i \).\(^{13}\) Since countries can have different fiscal rules, we model heterogeneity across countries by allowing each country’s policy parameters to differ randomly from the EMU panel policy parameters. Letting \( \zeta_i \) be the \( i^{th} \) country’s vector of parameters, and \( \zeta \) the vector of EMU parameters, we assume that \( \zeta_i = \zeta + \epsilon_{\zeta i} \), where the \( \epsilon_{\zeta i} \) have zero-means and constant variances for all \( i \).

We begin with two group-mean panel cointegration \( t \) tests suggested by Pedroni (1999, 2004).\(^{14}\) Both tests are residual-based cointegration tests, which test the null hypothesis that the variables of interest are not cointegrated for all the countries in the panel against the alternative hypothesis that there exists a heterogeneous cointegration vector for all the countries in the panel. Under the null hypothesis of no cointegration, the residuals from (17), \( \hat{e}_{it} \), are \( I(1) \). Denoting the autoregressive coefficient of the \( i^{th} \) country’s residuals by \( \varrho_i \), the group-mean statistics test the null hypothesis of no cointegration, \( H_0 : \varrho_i = 1 \) for all \( i \), versus the heterogeneous alternative hypothesis, \( H_A : \varrho_i < 1 \) for all \( i \). The alternative does not presume the same value for \( \varrho_i \). The first group-mean test uses semi-parametric corrections, while the second is a parametric ADF test. The tests are extensions of the single time series Phillips-Perron (1988) \( t \)-test and the ADF \( t \)-test. We account for cross-

\(^{13}\)Since we estimate the model in levels, the cointegrating relationships will contain constants, derived in Appendix A.

\(^{14}\)The group-mean statistics allow modeling an additional source of potential heterogeneity across individual members of the panel. Moreover, Gutierrez (2003) finds that Pedroni’s tests have higher power than the system test proposed by Larsson et al. (2001). Additionally, Banarjee et al. (2004) show that for small \( N \) the size distortions of Pedroni’s tests are lower than those of Larsson and Lyhagen (1999) test.
sectional dependence and cross-member cointegration by using common time effects. For the semi-parametric test we use the Bartlett kernel and the Newey-West (1994) bandwidth selection procedure, and for the parametric ADF-type test, we use the step-down procedure to estimate the number of lags. The results, reported in Table 2, indicate that both tests reject the null hypothesis of no cointegration at the 5 percent level. Therefore, Pedroni’s tests provide strong evidence that $s_{it}$ and $b_{it}$ are cointegrated and that $s_{it}$ and $y_{it}$ are also cointegrated.

We confirm that there are two cointegrating relations in the trivariate model using the system panel cointegration test proposed by Larsson et al. (2001). Their test is a panel version of Johansen’s (1988, 1995) full information maximum likelihood method. The null hypothesis is that all of the $N$ countries in the panel have at most $q$ cointegrating relationships among the 3 variables, $H_0 : q$ cointegrating relations for all $i$, and the alternative is that all the countries have 3, $H_A : 3$ cointegrating relations for all $i$. This is a sequential procedure where the first test is the null $q = 0$. If this hypothesis is rejected, then the null $q = 1$ is tested. The sequential procedure continues until the null is not rejected or the hypothesis $q = 2$ is rejected.

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15Common time effects allow us to model a limited form of cross-sectional dependence and cross-member cointegration (which is a form of long-run cross-sectional dependence). Common time effects assume that the cross-sectional dependence correlation between country $i$ and $j$ is identical for all $i, j$. Thus, in the presence of heterogeneous cross-sectional dependence, subtracting off the cross-sectional average does not completely eliminate cross-sectional dependence. The method by which cross-sectional dependence is modeled in panels is still an active area of research. Bai and Ng (2002) and Moon and Perron (2004) consider models in which the error terms have a factor structure in panel unit root tests, however the implications for such factor models have not been studied in the panel cointegration context. Notice though that time effects are a special case of a factor model where there is a single common factor and the response of each country is similar. Therefore, time effects account for both cross-sectional dependence and cross-member cointegration when the source of dependency is due to a single common time specific shock such as a common global business cycle shock.

16We also reject the null of no cointegration at the 5 percent level when we use all seven panel statistics of Pedroni (1999, 2004). Additionally, we reject the null of no cointegration at the 5 percent level when we do not use the common time effects.
rejected. The test is computed using one lag in first differences and it includes individual specific fixed effects. In Table 3 we verify that there are two cointegrating relations in the model. The panel test statistic indicates that \( q = 2 \) in the model with three variables \( s_{it} \), \( b_{it} \) and \( y_{it} \). This implies that there is a single stochastic trend in the data for each country, implying that there are cointegrating relations between \( s_{it} \) and \( b_{it} \), and between \( s_{it} \) and \( y_{it} \), consistent with previous results.

