Negative Income Taxes, Inequality, and Poverty

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Abstract: We use a neoclassical growth model with heterogeneous agents to analyze the redistributive effects of a negative income tax system, which combines a flat rate tax with a fully refundable credit ("demogrant"). We show that changing the demogrant-to-output ratio causes significant changes in the distribution of income. Specifically, we find that increasing the demogrant-to-output ratio sharply reduces the level of inequality as well as both relative and absolute poverty (all measured in terms of post-tax total income). However, these reductions in inequality and poverty come at the expense of a significant reduction in output.

JEL Codes: D63, H23, I32

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1 Introduction

Our objective in this paper is to analyze the effects of a negative income tax (NIT) system on inequality and poverty within a dynamic optimizing general equilibrium model. A NIT, advocated by Friedman (1962) as a means of alleviating poverty, combines a (positive) flat tax rate with a fully refundable credit or "demogrant".[1] Although this type of tax policy has been the focus of a number of previous studies, the relationship between the parameters of this policy and the levels of inequality and poverty have not previously been analyzed in a dynamic general equilibrium framework. More generally, while such a framework has been used to study the effects of various tax reforms on specific inequality measures (e.g., the Gini coefficient), our paper broadens such analyses by considering (i) the (generalized) Lorenz dominance criterion for inequality comparisons (see Section 2), and (ii) various measures of poverty, which have largely been ignored in this strand of literature.

Specifically, we employ a standard neoclassical growth model with heterogeneous agents, as in Aiyagari and McGrattan (1998), henceforth AM. This model incorporates uncertainty in the form of uninsured idiosyncratic shocks to labor productivity. We show that altering the demogrant-to-output ratio causes significant changes in the distribution of hours worked, asset holdings, and income. In particular, we find that increasing the demogrant-to-output ratio reduces the level of inequality as well as both relative and absolute poverty (all measured in terms of post-tax total income). In this respect, our results reinforce the arguments of authors as diverse as Friedman (1962), Tobin (1968) and Rawls (1971) in favour of a NIT as a means of alleviating poverty. However, we also find that these reductions in inequality and poverty are accompanied by a significant contraction in economic activity and, beyond a demogrant-to-output ratio of 5.0%, a fall in mean lifetime utility (i.e., utilitarian social welfare).

More specifically, we find that relative poverty (given a fixed poverty line equal to one-half of the median level of post-tax income) is eliminated when the demogrant-to-output ratio is greater than or equal to 19.7%, which requires a flat tax rate of 50.9%. Moreover, at this threshold level of the demogrant-to-output ratio, the Gini coefficient for post-tax total income is 0.384, as compared to 0.504 when the demogrant-to-output ratio is zero. Unsurprisingly, however, the high tax rate required to finance this demogrant-to-output ratio severely distorts the labor-leisure choice and discourages investment. This results in large reductions in both output and mean lifetime util-

[1] See also Moffitt (2003) for a general discussion.

ity: at this threshold level of the demogrant-to-output ratio, output is 68.7% of its level with a demogrant-to-output ratio of zero, while the loss in mean lifetime utility is 4.0% of annual consumption. Nonetheless, we find that *absolute* poverty (given a fixed poverty line equal to the relative poverty line when the demogrant-to-output ratio is equal to zero) is substantially reduced: the headcount ratio (i.e., the proportion of the population which is poor) is 19.8% at this threshold level of the demogrant-to-output ratio, while it is 29.1% with a demogrant-to-output ratio of zero.

1.1 Existing Literature

In the past, the effects of various income tax reforms on the level of inequality and poverty have primarily been analyzed using static partial equilibrium models. In such models, it is typical for the wage distribution to be given exogenously, with agents choosing only how much labour to supply. For example, Lambert (1985) and Creedy (1997) use such models to examine the effects of flat tax reforms on the level of inequality.[2] Creedy (1997) also considers the effect of such reforms on the level of poverty. Kanbur et al. (1994) use a similar model to find the non-linear income tax policy which minimizes poverty. While both Creedy (1997) and Kanbur et al. (1994), focus on fixed poverty lines, Thompson (2012) considers the effect of various flat tax reforms on the level of poverty when the poverty line is relative (i.e., some fraction of either the mean or median post-tax income level).

More recently, tax policy reforms have been analyzed using dynamic stochastic general equilibrium models with heterogenous agents. Heathcote (2005) considers a variety of alternative model economies with both idiosyncratic labor productivity risk and aggregate uncertainty in order to quantify the short-run effects of changes in the timing of proportional income tax rate changes under capital market imperfections. Heathcote finds that temporary changes have significant real effects. In particular, compared to a complete-markets economy, income tax cuts cause a larger response in consumption and a smaller response in investment when asset markets are incomplete.

Ventura (1999) uses an overlapping generations model to analyze the effects of switching from a progressive tax system that approximates the actual U.S. system to a flat rate tax system with differential rates for labour and capital income. The author finds that the flat tax reform leads to a substantial increase in capital accumulation and a significant change in the distribution of hours worked. These changes lead to large increases in inequality for both labour and capital income as

[2]Davies and Hoy (2002) consider the effect of tax reforms on the level of inequality using the Lorenz dominance criterion (see Section 2). Their paper, however, uses a static model in which the pre-tax income distribution is exogenously given (i.e., agents supply labour inelastically).

well as wealth. A similar model is used by Conesa and Krueger (2006) to find the flat rate tax system which maximizes utilitarian social welfare. The authors show that long-run welfare gains from a fundamental reform towards the flat tax are significant.

Lopez-Daneri (2015) analyzes the macroeconomic effects of moving from progressive tax and welfare systems that approximate their actual U.S. counterparts to a NIT using an overlapping generations model. As in our case, uncertainty arises due to the presence of idiosyncratic shocks to labor productivity. However, an additional source of uncertainty involves lifetime uncertainty in the sense that there is an age-dependent probability that the agent will not survive to the next period; the assets of agents that do not survive to the next period are expropriated by the government. Also, the NIT is accompanied by a social security system in which transfers are evenly distributed amongst all agents in a given post-retirement cohort.

