International Trade and Labor Market Discrimination*

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Abstract

We embed a competitive search model with labor market discrimination into a two-sector, two-country framework in order to analyze how labor market discrimination and international trade interact. Discrimination reduces the matching probability and output in the skill intensive differentiated-product sector so that discrimination-induced comparative advantage may overshadow technological comparative advantage in determining the pattern of trade. Trade liberalization generates a decrease in the skilled-worker wage gap in the country that is an exporter of goods from the simple sector but increases it in the country that is a net exporter of differentiated products. Trade liberalization has an opposite effect on firms. In the country that is an exporter of simple goods, trade liberalization reduces the profits of the non-discriminatory firms by more than those of the discriminatory firms.

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1 Introduction

Gary Becker's (1957) seminal work on taste-based discrimination in the labor market has, at least, two implications for international trade. First, if discrimination is costly to the firms that practice it, then a sector or an entire country where discrimination is more prevalent should have lower productivity. Second, if discrimination is costly, then the pro-competitive effects of international trade should mitigate discrimination. Thus, trade liberalization may help ameliorate discrimination.

Recent empirical analysis of the effect of discrimination on aggregate productivity and growth supports the first implication. For example, Hsieh, Hurst, Jones, and Klenow (2013) show that between fifteen and twenty percent of the growth in US output per worker between 1960 and 2008 can be explained by allowing blacks and women into skilled occupations in which they were formerly very poorly represented.[1] The negative effect of the gender wage gap on growth has also been demonstrated by Galor and Weil (1996), Lagerlöf (2003), UNCTAD (2004), and Cavalcanti and Tavares (2016).

The empirical evidence on the effect of international trade on discrimination, however, suggests implications that go far beyond the argument in Becker (1957). While Oostendorp (2009) finds that trade liberalization decreases the gender wage gap for countries in which the gender wage gap has initially been large, Saure and Zoabi (2014) show that NAFTA membership has increased the US gender wage gap in industries that were more exposed to imports from Mexico. Juhn, Ujhelyi and Villegas-Sanchez (2014) show that tariff reductions due to NAFTA membership have reduced gender discrimination in Mexican blue collar jobs and Black and Brainerd (2004) find evidence that globalization reduces the gender wage gap in more concentrated industries.

The main contribution of our paper is to provide a theoretical model that rationalizes the empirical findings on (*i*) how trade liberalization impacts the wage gap in skill-intensive industries, *(ii)* how globalization affects the profits of (non-)discriminatory firms, and *(iii)* how discrimination impacts aggregate productivity. Moreover, our paper also shows that discrimination influences comparative advantage and the pattern of trade.

In our analysis trade affects discrimination in two ways. First, the wage gap between identically productive workers of different genders (or other identifiable characteristics which may serve as a

[1] Likewise, a nepotist will incur a cost when excluding non-relatives. In fact, Bloom and Van Reenen (2007) show that much of the long tail of very-poorly managed firms can be explained by primogeniture.

source of discrimination) can decrease or increase by trade liberalization. Second, it differentially impacts the profits of discriminating and non-discriminating firms. The direction of these effects depends in a predictable manner on a country's initial conditions.

We embed a directed (competitive) search model into a general equilibrium framework. There are two sectors in the economy: a simple sector that uses only labor and a sophisticated sector in which each firm produces a differentiated product using labor and a manager. Firms in this second sector can only produce if they successfully hire a manager. In order to locate a manager, firms post a payment for the manager and the skilled workers decide where to apply (unskilled workers cannot become managers). Any skilled worker who does not find a match as a manager can work with the unskilled workers as labor in either sector. We take as given that some forms of labor market discrimination exist and ask how does this discrimination affect the structure of the economy.[2] Our modeling of discrimination in a competitive search environment follows Lang, Manove and Dickens (2005), henceforth LMD. In particular we take their one-good, one-country, model and extend it to a monopolistically-competitive two sector general equilibrium framework with international trade and, of greatest importance, the inclusion of discriminatory and non-discriminatory firms in the same sector and country.[3] We start by assuming that all firms prefer to hire a manager of a certain label. That is, productivity of either label of skilled worker is the same, but every firm has a very slight preference for an A-label over a B-label manager. Our assumption that discrimination only matters in managerial positions in the sophisticated sector mirrors the ILO (2016) finding that discrimination is of a much greater concern in high wage occupations.[4]