Therefore, the data satisfy restrictions necessary for existence of a stationary long-run relationship between the primary surplus and debt, and between the primary surplus and output for each country. Stationary long-run relationships are consistent with a globally stable model, implying a responsible fiscal rule.

### 3.3 Fiscal Response to Debt

Finally, we test whether the magnitude of the response of the primary surplus to lagged debt has been large enough to satisfy restrictions necessary for global stability, given by equations (16). Appendix A provides a derivation of the error correction model, under the assumption in the estimation that the data has a linear trend, yielding

\[
s_{it} - s_{it-1} = \tilde{\phi}_i \tilde{g}_i - \frac{\beta_{3i}}{r_i} \left[ s_{it-1} - r_i b_{it-1} + \tilde{\phi}_i \tilde{g}_i \left( \frac{1 + r_i}{r_i} \right) \right] - \frac{\beta_{2i}}{\tilde{\phi}_i} \left[ s_{it-1} - \tilde{\phi}_i y_{it-1} + \tilde{\phi}_i \tilde{g}_i \left( 1 - \frac{\beta_{3i}}{\beta_{2i}} \left( \frac{1 + r_i}{r_i} \right) \right) \right] + \nu_{it},
\]

where \( \tilde{g}_i \) is average change in output, and \( \tilde{\phi}_i \) is a combination of the parameters in the fiscal rule such that \( \tilde{\phi}_i \tilde{g}_i \) has the interpretation of the long-run average change in the primary surplus. We estimate the parameters of the above error correction model using a two-step
procedure, in which we initially estimate the cointegrating parameters. In the second step, we estimate the coefficients on the error correction terms.

To estimate the cointegrating parameters, we use Pedroni’s (2001) group-mean dynamic OLS (DOLS) procedure for cointegrated panels, which is based on equation (17). The group-mean DOLS procedure accommodates the heterogeneity that is typically present both in the transitional serial correlation dynamics and in the long-run cointegrating relationships. It is a parametric approach that adjusts for the effects of endogenous regressors and short-run dynamics of the errors by using lead and lag differences of the regressor. We account for the cross-sectional dependence between the countries using common time effects as suggested by Pedroni (2000, 2001). Table 4 indicates that the group-mean panel estimates of the cointegrating parameters are \( r = 0.0445 \) and \( \varphi = 0.0379 \).

In the final step we consider estimation of the group-mean panel fiscal rule parameters \( \beta_1, \frac{\beta_2}{r}, \) and \( \frac{\beta_3}{r} \). It is important to recognize that the residuals in equation (18) could be autocorrelated in the data. If so, then the residuals could be correlated with the right-hand-side variables, biasing estimates of the coefficients on the error correction terms. Also, a

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17Bohn (2008) considers a similar equation to (17) with \( x_{it} \) given by debt, using over two centuries of US data on the primary surplus, output, and debt. His real data series have severe heteroskedasticity, due to two centuries of growth in real GDP. He reduces, but does not eliminate, these problems by dividing by real GDP. Standard deviations for real variables are 64 to 98 times as high in the second period as in the first. For variables expressed as a fraction of GDP, this number falls to about 2. We have a very different data set from Bohn’s—a relatively short time dimension and eleven countries. The shorter time dimension implies that we do not have Bohn’s heteroskedasticity problem. Ratios of standard deviations in the second half of the sample relative to the first half average to about 2, similar to his adjusted data.

18Since DOLS is designed to reduce bias associated with short-run dynamics and the estimates in equation (17) are super-consistent, it is not necessary to add stationary variables, like HP-filtered measures of the data, as in Bohn’s model with stationary data (2008).

19The DOLS results are robust to the choice of leads and lags. Additionally, we obtain similar estimates when we use the group-mean fully modified OLS (FMOLS) procedure of Pedroni (2000, 2004) and the two-step estimator of Breitung (2005).
sufficient number of lags must be included in order to ensure that the fiscal shocks are white noise. Therefore, we use Sims’ (1980) likelihood ratio test to determine the appropriate number of lags to fully capture the dynamics for each country. For some countries, the test implies that equation (18) is appropriately specified. However, for others, an additional lag is chosen. Therefore, we estimate the model with an additional lag, \( s_{it} - s_{it-1} = \tilde{\varphi}_i g_i - \frac{\beta_3 i}{r_i} \left( \tilde{B}_{it} \right) - \frac{\beta_2 i}{\tilde{\varphi}_i} \left( \tilde{Y}_{it} \right) + \beta_4 i \left( s_{it-1} - s_{it-2} - \tilde{\varphi}_i g_i \right) \) (19)

\[ + \beta_5 i \left( b_{it-1} - b_{it-2} - \frac{\tilde{\varphi}_i g_i}{r_i} \right) + \beta_6 i \left( y_{it-1} - y_{it-2} - \tilde{g}_i \right) + \nu_{it}. \]

where \( \tilde{B}_{it} \) and \( \tilde{Y}_{it} \) are the cointegrating vectors given by their linear counterparts and \( \tilde{g}_i \) is the average change in output, all derived in Appendix A. A persistent, but negative fiscal shock, perhaps created by a war, would imply a negative error in the first cointegrating relationship and rising lagged debt, implying a negative correlation between the two terms. Therefore, failure to include the lagged change in debt could bias the estimate on the coefficient on the error correction terms.\(^{21}\)