Similar to our results, Lopez-Daneri finds that, within a NIT, an increase in the demogrant-to-output ratio leads to a significant contraction in economic activity, a substantial reduction in capital accumulation and a fall in labor supply. However, the author finds that the NIT which maximizes utilitarian social welfare involves a demogrant-to-output ratio of 11.0%, which is substantially higher than our finding of 5.0%. This difference can be traced back to the life-cycle features of Lopez-Daneri's model: In his overlapping generations model, the losses resulting from the reduction in economic activity are not borne equally across different generations, while in ours, these losses are borne by agents who are infinitely-lived.

Our focus on the redistributive effects of a NIT, in terms of both inequality and poverty, is motivated by the possibility that social preferences are not necessarily utilitarian. Interestingly, several other recent studies on redistributive fiscal policy consider non-utilitarian social welfare functions. Wane (2001) uses a static partial equilibrium model to find the income tax schedule which maximizes a social welfare function in which poverty enters as a public 'bad'. Heathcote and Tsujiyama (2015) use a static model to analyze several tax policies, including a NIT,[3] while assuming a social welfare function which incorporates a parameter governing the social preference for redistribution. Hsu and Yang (2013) use a dynamic stochastic general equilibrium model, based on AM, to find the linear and two-bracket tax schedules which maximize either a utilitarian or a Rawlsian social welfare function.

This paper is organized as follows. Section 2 provides a brief discussion of income inequality and

[3]Heathcote and Tsujiyama (2015) refer to the NIT as simply an 'affine tax function'.

poverty measurement. Section 3 describes the model used in our quantitative exercise. Section 4 presents our results, and Section 5 concludes.

2 Inequality and Poverty Measurement

In measuring inequality, we take as a focal point the Lorenz curve, which indicates the share of total income earned by any quantile of the population. Formally, the Lorenz curve at point $p \in [0,1]$ is defined as

$$L(p) = \frac{1}{\mu_X} \int_0^p F_X^{-1}(t)dt,$$

where F_X is the distribution function and μ_X is the mean. Specifically, to compare the levels of inequality for two different distributions, we use the concept of Lorenz dominance. We say that distribution A Lorenz dominates distribution B if $L_A(p) \geq L_B(p)$ for all $p \in [0,1]$, with strict inequality holding for at least some p.

It is interesting to note that, if distribution A Lorenz dominates distribution B, then any inequality measure satisfying the widely-accepted Pigou-Dalton "principle of transfers" (e.g., the Gini coefficient, $I = 1 - 2\int_0^1 L(p)dp$, or the coefficient of variation), will be lower for distribution A than for distribution B.[4] Thus, if (and only if) distribution A Lorenz dominates distribution B, we will say that the level of inequality in distribution A is *unambiguously* lower than in distribution B.[5]

If distribution A Lorenz dominates distribution B and $\mu_A \geq \mu_B$, then distribution A can be ranked above distribution B by a wide class of social welfare functions (see Atkinson, 1970). On the other hand, the generalized Lorenz curve, given by $GL(p) = \mu_X L(p)$ for $p \in [0,1]$, may be of some use in producing a welfare ranking under more general conditions. For example, if there exists a p^* such that $GL_A(p) \geq GL_B(p)$ for all $p \in [0, p^*)$, then distribution A will be ranked above distribution B using the Rawlsian maxi-min criterion, regardless of the ordering of means (see Dardanoni and Lambert, 1988).

[4] An inequality measure is said to satisfy the Pigou-Dalton "principle of transfers" if it increases in response to a regressive transfer (i.e., a transfer from a poorer individual to a richer individual) and decreases in response to a progressive transfer (i.e., a transfer from a richer individual to a poorer individual).

[5] Of course, when the Lorenz curves for two distributions intersect, we can not use the concept of Lorenz dominance to rank them, even though some particular inequality measure may be higher for one of them. That is, we may be able to rank distributions using, e.g., the Gini coefficient, but not using Lorenz dominance.

We measure the level of poverty using the Foster et al. (1984) class of measures,

$$P_\epsilon = \int_0^z \left(\frac{z-x}{z}\right)^\epsilon dF_X(x),$$

where z is the poverty line (which may be relative or absolute) and ϵ is a parameter representing the degree of poverty aversion (of a social planner). For example, $P_0 = F_X(z)$ is the well-known headcount ratio (i.e., the proportion of the population which is poor), while P_1 is the so-called poverty gap ratio (i.e., the mean relative distance below the poverty line for the entire population). For any $\epsilon > 1$, P_ϵ satisfies the monotonicity and transfer axioms of Sen (1976).[6] More generally, higher values of ϵ put greater emphasis on individuals who are further below the poverty line. Note that, as $\epsilon \to \infty$, poverty comparisons based on P_ϵ are equivalent to those using the Rawlsian maximin criterion.

3 The Model

We consider a closed economy that consists of a large number of infinitely lived agents. These agents supply labor elastically and are subject to idiosyncratic shocks to their labor productivity. Let e_t denote an agent's labor productivity. We assume that this level of productivity is identically and independently distributed across all agents and follows a Markov process over time. Per capita labor productivity is normalized to unity, which implies that $E(e_t) = 1$. The saving behavior of these agents is heavily influenced by borrowing constraints and a precautionary savings motive. In the absence of insurance markets, agents try to smooth their consumption over time by trades in the two risk-free assets in the model: capital and government bonds.

The aggregate production function exhibits constant-returns to scale and is given by $Y_t = F(K_t, z_t N_t)$, where Y_t denotes per capita output, K_t denotes the capital stock per capita and N_t denotes the per capita labor input. The exogenous labor-augmenting measure of technological progress z_t grows at rate g, which implies that $z_t = z_0 (1+g)^t$.