Labels may refer to differences in skin color, eye color, gender, religion, caste, ancestral origin, native language, regional accent, or familial connections. This preference only matters if skilled workers of both labels apply to the same firm. In that case a firm would always hire an A-label

[2] A very nice overview of the literature on discrimination is provided by Lang and Lehman (2012), who discuss an overwhelming number of papers that provide significant empirical evidence of labor-market discrimination. As Gary Becker noted about his (1957) book: "For several years it had no visible impact on anything. Most economists did not think racial discrimination was economics, and sociologists and psychologists generally did not believe I was contributing to their fields," as quoted in Murphy (2014). The eventual realization that discrimination is an important topic for economists is echoed in the words of Kevin Murphy (2014): "Now the impact is clear. Not only is racial discrimination viewed as a subject about which economics has something useful to say, but economists are among the top academics in any field researching the topic."

[3] In fact, our first proposition directly follows from our extension of LMD to general equilibrium. Still, we discuss our first proposition in detail since intimate familiarity with this first proposition is necessary for the reader to understand our further extensions, which are allowing for international trade and the co-existence of discriminatory and non-discriminatory firms. In addition, we provide additional figures that not only illustrate our extensions but also help clarify, and illustrate, their model.

[4] In their survey article, Blau and Kahn (2017) argue that the US gender wage gap is much more persistent in higher-wage firms. Ben Yahmed (2018) reports for Brazil that the gender wage gap is highest among highly skilled workers.

manager and they would hire a B-label only if no A-labels apply. Hence, there are two posted payments in equilibrium: a higher one by firms that attract A-labels and a lower one by those that attract B-labels. This difference in posted payments is the skilled wage gap that we analyze. Thus, our theoretical setting mirrors the ample evidence from country studies, suggesting that labor market discrimination leads to labor market segregation.[5]

Interestingly, because the two groups are divided, which increases oligopsonistic power of firms in the labor market, both posted payments are lower than in the label-blind equilibrium (i.e. in the equilibrium without discrimination). Furthermore, because the posted payments are different, the proportion of firms posting each payment differs from the proportion of each label in the population. There is then an asymmetric arrival rate at the two groups of firms and, consequently, the overall arrival rate of potential managers at firms is lower than in the label-blind equilibrium. Hence, the matching rate is lower in the discriminatory equilibrium.

This difference in match success rates can drive the pattern of trade in our framework. When liberalizing trade with a label-blind country, the discriminatory country will be an exporter of the simple good that does not require a manager or a skilled worker. It is through the induced distortion in the matching process that discrimination inhibits development of the sophisticated differentiated product sector and generates comparative advantage in the simple sector. Discrimination-induced comparative advantage can also overshadow technological comparative advantage in determining the pattern of trade. This implies that discrimination can have an additional negative effect on long-run growth by generating reliance on exports with limited growth potential.

In order to consider the effect of trade on discrimination we introduce a second type of firm. These firms are non-discriminatory and it is common knowledge that they do not discriminate. Because they are known to not show hiring preference to either label of manager, they can offer a higher payment to B-label managers than can the existing discriminatory firms. This higher payment by a discriminatory firm would attract A-labels because they would be hired with certainty, however, they would only be hired with an equal probability by the non-discriminatory firms. The presence of these non-discriminatory firms partially mitigates the discrimination induced matching inefficiencies in the resulting equilibrium. In addition, these firms have higher expected profits than

[5] For example, Ghani, Goswami, Kerr, and Kerr (2016) show for the Indian informal labor market that close to 100% of male (female) workers work at firms with male (female) owners. Abel (2017) shows for South Africa that those who are discriminated against search in more remote areas for a job. While Blau and Kahn (2017) do not establish a causal link from discrimination to segregation, their empirical evidence suggests that about half of the gender wage gap in the US can be explained by employment segregation.

the discriminatory firms because they have a higher matching probability.[6]

Trade liberalization has the following effect on the skilled-worker wage gap. The payment to skilled workers is an increasing function of the sophisticated firms' profits. Crucially, the A-labels receive a larger portion of these profits than do the discriminated against B-labels. When the home country has a comparative disadvantage in the sophisticated sector (whether induced by technological or discriminatory comparative advantage), trade liberalization decreases the profits of Home's sophisticated firms. The payments to A-labels then decline by more than those to B-labels. Hence, the decrease in firm profits due to trade liberalization shrinks the skilled-worker wage gap in the country that is an exporter of goods from the simple sector.[7]

In the country with a comparative advantage in the sophisticated sector, trade liberalization accordingly increases the wage gap. These mechanisms have also been identified in the empirical literature: Oostendorp (2009) shows that trade liberalization decreases the gender wage gap in countries in which the gender wage gap has been large, while Barth, Kerr, and Olivetti (2017) and Blau and Kahn (2017) show that any changes in the US gender wage gap have not occurred in high-skilled sectors and were mostly due to factor relocations across, as opposed to within, sectors.