Now, consider estimation of the coefficients on the error correction terms in equation (19). First, we use the estimated cointegrating parameters of the real interest rate and long-run value of the primary surplus to construct the error correction terms for each country, \(^{20}\)The additional dynamics do not change the criterion for stability, given by equation (16), \( \frac{\beta_3}{r} > 1 - \beta_1 \), but modify slightly the criterion for \( \beta_1 \), which becomes \( \beta_1 \left( 1 + r \right) - \beta_5 < 1 + \beta_4 r \), and add a third criterion, given by \(-1 < \beta_4 \left( 1 + r \right) < 1.\)

\(^{21}\)Using long US samples of one and two centuries that highlight the role of wars, Bohn (2008) expresses the fiscal rule in terms of the primary surplus and debt as a fraction of output, and finds it necessary to add HP-filtered measures of transitory values of the variables to distinguish between the response of the primary surplus to permanent and transitory shocks and reduce "omitted variables bias." Transitory, but persistent government spending, as associated with a war, would be accompanied by rising debt implying that the residual would be low when debt is high relative to the surplus. The error correction model deals with this by adding lagged changes of the model variables. In this example, the lagged change in debt would be high in a war and would play the role of Bohn’s transitory military spending, and the lagged change in output would play the role of Bohn’s HP-filtered output.
yielding an equation in which all the variables are stationary. Asymptotically, the fact that we use the estimated error correction terms rather than the true error correction terms in (19) does not affect the standard properties of our estimates due to the super-consistency properties of the estimator of the cointegrating relationships.\footnote{See Engle and Granger (1987), Toda and Phillips (1993) and Urbain (1992) for the properties of estimators in cointegrated systems.} After constructing the error correction terms, we estimate the coefficients of the error correction model, augmented with lagged changes in variables, providing estimates for \( \beta_1, \frac{\beta_2}{\varphi}, \) and \( \frac{\beta_3}{r} \). We use the group-mean procedure recommended by Pesaran and Smith (1995).\footnote{They show that when the parameters of interest are heterogeneous, the group-mean procedure provides consistent estimates, whereas the pool panel procedures give inconsistent estimates.} 

Table 5 indicates that the panel estimates are: \( \beta_1 = 0.5470, \frac{\beta_2}{\varphi} = -1.1812 \) and \( \frac{\beta_3}{r} = 1.6341 \). Each coefficient is statistically different from zero at the 5% level, and \( \frac{\beta_2}{\varphi} \) is statistically significantly larger than \( 1 - \beta_1 \).\footnote{We obtain the same result when we use the hierarchical Bayes procedure of Hsiao et al. (1999) and the weighted method of Swamy (1980) (also referred as the empirical Bayes procedure). Additionally, our results satisfy the criterion on \( \beta_1 \left( \beta_1 (1 + r) - \beta_4 r - \beta_5 = 0.6019 < 1 \right), \) and the criterion on \( \beta_4 \left( \beta_4 (1 + r) = 0.0617 < 1 \right), \) since these criteria are necessary but not sufficient, we compute the three roots with our estimates of parameter values, which satisfy these restrictions, confirming that all roots are real, positive, and less than unity.} Additionally, our results imply that the coefficient on lagged debt, \( \beta_3 \), is 0.0727, which is similar in magnitude to those obtained by Bohn (2008) and Mendoza and Ostry (2008).\footnote{Bohn’s (2008) estimates for \( \beta_3 \) using US data over 1792-2003 range from 0.028 to 0.147. Mendoza and Ostry (2008) estimates for \( \beta_3 \) using 22 industrial countries over 1970-2005 range from 0.020-0.038 and using 34 emerging economies over 1990-2005 range from 0.035 to 0.106.} Moreover, our estimates imply that that the long-run value of the primary surplus, \( \varphi = \frac{-\beta_2}{(1+g)\left[\frac{\beta_1}{1+g} + \frac{\beta_3}{r-g} \right]} \), is 1.54% of GDP. The long-run value of debt, \( \frac{\varphi(1+g)}{(r-g)} \), is 71.62% of GDP, which is below reasonable values for the fiscal limit.