Note that there are no aggregate shocks. Along the balanced growth path (BGP) equilibrium, there are fluctuations in an agent's assets, consumption, income and labor (leisure). However, per

[6] Sen's monotonicity axiom requires that a reduction in the income of an individual with income below the poverty line must lead to an increase in the poverty measure. Sen's transfer axiom requires that a regressive transfer from a poor individual (i.e., a transfer of income from an individual with income below the poverty line to any richer individual) must lead to an increase in the poverty measure. It is noteworthy that P_0 does not satisfy either of these axioms, while P_1 satisfies only the monotonicity axiom.

capita variables are growing at the constant rate g, with the exception of the per capita labor input (N) which remains constant. In addition, all cross-sectional distributions (relative to per capita values) are also constant over time along the BGP equilibrium.

Assuming competitive product and factor markets, the pre-tax wage rate w_t and interest rate r_t are given by

$$w_t = z_t F_2 \left(K_t, z_t N_t\right), \tag{1}$$

and

$$r_t = F_1 \left(K_t, z_t N_t\right) - \delta, \tag{2}$$

respectively, where δ is the depreciation rate of capital. Note that along the BGP equilibrium, Y_t and K_t are growing at the rate g, while N_t and r_t are constant: i.e., $N_t = N$ and $r_t = r$, $\forall\ t$. Both the labor supply and real interest rate are determined endogenously.

We now describe the individual agent's problem. In each period the agent has a fixed time endowment normalized to unity. In period t, fraction l_t of this endowment is allocated to leisure, and the remainder is allocated to supplying labor. Pre-tax wage income in period t is thus given by $w_t e_t (1 - l_t)$. Letting a_t denote the agent's assets at the beginning of period t, the agent's pre-tax interest income in period t is given by $r a_t$. Both wage income and interest income are taxed at the flat rate $0 < \tau < 1$. The agent also receives the demogrant $D_t \geq 0$ in period t, which is not taxed.

Let c_t denote the agent's consumption in period t. The agent's utility in period t is given by $(c_t^{\eta} l_t^{1-\eta})^{1-\mu}/(1-\mu)$, where $\mu > 0$ is the coefficient of relative risk aversion and $0 < \eta < 1$ is the share of private consumption in full-consumption. Due to the uncertainty of future labor productivity shocks, the agent chooses stochastic processes for consumption, leisure (labor) and asset holdings in order to maximize the expected discounted sum of utilities given some initial assets a_0 and an initial productivity shock e_0. Formally, the agent's problem is given by

$$\max_{\{c_t, l_t, a_{t+1}\}} \quad E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\eta} l_t^{1-\eta}\right)^{1-\mu}}{1-\mu} \middle| a_0, e_0 \right],$$

$$\text{subject to} \quad c_t + a_{t+1} \leq \overline{w}_t e_t \left(1 - l_t\right) + \left(1 + \overline{r}\right) a_t + D_t,$$

$$c_t \geq 0, \quad 0 \leq l_t \leq 1, \quad a_t \geq 0,$$

where $\overline{w}_t = (1-\tau)w_t$ and $\overline{r} = (1-\tau)r$ are the post-tax wage rate and the post-tax interest rate, respectively. Note that the constraint $a_t \geq 0$ prevents the agent from borrowing.

We now consider the government's budget constraint. Let G_t represent per capita government consumption. Following AM, we assume that government consumption equals a fixed fraction γ of output: i.e., $\gamma = G_t/Y_t$. Letting B_t denote the per capita stock of government debt at the beginning of period t, the government's budget constraint is given by

$$G_t + D_t + rB_t = B_{t+1} - B_t + \tau\left(w_t N + rA_t\right). \tag{3}$$

Next, let A_t denote the per capita assets held by the agents. The asset market equilibrium condition is

$$A_t = K_t + B_t. \tag{4}$$

In addition, the labor market clearing condition is given by

$$N = E\left[e_t\left(1 - l_t\right)\right], \tag{5}$$

where the expectation is taken with respect to the steady-state distribution. Note that, along the BGP equilibrium, D_t, B_t and A_t will also be growing at the constant rate g.

In solving the model, it is convenient to perform a stationary transformation by dividing all variables that grow at the rate g by output Y_t. Let

$$k = \frac{K_t}{Y_t}, \quad \overline{w} = \frac{\overline{w}_t}{Y_t} = \frac{(1-\tau)\,w_t}{Y_t}, \quad \widetilde{c}_t = \frac{c_t}{Y_t},$$

$$\widetilde{a}_t = \frac{a_t}{Y_t}, \quad b = \frac{B_t}{Y_t}, \quad \overline{a}_t = \frac{A_t}{Y_t}, \quad \chi = \frac{D_t}{Y_t}.$$

Note that along the BGP equilibrium, all variables included in the ratios above grow at rate g, which is the growth rate of the exogenous labor-augmenting technological progress z_t.

The agent's problem can now be expressed as

$$\max_{\{\widetilde{c}_t, l_t, \widetilde{a}_{t+1}\}} \quad E\left[Y_0^{\eta(1-\mu)} \sum_{t=0}^{\infty} \left[\beta (1+g)^{\eta(1-\mu)}\right]^t \frac{\left(\widetilde{c}_t^{\eta} l_t^{1-\eta}\right)^{1-\mu}}{1-\mu} \middle| \widetilde{a}_0, e_0\right],$$

$$\text{subject to} \quad \widetilde{c}_t + (1+g)\,\widetilde{a}_{t+1} \leq (1+\overline{r})\,\widetilde{a}_t + \overline{w} e_t (1-l_t) + \chi,$$

$$\widetilde{c}_t \geq 0, \quad 0 \leq l_t \leq 1, \quad \widetilde{a}_t \geq 0.$$