Trade liberalization has the opposite effect on firms. Because the expected profits of a non-discriminatory firm are greater than those of a discriminatory firm, they see a bigger change as a result of opening to trade. In particular, because of their higher match probability, any change in realized firm profits has a magnified effect on their expected profits. In the country that is an exporter of simple goods, trade liberalization reduces the profits of the non-discriminatory firms by more than those of the discriminatory firms.

Identifying these two opposing factors is the most important contribution of our paper. Developing countries, which are more likely to export goods from the simple sector, could see globalization

[6]If entry were costless, then these firms would come to dominate the market which would substantiate the hypothesis first mentioned in Becker (1957) and substantiated in Arrow (1972). Alternatively, if firms had to pay entry costs and if firms had differing entry costs (or differing variable costs), then non-discriminatory firms would be able to enter for higher entry (or variable) costs, but they would not take over the market. As our focus is on how trade affects each type of firm and the payment to their managers, we limit their numbers and instead analyze how the relative profits of discriminatory and non-discriminatory firms are affected by trade.

[7]The empirical research on the impact of trade on labor market discrimination typically regresses a wage gap on a trade openness measure and is therefore silent on whether a change in the wage gap is because the high-wage earners or the low-wage earners are affected disproportionately by trade liberalization. Still, there are two studies on Mexico and on Brazil, respectively, that indicate that the high-wage earners are affected disproportionately by trade liberalization and that this leads to a change in the wage gap due to trade liberalization (see Artecona and Cunningham, 2002, for Mexico and Arbache and Santos, 2005, for Brazil.)

reducing the wage gap but at the same time reducing the profit advantage of non-discriminatory firms. Industrialized countries, on the other hand, could see an increase in the portion of non-discriminatory firms but no corresponding reduction in the wage gap. Although there has been initial improvements in the gender wage gap since the fifties, it has held constant at about eighty percent for the last fifteen years for both the US (Graf, Brown and Patten, 2018) and the UK (The Economist, 2018). For black and Hispanic men in the US it has remained constant at around seventy percent for the last thirty-five years (Patten, 2016).

Our paper is related to several distinct strands of the literature.

Starting with Black (1995) and Rosen (1997, 2003), economists have analyzed discrimination as the equilibrium of a model with random search.[8] Recognizing that firms may want to strategically post a payment, LMD analyze discrimination as the equilibrium of a competitive search framework. We extend this literature by considering a mix of non-discriminatory and discriminatory firms in a competitive search framework, embedding it into a two-sector, two-country general equilibrium environment, and allowing for international trade. Our paper contributes to the literature on the wage gap as discussed in this introduction, by providing a theoretical model that addresses the source and persistence of the wage gap, by showing how it can be affected by globalization, and by studying why the results may differ in predictable ways across countries. We also extend a literature that shows how comparative advantage may be determined by reputation (Chisik, 2003) or institutional quality (Nunn, 2007, Vogel, 2007, Costinot, 2009), by showing that it may also be determined by labor market discrimination and, in turn, how globalization, can exacerbate or mitigate the market imperfection that generated the pattern of trade. Finally, our paper is related to the broad literature on international trade with labor market frictions, such as Davidson, Martin and Matusz (1999), Davidson, Matusz and Shevchenko (2008), Egger and Kreickemeier (2009), Helpman and Itskhoki (2010), Helpman, Itskhoki and Redding (2010), Grossman, Helpman and Kircher (2013), and King and Stähler (2014). We extend this literature in three ways. First, like King and Stähler (2014), we analyze a competitive instead of a random search framework. Second, we analyze discrimination as a source of comparative advantage. Third we analyze how trade liberalization affects the prevalence of discrimination.