We also present parameter estimates for each individual country in Table 6. Our inference for each country’s fiscal rule is weaker. We find that each country’s \( \frac{\beta_2}{\beta_3} \) is larger than \( 1 - \beta_1 \).
although standard errors are sometimes too large to yield significance for each member in the panel. The large individual-country standard errors illustrate the reasons for using panel estimation instead of time series estimation. Although the individual country results are not all significant, signs are correct, providing some individual country corroboration for our panel results.

At first glance the estimated coefficient on output, or equivalently the estimated coefficient on the long-run value for the primary surplus, $\beta_2$, might seem to have the wrong sign. An alternative and equivalent expression of the error correction model is more intuitive. Using results in Appendix A, the fiscal rule in equation (18) can also be written as

$$s_{it} - s_{it-1} = (1 - \beta_1) (\varphi_i y_{it-1} - s_{it-1}) + \beta_3 (r_i b_{it-1} - \varphi y_{it-1}) + v_{it},$$

which implies that the primary surplus increases whenever the lagged primary surplus is below its long-run value and whenever lagged debt service is above its long-run value. As shown in the Appendix A, the criterion $\frac{\beta_3}{\varphi_i} > 1 - \beta_1$ is equivalent to $\frac{\beta_2}{\varphi_i} < 0$. Given that $\varphi_i > 0$, the value for $\beta_2$ must be negative.

We have not allowed estimates of the parameters of the error correction model to change as the monetary union has evolved over time. However, if the coefficients in the cointegrating relationships had changed and no account were taken for the change, then we should not have rejected the null of no cointegration. The values for the coefficients on the error correction terms could have changed. To test for a change in $\beta_1$, $\frac{\beta_2}{\varphi_i}$, and $\frac{\beta_3}{r_i}$, we break the sample into two sub-periods, the pre-EMU era (1970 – 1998) and the post-EMU era (1999 – 2011). Table 7

26 For example, assume that there is an unmodelled change in $\varphi$. The residuals in (17) will be non-stationary and cointegration would be rejected.
shows that $\frac{\beta_4}{r} > 1 - \beta_1$ in both sub-samples, and there is some evidence that $\frac{\beta_4}{r}$ is statistically significantly larger than $1 - \beta_1$ in the post-EMU era. Testing the hypothesis whether $\frac{\beta_4}{r}$ is statistically significantly larger than $1 - \beta_1$ is equivalent to testing the hypothesis that $\frac{\beta_4}{\phi}$ is significantly less than zero. In the pre-EMU era, the standard errors are too high to yield significance for both hypothesis tests.\footnote{The sub-samples are not long enough to allow reliable inference. Most of our panel techniques require the time series dimension to be substantially larger than the cross-section dimension, and we lose this when we split the sample.} In the post-EMU era, $\frac{\beta_4}{r}$ is statistically significantly larger than $1 - \beta_1$ at the 5 % level, but $\frac{\beta_4}{\phi}$ is not significantly less than zero. Therefore, the signs and magnitudes in the split-samples are consistent with our hypothesis that the fiscal rule has been responsible, but the sample sizes are too short to yield significance.\footnote{Afonso (2008) and Annett (2006), who use a similar EMU data set, but different empirical techniques, do not find evidence for a change in fiscal policy as the monetary union evolved over time.}

In summary, our group-mean panel results imply that primary surpluses in the EMU countries have been sufficiently responsive to lagged debt to satisfy the criteria for the fiscal rule to have been responsible. When we reduce the size of the sample, either by estimating individual-country fiscal rules or by splitting the sample, we retain the magnitudes necessary for responsibility, but we do not always get significance.

4 Conclusion

The first contribution of the paper is theoretical. We assume that the fiscal authority follows a rule and establish criteria for the rule to satisfy the government’s intertemporal budget constraint and eliminate explosive behavior of debt and the primary surplus relative to output. We define this rule as responsible, and we solve for the restrictions on parameter

\footnote{Afonso (2008) and Annett (2006), who use a similar EMU data set, but different empirical techniques, do not find evidence for a change in fiscal policy as the monetary union evolved over time.}
values necessary for the fiscal rule to be responsible. We find that the response of the primary surplus to debt must be large enough to render the system, expressed as long-run equilibrium deviations (equivalently cointegrating vectors), globally stable. This modifies Bohn’s (1998, 2008) condition, in which he defines a sustainable fiscal rule in the absence of fiscal limits as one which satisfies the government’s intertemporal budget constraint. Our additional criterion in the presence of fiscal limits, requires a larger response of the primary surplus to government debt. Moreover, consideration of non-explosive behavior, required by fiscal limits, restores cointegration as a requirement for a responsible fiscal rule, since debt, the primary surplus, and GDP must reach stable long-run relationships with each other. These results imply two empirically testable criteria for a responsible fiscal rule. The first is cointegration, and the second is the magnitude of the responsiveness of the primary surplus to lagged debt in the fiscal rule.