Combining the asset market clearing condition (4) with the government's budget constraint (3), and using the equilibrium conditions (1) and (2), allows us to rewrite the government's budget constraint as

$$\gamma + \chi + (\overline{r} - g)\, b = \tau\,(1 - \delta k)\,. \tag{6}$$

Finally, dividing through the asset market equilibrium condition (4) by Y_t yields

$$\overline{a} = k + b. \tag{7}$$

The BGP equilibrium of the original economy corresponds to the steady state of the transformed economy. Using the equation for the pre-tax interest rate (2), we can express the capital stock per unit of effective labor $K_t/(z_t N)$ as a function of r. From the properties of the production function, it follows then that the capital-output ratio along the BGP equilibrium $k = K_t/Y_t$ can be written as a function of r. Letting this function be denoted as $\kappa(r)$, we have

$$k = \kappa(r)\,. \tag{8}$$

Furthermore, using equation (1) we can express the share of wage income $\widetilde{w} = w_t N / Y_t$ as a function of r. Letting this function be denoted as $\omega(r)$, we have

$$\widetilde{w} = \omega(r)\,. \tag{9}$$

It follows then that $\overline{w} = \overline{w}_t / Y_t$ can be expressed as

$$\overline{w} = (1-\tau)\,\omega(r)\,/N. \tag{10}$$

The steady state is fully characterized by an interest rate r^* and a per capita effective labor input N^* that solve

$$\begin{aligned}
\overline{a}\left(r, N; \gamma, b, g, \chi\right) &= \kappa\left(r\right) + b, &\quad (11)\\
\overline{\varphi}\left(r, N; \gamma, b, g, \chi\right) &= N. &\quad (12)
\end{aligned}$$

Equation (11) corresponds to the asset market equilibrium condition (7). The lefthand side of this equation represents the per capita demand for assets by the individuals relative to output per capita. The righthand side represents the per capita supply of assets relative to per capita output. The supply of assets equals the sum of capital $\kappa\left(r\right)$ and government bonds b.

The lefthand side of (11) is obtained as follows. Solving the household's problem yields a decision rule for the optimal asset accumulation that can be expressed as $\widetilde{a}_{t+1} = \alpha\left(\widetilde{a}_t, e_t; r, N, \gamma, b, g, \chi\right)$. Given a Markov process for the labor productivity shock e_t, this decision rule can be used to compute the stationary joint distribution of assets and the productivity shock denoted by $H \equiv H\left(\widetilde{a}, e; r, N, \gamma, b, g, \chi\right)$. This stationary joint distribution can be used to calculate per capita assets as $\overline{a} = \int\int \widetilde{a} dH = \overline{a}\left(r, N; \gamma, b, g, \chi\right)$, which provides us with the lefthand side of (11). The righthand side of (11) is obtained from (7) using (8).

Equation (12) represents the per capita effective labor supplied by the individuals. This expression is obtained as follows. From (5), we know that $N = 1 - E\left(e_t l_t\right)$. After deriving the first-order condition with respect to leisure from the household's problem and taking expected values, we can express $E\left(e_t l_t\right)$ as a function of $E\left(\widetilde{c}_t\right)$ and $\overline{w}$ from (10). Consolidating the budget constraints of the individuals and the government's budget constraint yields the economy's resource constraint. This constraint allows us to express per capita consumption $E\left(\widetilde{c}_t\right)$ as a function of parameters of the model such as γ and g, as well as r. Combining $E\left(\widetilde{c}_t\right)$ with $\overline{w}$ yields $E\left(e_t l_t\right)$ and, hence, N.

Given values for r, N, γ, b, g and χ, the government's budget constraint (6) can be solved to yield τ. Then, $\overline{r} = (1 - \tau)r$ and $\overline{w}$ can be determined from (10). The household's problem is then solved by obtaining the decision rules for consumption, leisure and assets by applying the finite element method. McGrattan (1996) provides a detailed description of the finite element method that can be used in solving dynamic stochastic general equilibrium models.

3.1 Parameterization

With the exception of one of the parameters in the process for idiosyncratic productivity shocks (explained below), all parameter values are identical to those used by AM (which are based on Aiyagari, 1994). The parameter values describing aggregate variables and technology were calibrated to the first four decades of the post-WWII U.S. economy. The values for all preference parameters are commonly used in growth and business cycle models, and are consistent with long-run macroeconomic features of the U.S. economy (see, e.g., Prescott, 1986). Furthermore, they are very similar to those commonly used elsewhere in the literature on heterogenous agents models (e.g., Heathcote, 2005).

The growth rate g is set equal to 1.85%. The production function is assumed to be Cobb-Douglas and the share of capital, denoted by θ, is set equal to 0.30. The depreciation rate δ is set equal to 0.075. The ratio of government spending to output γ is set equal to 21.7%, while the debt-to-output ratio b is set equal to 66.66%.[7] In what follows, we allow the value of the demogrant-to-output ratio χ vary between zero and 20%.[8]

The risk-aversion coefficient μ is set equal to 1.5, while the discount factor β is set equal to 0.991. The value of η is set equal to 0.328, which corresponds to a labour elasticity of 2% (see AM, pp. 460–461). Note that the calibrated values of β, g, η and μ imply an effective discount factor of 0.988 in the stationary economy.

Finally, the idiosyncratic productivity shock e_t is assumed to follow the process

$$\ln(e_t) = \rho \ln(e_{t-1}) + v_t, \tag{13}$$

where $E(v_t) = 0$ and $\mathrm{Var}(v_t) = \sigma^2$. We follow AM in setting the autocorrelation coefficient ρ equal to 0.6, but set the standard deviation of the error term σ equal to 0.4 (rather than 0.3), which is at the top end (rather than the middle) of the range considered by Aiyagari (1994). As shown in Table 1, changing the value of σ in the benchmark parameterization of AM from 0.3 to 0.4 results in a distribution of wage income that better matches the estimates of Díaz-Giménez et al. (2011) for the U.S.

[7] While this value is based on observed U.S. data, AM actually find that it is optimal in the sense of maximizing utilitarian social welfare (under their benchmark parameterization).

[8] In their benchmark parameterization, AM set the transfers-to-output ratio – which is analogous to our demogrant-to-output ratio – equal to 8.2%.