In the next section we analyze autarky with and without discrimination and then allow for international trade. In the third section we introduce a mix of non-discriminatory and discriminatory

[8]Fang and Moro (2010) contains a review of many additional theoretical papers on discrimination that are not covered in Lang and Lehman (2012).

firms. Our most important results, on how trade affects discrimination, are contained in the fourth section. Section 5 contains our conclusions.

2 Economic environment

2.1 Preferences and technology

There are two countries: Home and Foreign. We denote foreign variables with an (*). In each country there are two sectors. The **simple** sector produces perfectly substitutable goods with a constant returns to scale technology using only labor. The **sophisticated** sector produces differentiated goods using labor and a manager. Upper tier preferences over goods from the two sectors are represented by a Cobb-Douglas utility function:

$$U(C_M, C_0) = C_M^{\alpha} C_0^{1-\alpha}. \tag{1}$$

Preferences over goods in the sophisticated sector are represented by a constant elasticity of substitution sub-utility function:

$$C_M = \left(\sum_{z=0}^{\infty} c_z^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where the elasticity of substitution between varieties is σ and $\sigma > 1$. Therefore, none of these varieties is essential to consumption. Although preferences are defined over a potentially infinite number of varieties, only a finite number will be available to consume. Agents derive income from working as either labor, or if they are skilled and successfully locate a match, as a manager. In addition, all agents are equal owners of each of the firms and they equally share any firm profits. Each firm producing in the sophisticated sector has the same technology:

$$\ell_z = \begin{cases} q_z + f & if\ m_z = 1 \\ \xi & if\ m_z = 0 \end{cases}, \tag{3}$$

where ℓ_z is the amount of **labor** used in producing variety z, q_z is the quantity of variety z, m_z is a **manager** for the firm producing variety z, f denotes the fixed input requirement, and ξ is an arbitrary large constant that makes production infeasible if firm z is not successful in hiring a manager, i.e. if $m_z = 0$. We use the convention that all fixed costs are paid in terms of labor.

The technology for producing the simple good is $\ell_0 = q_0$, and the labor supply of each country, $L = L^*$, is assumed to be large enough so that there is positive production of the simple good in each country. We choose the simple good as our numeraire, which implies that its price P_0 and the wage of unskilled workers in either sector are equal to unity. Still, we include P_0 in several equations to help the reader to follow more easily the analysis.

We are interested in the composition of firms in the sophisticated sector, rather than in their absolute number, therefore, the number of potentially active firms in this sector, $N = N^*$, is taken as exogenous. As a result of search frictions only M of the N (M^* of the N^*) firms are successful in hiring a manager and producing.[9] Still, the size of the economy is large enough so that the number of operating sophisticated firms is large and, therefore, the effect of each firm's output on the price and quantity of other firms is negligible.

For each sophisticated firm that successfully hires a manager, the product market is described by monopolistic competition. As shown by Dixit and Stiglitz (1977), the set of purchased sophisticated goods can be considered as a composite good C_M with corresponding aggregate price

$$P_M = \left(\sum_{z \in M} {p_z}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{4}$$

Consumer maximization of the first stage utility function yields the following demand functions:

$$C_M = \frac{\alpha I}{P_M}, \qquad C_0 = \frac{(1-\alpha)I}{P_0}, \tag{5}$$

where I denotes aggregate income which we will derive below. Consumer maximization of the sub-utility function yields demand for each variety as

$$c_z = C_M \left(\frac{p_z}{P_M} \right)^{-\sigma} . \tag{6}$$

Each sophisticated firm chooses output to maximize profits, taking the output of other firms and the aggregate price index and C_M as given. This leads to the following pricing rule: $p_z = \frac{\sigma}{\sigma-1}$. Hence,

[9] Although $N = N^*$, it is not necessarily the case that $M = M^*$.

$$c_z = \frac{\alpha I}{M p_z} = \frac{\alpha I}{M \frac{\sigma}{\sigma-1}}, \tag{7}$$

and, denoting firm revenue as r_z, the gross profits of operating each firm are given by:

$$\pi_z = r_z - l_z = p_z q_z - q_z - f = \frac{r_z}{\sigma} - f = \frac{\alpha I}{M\sigma} - f. \tag{8}$$

Agents in each country are either **skilled** or **unskilled**. Unskilled ones work either in the simple sector or as laborers in the sophisticated sector. Skilled workers can work as a manager if they are offered a managerial job and they can also work as unskilled laborers if their managerial search is unsuccessful.