Our theoretical contribution brings the literature on fiscal sustainability full circle. Trehan and Walsh (1991) summarized requirements for fiscal policy to satisfy the government’s intertemporal budget constraint as those which we present to satisfy both the intertemporal budget constraint and fiscal limits. Bohn (2007) argued that those requirements were too strong to satisfy the government’s intertemporal budget constraint alone. We show that adding fiscal limits restores those original criteria.

The second contribution of the paper is empirical. We test for cointegration, and we estimate the parameters of the fiscal rule for a panel of eleven EMU countries over the period of 1970-2011, using panel techniques that allow for heterogeneity across the countries.
We find that over the sample period, the data exhibits the required cointegration and the required responsiveness of the primary surplus to lagged debt for the fiscal rule to have been responsible.

However, even a country following a responsible fiscal rule can experience a fiscal solvency crisis. Bad luck, experienced as a string of negative fiscal shocks together with the policy response to those shocks, can send the system onto a path which is expected to violate the fiscal limit, creating a solvency crisis. Or policy makers can make future promises which create an insolvent fiscal position into which creditors refuse to lend. This analysis suggests that the recent financial turmoil in several EMU countries is not a consequence of explosive fiscal rules, but rather of adverse fiscal shocks or insolvent future promises.

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Appendix
A. Model with a Linear Trend

The model consists of equations (1), and (6), with the equation for output growth is expressed as

\[ y_t - y_{t-1} - \bar{g} = \rho [y_{t-1} - y_{t-2} - \bar{g}] + \eta_t, \quad (20) \]

where \( \bar{g} \) is the average change in output, such that \( \bar{g} = \frac{\Delta y}{y} = g \). To solve for long-run equilibrium changes, first difference equations (1) and (6) and drop time subscripts to yield

\[ \Delta s = r \Delta b = \bar{\varphi} \Delta y = \bar{\varphi} \bar{g} \]

where

\[ \bar{\varphi} = \frac{-\beta_2}{\beta_1 + \beta_3/r - 1} < \bar{\varphi}_{\text{max}}. \]

Since output changes on average by \( \bar{g} \), the primary surplus changes on average by \( \bar{\varphi} \bar{g} \), and debt changes on average by \( \frac{\bar{\varphi}}{r} \). Differentiating and subtracting the linear trend in the surplus and debt from equations (1) and (6), respectively, yields

\[ s_t - s_{t-1} - \bar{g} \bar{\varphi} = (\beta_1 - 1) s_{t-1} + \beta_2 y_{t-1} + \beta_3 b_{t-1} - \bar{g} \bar{\varphi} + \nu_t \quad (21) \]

\[ b_t - b_{t-1} - \frac{\bar{g} \bar{\varphi}}{r} = -\beta_1 s_{t-1} - \beta_2 y_{t-1} + (r - \beta_3) b_{t-1} - \frac{\bar{g} \bar{\varphi}}{r} - (\gamma_t - E_{t-1} \gamma_t) - \nu_t \quad (22) \]

To solve for long-run equilibrium relationships, set the equations above equal to zero, with disturbances at their expected values of zero, to yield

\[ s_t = rb_t - \bar{\varphi} \bar{g} \left( \frac{1 + r}{r} \right) = \bar{\varphi} y_t - \frac{\bar{\varphi}^2 \bar{g}}{\beta_2} \left( 1 - \frac{\beta_3}{r} \left( \frac{1 + r}{r} \right) \right) \quad (23) \]
Therefore, compared with the theoretical model, which has a geometric trend, the empirical model, estimated with a linear trend over the finite sample, yields cointegrating vectors with constants and different coefficients. At mean values, the cointegrating relationships are identical to those in the model with a geometric trend.

To prove this, express the first equilibrium relationship at the mean as

\[ \bar{s} = r\bar{b} - \bar{g}\bar{\varphi} \left( \frac{1 + r}{r} \right) = r\bar{b} - \Delta \bar{s} = \Delta \bar{b} \]

Dividing both sides by \( \bar{b} \) yields

\[ \frac{\bar{s}}{\bar{b}} = r - \left( \frac{\Delta \bar{s}}{\bar{s}} \right) \left( \frac{\bar{s}}{\bar{b}} \right) - \frac{\Delta \bar{b}}{\bar{b}} \]

Since

\[ \left( \frac{\Delta \bar{s}}{\bar{s}} \right) = \frac{\Delta \bar{b}}{\bar{b}} = g \]

\[ \bar{s} = \frac{r - g\bar{b}}{1 + g} \]

(24)