The autoregression in (13) is approximated using the method of Tauchen (1986) with a first-order Markov chain that has seven states. The parameters for this Markov process are the values for the seven states,

$$\{0.2865, 0.4346, 0.6592, 1.0000, 1.5169, 2.3010, 3.4903\}$$

and a probability transition matrix,

$$\begin{pmatrix} 0.2329 & 0.3897 & 0.2895 & 0.0796 & 0.0080 & 0.0003 & 0.0000 \\ 0.0878 & 0.2895 & 0.3897 & 0.1947 & 0.0358 & 0.0024 & 0.0001 \\ 0.0239 & 0.1504 & 0.3672 & 0.3326 & 0.1116 & 0.0137 & 0.0006 \\ 0.0046 & 0.0545 & 0.2422 & 0.3975 & 0.2422 & 0.0545 & 0.0046 \\ 0.0006 & 0.0137 & 0.1116 & 0.3326 & 0.3672 & 0.1504 & 0.0239 \\ 0.0001 & 0.0024 & 0.0358 & 0.1947 & 0.3897 & 0.2895 & 0.0878 \\ 0.0000 & 0.0003 & 0.0080 & 0.0796 & 0.2895 & 0.3897 & 0.2329 \end{pmatrix}.$$

In summary, we use the following parameter values:

$$g = 0.0185, \quad \theta = 0.3, \quad \delta = 0.075, \quad \gamma = 0.217, \quad b = 0.6666$$

$$\mu = 1.5, \quad \beta = 0.991, \quad \eta = 0.328, \quad \rho = 0.6, \quad \sigma = 0.4$$

In Section 4.1, we undertake a sensitivity analysis in which we vary the value of μ. We find that our quantitative results, particularly in terms of relative poverty alleviation, are reasonably robust.

4 Results

From the government's flow budget constraint (6), it is clear that, by keeping the debt- and government spending-to-output ratios constant, a rise in the demogrant-to-output ratio will cause an increase in the tax rate, τ. Indeed, by increasing the demogrant-to-output ratio from zero to 20%, we find that the tax rate rises monotonically from 27.9% to 51.3% (see Table 2 for a summary of our results). It is interesting to note that, at a demogrant-to-output ratio of 20%, 48.8% of the population is faced with a non-positive tax liability (the demogrant is greater than or equal to the tax paid on the wage and interest income of these agents).

The increase in the tax rate distorts the labor-leisure choice and results in a fall in aggregate labor supply from 0.380 to 0.280 as the demogrant-to-output ratio is increased from zero to 20%. This reduction in labour supply causes an increase in the pre-tax wage rate from 1.843 to 2.497. On the other hand, the increase in the tax rate is sharp enough that the post-tax wage rate falls from 1.329 to 1.217.

Similarly, the rising tax rate caused by the increase in the demogrant-to-output ratio distorts investment, resulting in lower asset accumulation. Specifically, the assets-to-output ratio falls from 3.522 to 3.045 as the demogrant-to-output ratio is increased from zero to 20%. Due to the decrease in investment, the pre-tax interest rate increases sharply from 3.0% to 5.1%. However, despite the large increase in the tax rate, the post-tax interest rate rises only slightly, from 2.2% to 2.5%.

As a result of the large decreases in both the labour supply and investment, output at a demogrant-to-output ratio of 20% is only 68.2% of its level with a demogrant-to-output ratio of zero. Moreover, mean lifetime utility at a demogrant-to-output ratio of 20% is reduced by 4.2% of annual consumption (as compared to a demogrant-to-output ratio of zero). However, as seen in Figure 1, mean lifetime utility does not decrease monotonically with the demogrant-to-output ratio (as output does). Indeed, mean lifetime utility is maximized at a demogrant-to-output ratio of 5.0%.

We now turn to the redistributive effects of increasing the demogrant-to-output ratio. In Figure 2a, we plot the Lorenz curves for wage income corresponding to demogrant-to-output ratios of zero and 20%. Note that the Lorenz curves for pre- and post-tax wage income, as well as for hours worked, are all identical since they are constant multiples of one another. Clearly, the distribution of wage income at a demogrant-to-output ratio of zero Lorenz dominates that of wage income at a demogrant-to-output ratio of 20%, which implies an unambiguously higher level of inequality in the latter case; see Section 2. We also confirm this numerically for all intermediate levels of the demogrant-to-output ratio. Indeed, the Gini coefficient for wage income (which, like the Lorenz curve, is the same pre- and post-tax) monotonically increases from 0.580 to 0.680 as the demogrant-to-output ratio is increased from zero to 20%. This unambiguous and substantial increase in wage income inequality can be explained by the fact that, although all individuals reduce the number of hours they work, higher productivity individuals reduce their labor supply by a lesser amount compared to lower productivity individuals.

Similarly, although investment in assets falls for all individuals, it falls by slightly less for the higher productivity individuals who work more relative to the lower productivity individuals and, thus, have relatively higher disposable income to invest. Figure 2b displays the Lorenz curves for interest income corresponding to demogrant-to-output ratios of zero and 20%. Note that the Lorenz curves for pre- and post-tax interest income, as well as for wealth, are all identical since they are constant multiples of one another. Although it is difficult to detect visually, the distribution in interest income at a demogrant-to-output ratio of zero Lorenz-dominates that of the interest income at a demogrant-to-output ratio of 20% (again, we confirmed this numerically for all intermediate cases). This is also reflected in the behavior of the Gini coefficient for interest income, which, as seen in Table 2, increases slightly from 0.425 to 0.441 as the demogrant-to-output ratio is increased from zero to 20%. Of course, since interest income is just a multiple of asset holdings, the Lorenz curve for asset holdings is identical to that shown for interest income in Figure 2b.

In contrast to what we saw for wage and interest income, the Lorenz curves for pre- and post-tax total income (see Figures 2c and 2d, respectively) are different from one another. The reason for this is the inclusion of the demogrant in total income, which makes post-tax total income an *affine* function of pre-tax total income. However, the Lorenz curves, and thus the Gini coefficients, for pre- and post-tax total income are equal when the demogrant-to-output ratio is zero. As the demogrant-to-output ratio is increased, the resulting pre-tax total income distribution is always Lorenz-dominated by the existing one. However, the resulting post-tax total income distribution always Lorenz-dominates the existing one.