The second cointegrating vector is obtained by setting equation (21) equal to zero, with shocks at their expected value of zero. Evaluating at the mean yields

\[ (\beta_1 - 1) \bar{s} + \beta_2 \bar{y} + \beta_3 \bar{b} - \bar{g}\bar{\varphi} = 0. \]

Substitute for \( \bar{b} \) from equation (24) and use \( \bar{g}\bar{\varphi} = \Delta \bar{s} \) to yield

\[ (\beta_1 - 1) \bar{s} + \beta_2 \bar{y} + \beta_3 \frac{1 + g}{r - g} \bar{s} - \Delta \bar{s} = 0. \]

Dividing by \( \bar{y} \) and rearranging yields

\[ \left( \beta_1 - 1 + \beta_3 \frac{1 + g}{r - g} \right) \frac{\bar{s}}{\bar{y}} + \beta_2 - \left( \frac{\Delta \bar{s}}{\bar{s}} \right) \left( \frac{\bar{s}}{\bar{y}} \right) = 0. \]
Using $\frac{\Delta s}{s} = g$ yields
\[
\left(\beta_1 - 1 - g + \frac{\beta_3}{r - g} \frac{1 + g}{r} \right) \frac{s}{y} + \beta_2 = 0
\]
\[
\frac{s}{\beta_1 + \beta_3 \left(\frac{1 + g}{r - g} \right) - (1 + g)}\frac{y}{\varphi} = \varphi \bar{y}.
\]

Using equations (21), (22), and (23), the error correction model can be expressed as
\[
s_t - s_{t-1} = \varphi \bar{g} - \frac{\beta_3}{r} \left(s_{t-1} - rb_{t-1} + \varphi \bar{g} \left(\frac{1 + r}{r}\right)\right) - \frac{\beta_2}{\varphi} \left(s_{t-1} - \varphi y_{t-1} + \frac{\varphi^2 \bar{g}}{\beta_2} \left(1 - \frac{\beta_3}{r} \left(\frac{1 + r}{r}\right)\right)\right) + \nu_t
\]
\[
b_t - b_{t-1} = \frac{\varphi \bar{g}}{r} - \left(1 - \frac{\beta_3}{r}\right) \left(s_{t-1} - rb_{t-1} + \varphi \bar{g} \left(\frac{1 + r}{r}\right)\right) + \frac{\beta_2}{\varphi} \left(s_{t-1} - \varphi y_{t-1} + \frac{\varphi^2 \bar{g}}{\beta_2} \left(1 - \frac{\beta_3}{r} \left(\frac{1 + r}{r}\right)\right)\right) - \nu_t - (\gamma_t - E_{t-1} \gamma_t)
\]

Defining deviations from cointegrating vectors and long-run relationships as
\[
\check{Y}_t = s_t - \varphi y_t + \frac{\varphi^2 \bar{g}}{\beta_2} \left(1 - \frac{\beta_3}{r} \left(\frac{1 + r}{r}\right)\right)
\]
\[
\check{B}_t = s_t - rb_t + \varphi \bar{g} \left(\frac{1 + r}{r}\right)
\]
\[
\check{\Gamma}_t = y_t - y_{t-1} - \bar{g},
\]

we can rewrite the dynamic system in terms of the cointegrating vectors and the long-run relationship of output as
\[
\check{Y}_t = \left(1 - \frac{\beta_2}{\varphi}\right) \check{Y}_{t-1} - \frac{\beta_3}{r} \check{B}_{t-1} - \check{\varphi} \rho \check{\Gamma}_{t-1} + \nu_t - \check{\varphi} \eta_t
\]
\[
\check{B}_t = - (1 + r) \frac{\beta_2}{\varphi} \check{Y}_{t-1} + \left(1 - \frac{\beta_3}{r}\right) \left(1 + r\right) \check{B}_{t-1} + (1 + r) \nu_t + r (\gamma_t - E_{t-1} \gamma_t)
\]
\[
\check{\Gamma}_t = \rho \check{\Gamma}_{t-1} + \eta_t.
\]
Substituting for $\tilde{\varphi}$, the characteristic equation is identical to that for the theoretical model with the geometric trend. Therefore, the requirements for global stability are identical. The requirement that $\frac{\beta_3}{r} > 1 - \beta_1$ is simply the requirement that

$$\frac{-\beta_2}{\tilde{\varphi}} = \beta_1 + \beta_3/r - 1 > 0.$$ 

The loading on the second error correction term in equation (25) must be positive.

B. Data Appendix

Data is primarily from the OECD database, and for missing years data is obtained from the ECB’s AMECO database. For the nominal primary surplus we use the general government primary balances (OECD Annex Table 29) and for nominal debt we use the general government gross financial liabilities (OECD Annex Table 33). The sample consists of annual data from 1970-2011 for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. For Germany we use the data for West Germany before unification and Germany after unification. The real values of the variables for each country are obtained by dividing the nominal values by the GDP deflator.
References


Table 1. Panel Unit Root Tests

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>$s_{it}$</th>
<th>$b_{it}$</th>
<th>$y_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{IPS}$</td>
<td>-0.98</td>
<td>1.78</td>
<td>-0.14</td>
</tr>
<tr>
<td>$t_{BD-rob}$</td>
<td>-1.38</td>
<td>-0.30</td>
<td>-0.33</td>
</tr>
<tr>
<td>$t_{MP-a}$</td>
<td>0.61</td>
<td>0.52</td>
<td>0.93</td>
</tr>
<tr>
<td>$t_{MP-b}$</td>
<td>0.38</td>
<td>0.64</td>
<td>0.63</td>
</tr>
<tr>
<td>$t_{CIPS}$</td>
<td>-2.23</td>
<td>-1.29</td>
<td>-1.43</td>
</tr>
</tbody>
</table>

Note: $t_{IPS}$ is the group mean $t$-statistic proposed by Im et al. (2003), $t_{BD-rob}$ is the OLS robust $t$-statistic proposed by Breitung and Das (2005), $t_{MP-a}$ and $t_{MP-b}$ are the $t$-statistics based on the factor model proposed by Moon and Perron (2004) and $t_{CIPS}$ is the test proposed by Pesaran (2007). They all test the null of a unit root against the alternative of stationarity. All tests include individual specific constants and trends. For the $t_{IPS}$, $t_{BD-rob}$, $t_{MP-a}$ and $t_{MP-b}$ tests, the null is rejected if $t < -1.64$ and for the $t_{CIPS}$ test, the null is rejected if $t_{CIPS} < -2.85$. No test statistic rejects the null at the 5 percent level.

Table 2: Residual Based Panel Cointegration Tests

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>$s_{it}$ and $b_{it}$</th>
<th>$s_{it}$ and $y_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group mean $t$ (semi-parametric)</td>
<td>-6.80**</td>
<td>-6.43**</td>
</tr>
<tr>
<td>Group mean $t$ (parametric)</td>
<td>-8.15**</td>
<td>-7.79**</td>
</tr>
</tbody>
</table>

Note: The statistics are distributed standard normal. They test the null hypothesis of no cointegration against the alternative of cointegration. All the tests include individual specific fixed effects and common time effects. The test statistics with ** reject the null hypothesis at the 5 percent level. The 5 percent critical value is -1.64.

Table 3: System Based Panel Cointegration Tests

<table>
<thead>
<tr>
<th>$s_{it}, b_{it}, y_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 0$</td>
</tr>
<tr>
<td>$q = 1$</td>
</tr>
<tr>
<td>$q = 2$</td>
</tr>
<tr>
<td>Cointegrating relations</td>
</tr>
</tbody>
</table>

Note: The panel cointegration statistic tests the null hypothesis that there are $q$ cointegrating relationships against the alternative that there are 3 cointegrating relationships. The test includes individual specific fixed effects. The test statistics with ** reject the null at 5 percent level. The panel test has a 5 percent critical value of 1.64. The moments used for the panel test are tabulated in Shiamptanis (2008).
### Table 4: Panel Estimates of the Cointegrating Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$r$</th>
<th>$\tilde{\varphi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>0.0445**</td>
<td>0.0379**</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.0030)</td>
<td>(0.0047)</td>
</tr>
</tbody>
</table>

Note: $r$ and $\tilde{\varphi}$ are the group-mean panel estimates of the cointegrating parameters. They include individual specific effects and common time effects. The ** indicate statistical significance at the 5 percent level.

### Table 5: Panel Estimates of the Fiscal Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\beta_1$</th>
<th>$\frac{\beta_2}{\tilde{\varphi}}$</th>
<th>$\frac{\beta_3}{r}$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$\beta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>0.5470**</td>
<td>-1.1812**</td>
<td>1.6341**</td>
<td>0.0591</td>
<td>-0.0332</td>
<td>0.3555</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.0930)</td>
<td>(0.2857)</td>
<td>(0.2951)</td>
<td>(0.0637)</td>
<td>(0.0577)</td>
<td>(0.1895)</td>
</tr>
</tbody>
</table>

Note: $\beta_1$, $\frac{\beta_2}{\tilde{\varphi}}$ and $\frac{\beta_3}{r}$ are the group-mean panel estimates for the loadings on the error correction terms. $\beta_4$, $\beta_5$ and $\beta_6$ are the group-mean panel estimates for the coefficients on the lagged change in surplus, the lagged change in debt and the lagged change in output, respectively. The ** indicate statistical significance at the 5 percent level.
### Table 6: Country Estimates of the Fiscal Parameters