The increase in pre-tax total income inequality follows directly from the combined increase in both wage and capital income inequality. On the other hand, the effect of the increasing demogrant is strong enough to cause the level of inequality in post-tax total income to fall. This result is consistent with the theoretical result of Davies and Hoy (2002), where the number of hours worked is fixed. Furthermore, it implies that the Gini coefficient for pre-tax total income is rising, from 0.504 to 0.564, while for post-tax total income it is falling from 0.504 to 0.383.

Generalized Lorenz curves for post-tax total income are shown in Figure 3 (the distributions of wage income, interest income, and pre-tax total income at a demogrant-to-output ratio of zero clearly dominate the corresponding distributions at any higher demogrant-to-output ratio in the generalized Lorenz order). Note that, in computing these Generalized Lorenz curves, the mean level of post-tax income has been scaled by the relative output level (i.e., the mean level of post-tax income is multiplied by 0.682 when the demogrant-to-output ratio is 20%). It is interesting to note that, at a demogrant-to-output ratio of 20%, the generalized Lorenz curve for post-tax total income lies above that corresponding to a demogrant-to-output ratio of zero for all p up to 0.600. Accordingly, we can conclude that the (post-tax) income of the poorest individuals actually increases as the demogrant-to-output ratio rises, thereby producing an improvement under the Rawlsian maxi-min criterion; see Section 2. Clearly, this finding has important implications for the level of poverty reduction, a topic to which we now turn.

Figure 4 displays the Foster et al. (1984) poverty measures, P_ϵ with $\epsilon \in \{0, 1, 2\}$, for post-tax total income (see also Table 2). We consider both relative (Figure 4a) and absolute (Figure 4b) poverty lines. As noted in Section 1, the relative poverty line is set equal to one-half of the median level of post-tax income (a level widely-used in studies of relative poverty; see Smeeding, 2006). The absolute poverty line is fixed at the level of the relative poverty line when the demogrant-to-output ratio is equal to zero. Changes in the level of absolute poverty thus reflect both the redistributive effects and the output effects of the change in tax policy.

From Figure 4, it is clear that all of these poverty measures are falling substantially as the demogrant-to-output ratio is increased. This is consistent with the static partial equilibrium results of Thompson (2012). It is interesting to note that, while the headcount ratio is equal to 29.1% when the demogrant-to-output ratio is zero, relative poverty is entirely eliminated (i.e., $P_\epsilon = 0$ for any ϵ) when the demogrant-to-output ratio is 19.7%, which corresponds to a tax rate of 50.9%. Moreover, despite the large decreases in output, absolute poverty is substantially reduced as the demogrant-to-output ratio is increased: the headcount ratio (i.e., the proportion of the population which is poor) is 19.8% at this threshold level of the demogrant-to-output ratio, while it is 29.1% with a demogrant-to-output ratio of zero.

Note also that the effect of relatively small increases in the demogrant-to-output ratio on both relative and absolute poverty is even more pronounced for the higher order poverty measures, P_1 and P_2. This is because these measures place a greater weight on "extreme" poverty. The demogrant essentially provides a minimum income guarantee, significantly increasing the incomes of the poorest individuals. Nevertheless, one needs to take into account that increasing the demogrant-to-output ratio reduces the incomes of many non-poor individuals, as is clear from the generalized Lorenz curves shown in Figure 3.

4.1 Sensitivity analysis

We now consider the sensitivity of our results with respect to the coefficient of relative risk aversion, μ. The inverse of μ is the intertemporal elasticity of substitution. As a consequence, it is a key parameter that affects the household's consumption/saving and labour supply decisions. Specifically, we consider two higher values of μ, namely 2.0 and 2.5, and compare them with our benchmark value of $\mu = 1.5$.

Qualitatively, the results of increasing the demogrant-to-output ratio on all variables of interest are the same as in the benchmark model. Therefore, in order to preserve space, we limit our attention to only the minimum demogrant-to-output ratio that eliminates relative poverty. We denote this threshold level of the demogrant-to-output ratio by χ^*.

As is seen in Table 3, higher values of μ require a larger demogrant-to-output ratio (and therefore a higher tax rate) in order to eliminate relative poverty. However, these increases are not large. Specifically, in increasing μ from 1.5 to 2.5, χ^* only increases from 19.7% to 22.0%. Thus, this particular quantitative result is quite robust to the value of μ. Table 3 also shows that the reductions in output at χ^* are slightly larger at higher values of μ, while the reductions in mean lifetime utility are substantially larger at higher values of μ.

5 Conclusion

We have shown that increases in the demogrant-to-output ratio (between zero and 20%) reduce the level of inequality, and both relative and absolute poverty (all measured in terms of post-tax total income). However, these reductions in inequality and poverty are accompanied by a significant contraction in output and, beyond a demogrant-to-output ratio of 5.0%, a fall in mean lifetime utility.

The model that we considered assumes an exogenously determined growth rate for labor-augmenting technological progress. As a consequence, the redistributive policies that we studied had no impact on the economy's long-run growth rate. The effect of these policy changes on the growth rate could be examined using an endogenous growth model, as in, e.g., Cassou and Lansing (2004, 2006). Another possible extension is to incorporate differential taxation between capital and labour, as in Ventura (1999) and Conesa et al. (2009), or a progressive income tax schedule, as in Hsu and Yang (2013) and Heathcote et al. (2014). We intend to pursue such extensions in future research.

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	AM Calibration		Estimate for
	$\sigma = 0.3$	$\sigma = 0.4$	U.S. in 2007
Gini coefficient	0.524	0.619	0.636
Share of 1st quintile	0.002	0.000	-0.001
Share of 2nd quintile	0.069	0.025	0.042
Share of 3rd quintile	0.145	0.110	0.117
Share of 4th quintile	0.277	0.269	0.208
Share of 5th quintile	0.507	0.596	0.635

NOTE: The estimates, from Díaz-Giménez et al. (2011), are for earnings (defined as "the rewards to all types of labor including entrepreneurial labor", p. 3).