<table>
<thead>
<tr>
<th>Countries</th>
<th>$\beta_1$</th>
<th>$\frac{\beta_2}{\varphi}$</th>
<th>$\frac{\beta_3}{\tau}$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$\beta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.6256**</td>
<td>-1.9854**</td>
<td>2.3598**</td>
<td>-0.1364</td>
<td>-0.1293</td>
<td>0.3989**</td>
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<tr>
<td></td>
<td>(0.1208)</td>
<td>(0.7709)</td>
<td>(0.7762)</td>
<td>(0.1586)</td>
<td>(0.0967)</td>
<td>(0.1164)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.7865**</td>
<td>-2.8416**</td>
<td>3.0551**</td>
<td>-0.2649</td>
<td>-0.1830</td>
<td>-0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.1046)</td>
<td>(0.8878)</td>
<td>(0.9061)</td>
<td>(0.1746)</td>
<td>(0.0948)</td>
<td>(0.1901)</td>
</tr>
<tr>
<td>Finland</td>
<td>0.5061**</td>
<td>-1.2844**</td>
<td>1.7783**</td>
<td>-0.1587</td>
<td>-0.3031</td>
<td>0.3223</td>
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<tr>
<td></td>
<td>(0.1712)</td>
<td>(0.5349)</td>
<td>(0.5108)</td>
<td>(0.2531)</td>
<td>(0.1672)</td>
<td>(0.1913)</td>
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<tr>
<td>France</td>
<td>0.3276</td>
<td>-1.1635**</td>
<td>1.8359**</td>
<td>0.0642</td>
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<td>0.4142</td>
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<td>(0.1671)</td>
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<td>(0.1725)</td>
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<td>(0.1446)</td>
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<td>Ireland</td>
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<td>-2.5751**</td>
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<td>2.0708**</td>
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<tr>
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<td>(0.2249)</td>
<td>(0.5166)</td>
<td>(0.4411)</td>
<td>(0.4205)</td>
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<td>(0.3047)</td>
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<tr>
<td>Italy</td>
<td>0.8078**</td>
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<td>0.0285</td>
<td>0.0977</td>
<td>0.1190</td>
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<tr>
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<td>(0.0877)</td>
<td>(0.2422)</td>
<td>(0.2664)</td>
<td>(0.1809)</td>
<td>(0.0604)</td>
<td>(0.1465)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.1272</td>
<td>-0.1552</td>
<td>1.0279**</td>
<td>0.0941</td>
<td>-0.1590**</td>
<td>0.5610**</td>
</tr>
<tr>
<td></td>
<td>(0.1889)</td>
<td>(0.1712)</td>
<td>(0.2361)</td>
<td>(0.1771)</td>
<td>(0.0743)</td>
<td>(0.2220)</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.8271**</td>
<td>-1.1426</td>
<td>1.3156</td>
<td>0.0442</td>
<td>0.0335</td>
<td>0.2289</td>
</tr>
<tr>
<td></td>
<td>(0.1660)</td>
<td>(1.0435)</td>
<td>(1.0610)</td>
<td>(0.2127)</td>
<td>(0.1831)</td>
<td>(0.1661)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.8440**</td>
<td>-0.9048**</td>
<td>1.0608**</td>
<td>0.5244**</td>
<td>-0.0632</td>
<td>-0.3623</td>
</tr>
<tr>
<td></td>
<td>(0.1164)</td>
<td>(0.4024)</td>
<td>(0.3968)</td>
<td>(0.1956)</td>
<td>(0.1533)</td>
<td>(0.2162)</td>
</tr>
</tbody>
</table>

Note: The ** indicate statistical significance at the 5 percent level.

### Table 7: Panel Estimates of the Fiscal Parameters Before and After EMU

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\frac{\beta_2}{\varphi}$</th>
<th>$\frac{\beta_3}{\tau}$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$\beta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-EMU</td>
<td>0.6783**</td>
<td>-1.0669</td>
<td>1.3836</td>
<td>-0.0303</td>
<td>0.0937</td>
<td>0.2031**</td>
</tr>
<tr>
<td></td>
<td>(0.0884)</td>
<td>(0.7803)</td>
<td>(0.7915)</td>
<td>(0.1079)</td>
<td>(0.0601)</td>
<td>(0.0665)</td>
</tr>
<tr>
<td>Post-EMU</td>
<td>0.0202</td>
<td>-1.9125</td>
<td>2.8973**</td>
<td>0.3277</td>
<td>-0.0850</td>
<td>0.5569</td>
</tr>
<tr>
<td></td>
<td>(0.2575)</td>
<td>(1.1821)</td>
<td>(1.0451)</td>
<td>(0.1989)</td>
<td>(0.1449)</td>
<td>(0.3254)</td>
</tr>
</tbody>
</table>

Note: The ** indicate statistical significance at the 5 percent level.