Table 1: Distribution of wage income under different parameterizations.

	Demogrant-to-output ratio (χ)				
	0	0.05	0.10	0.15	0.20
Tax rate	0.279	0.340	0.399	0.457	0.513
Tax non-payers	0	0.199	0.288	0.345	0.488
Hours	0.380	0.355	0.331	0.306	0.280
Wage rate					
Pre-tax	1.843	1.970	2.117	2.291	2.497
Post-tax	1.329	1.301	1.272	1.244	1.217
Assets-to-output ratio	3.522	3.416	3.301	3.178	3.045
Interest rate					
Pre-tax	0.030	0.034	0.039	0.044	0.051
Post-tax	0.022	0.023	0.023	0.024	0.025
Relative output	1	0.920	0.841	0.761	0.682
% change in mean lifetime utility	0	0.571	0.029	-1.525	-4.181
Gini coefficient					
Wage income / hours	0.580	0.603	0.627	0.653	0.680
Interest income / wealth	0.425	0.427	0.430	0.434	0.441
Pre-tax total income	0.504	0.518	0.533	0.548	0.564
Post-tax total income	0.504	0.475	0.444	0.414	0.383
Relative poverty					
P_0 (headcount ratio)	0.291	0.265	0.209	0.108	0
P_1 (poverty gap ratio)	0.147	0.092	0.043	0.009	0
P_2	0.090	0.040	0.012	0.001	0
Absolute poverty					
P_0 (headcount ratio)	0.291	0.279	0.251	0.220	0.189
P_1 (poverty gap ratio)	0.147	0.103	0.068	0.042	0.025
P_2	0.090	0.047	0.022	0.010	0.004

NOTE: The change in mean lifetime utility is measured in terms of annual consumption. Tax non-payers refers to the share of the population that is faced with a non-positive tax liability.

Table 2: Summary of results.

	Coefficient of Relative Risk Aversion (μ)		
	1.50	2.00	2.50
Min. χ eliminating relative poverty (χ^*)	0.197	0.210	0.220
Tax rate at χ^*	0.509	0.521	0.530
Relative output at χ^*	0.687	0.663	0.643
% change in mean lifetime utility at χ^*	-3.991	-5.719	-7.003

Table 3: Sensitivity analysis.

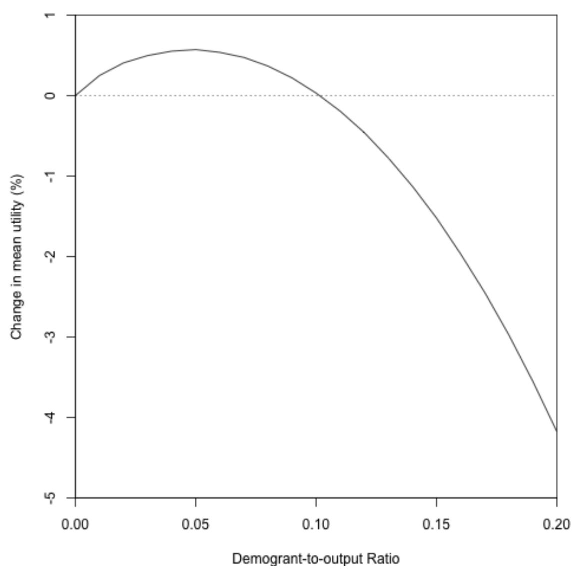
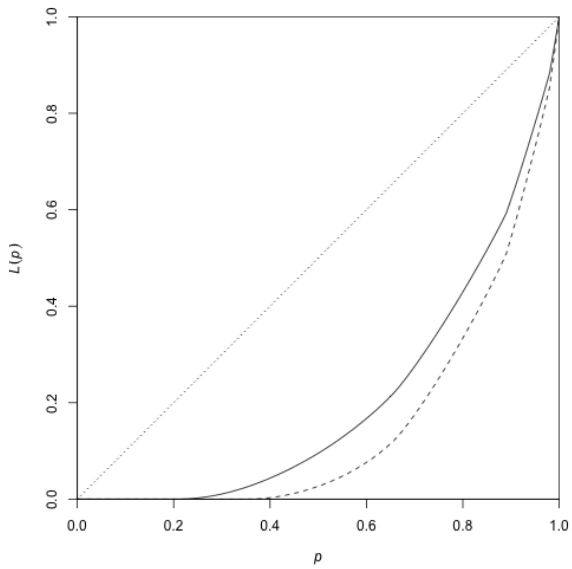
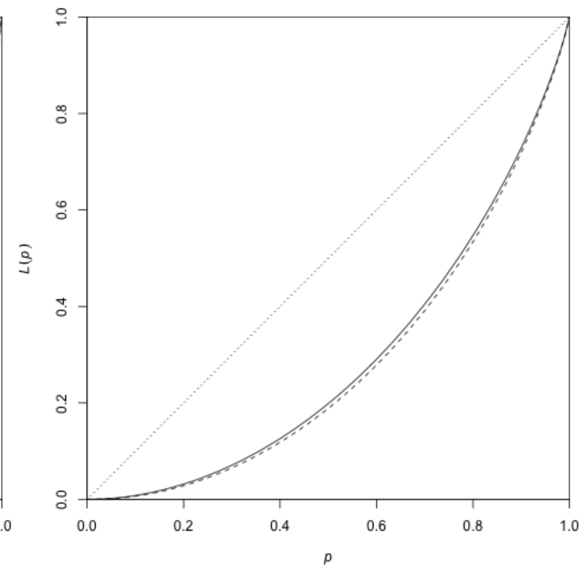


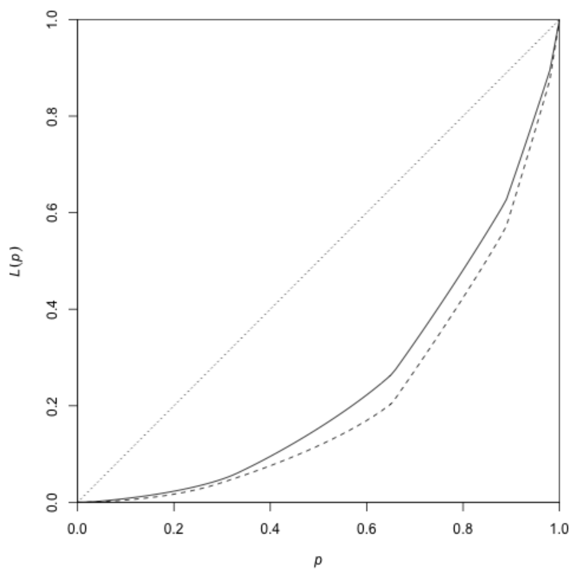
Figure 1: Change in mean lifetime utility (%).



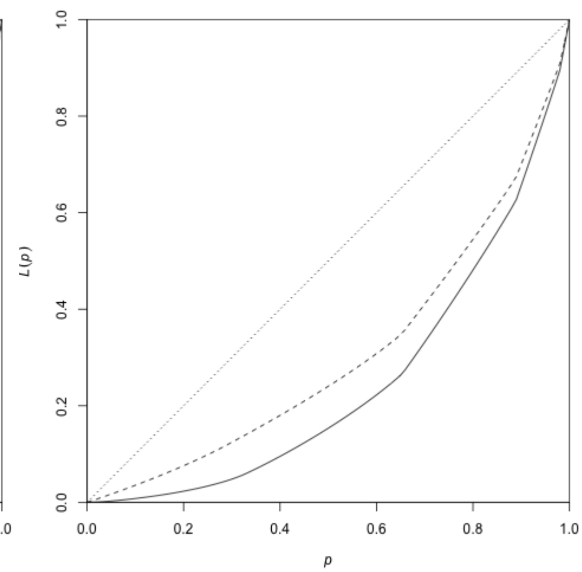
(a) Wage Income / Hours



(b) Interest Income / Wealth



(c) Pre-tax Total Income



(d) Post-tax Total Income

Figure 2: Lorenz curves: Solid line corresponds to a demogrant-to-output ratio of zero; dashed line corresponds to a demogrant-to-output ratio of 0.20.

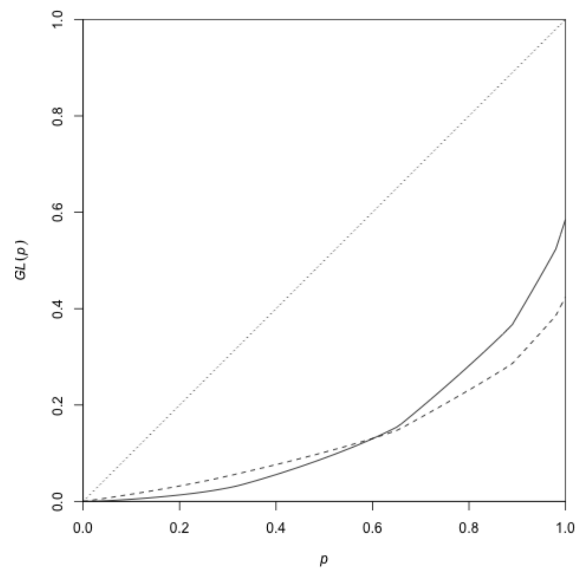
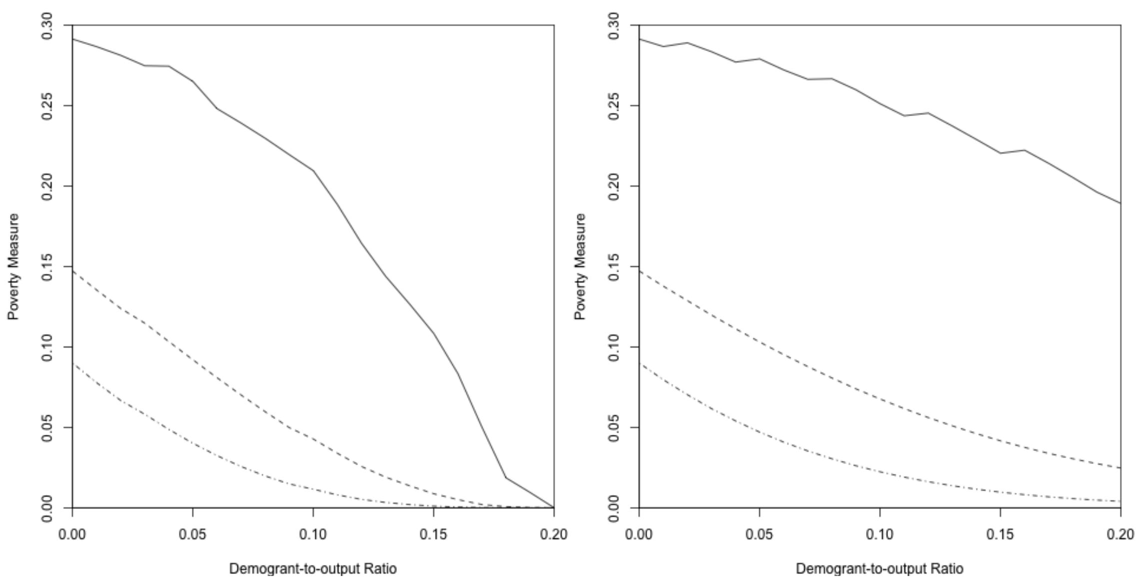


Figure 3: Generalized Lorenz curves for post-tax total income: Solid line corresponds to a demogrant-to-output ratio of zero; dashed line corresponds to a demogrant-to-output ratio of 0.20.



(a) Relative poverty line

(b) Absolute poverty line

Figure 4: Poverty measures: Solid line is P_0 (headcount ratio), dashed line is P_1 (poverty gap ratio), dash-dotted line is P_2.