In addition to their skill level (skilled versus unskilled), agents differ by their **label** $k \in \{A, B\}$. Labels may refer to differences in skin color, eye color, gender, religion, caste, ancestral origin, native language, regional accent, or familial connections. This label is also perfectly observable and it is common knowledge that productivity does not depend on the label. The number of skilled workers in each country with each label is given as $S_k = S_k^*$. The total number of unskilled workers in the home country is, therefore, $L - S_A - S_B = L - S$. Only a subset M of the S will find work as a manager and the remainder will work as laborers in either sector.

2.2 Firm decisions

Despite the identical productivity of all skilled workers, **discriminatory** firms prefer to hire an A-label manager.[10] Formally, the preferences of discriminatory firms are lexicographic. first, they prefer to hire a manager. Second, they prefer to hire an A-label manager. Hence, if skilled managers of each label apply to the same job with the same posted bonus, then a discriminatory firm will hire the A-label worker. A B-label manager will be hired by such a firm only if there are no A-label skilled applicants at the posted bonus. We use the term **bonus** for the payment

[10]These preferences may be solely based on an inherited prejudice. In this case it could be reflected as a utility function for firms such as $U_A = \pi - b_A$ and $U_B = \pi - \delta - b_B$, where U_k is the firm's utility from hiring a k-label manager, b_k is the payment paid to a k-label manager and δ is the (vanishingly small) disutility that discriminatory firms suffer when they hire a B-label manager. A similar firm utility function could result if the firm was not itself discriminatory, but if the board of directors felt that prospective shareholders hold discriminatory views towards B-label managers. If we allowed each firm to hire more than one manager it may also be the case that a non-discriminatory board of directors was concerned that the other managers hold discriminatory views and, therefore, the board would not wish to mix the labels in managerial positions. Although an important topic for further research, understanding the genesis, and persistence, of discrimination does not directly affect our results on how trade liberalization affects the extent of discrimination.

to managers in order to differentiate it from the payment to labor, which is the **wage**. We denote the portion of B-label skilled workers in each country as β, so that the number of A-label skilled workers is $S_A = (1-\beta)S$. We assume that $\beta \in (0,1)$. At times we will find it useful to write $\beta_B = \beta$ and $\beta_A = 1-\beta$ or, more generally, β_k. Discriminatory firms have no preferences over the label of unskilled workers. In addition to the N_D (N_D^*) discriminatory firms there can be $N_0 = N - N_D (N_0^* = N^* - N_D^*)$ **label-blind** (or: non-discriminatory) firms who have no preference over the label of a skilled manager.

The timing and information structure of the model is as follows. We write the case of the home country. The foreign country is similar. First, each of the N firms posts a bonus, b_z, for a manager. Second, skilled workers observe the vector of posted bonuses, $\mathbf{b} = \{b_z\}$, and decide where to apply. Skilled workers can only apply once and to only one firm.[11] Formally, from the perspective of firms, a worker's action at this stage is a collection of probabilities that they will apply to firm z, denoted as $a_z(\mathbf{b})$. The skilled workers' application strategy is restricted to those that assign equal probability to all firms offering the same bonus. Hence, the workers' strategies satisfy anonymity. Third, the M firms that have an applicant are successful and produce and sell their goods in the market. Unsuccessful firms do not produce. Unmatched skilled workers and all unskilled workers work as wage laborers in the sophisticated or the numeraire sector. We assume that the number of skilled workers is small in relation to the total labor supply so that any matched skilled worker receives a bonus that is larger than the wage they would receive as a wage laborer. Hence, all skilled workers apply for a managerial position.

Firm z's strategy consists of posting a bonus and choosing output. Each skilled worker's strategy is a vector of application probabilities $\mathbf{a}(\mathbf{b}) = \{a_z(\mathbf{b})\}$. If all skilled workers with the same label use the same strategy, then the expected number of workers of each label applying to firm z is given by $\lambda_{zk} = a_z(\mathbf{b})S_k$. Note that, because the application probabilities sum to one, we have that the market tightness $\frac{S}{N}$ for each label can be expressed as $\frac{1}{N}\sum^{N}\lambda_{zk} = \beta_k \frac{S}{N}$. Since the firms' and the skilled workers' payoff functions depend on whether or not firms discriminate across workers, we will derive them in the next subsection for these two cases.

[11] As long as there is some cost to additional applications, allowing skilled workers to apply to more than one firm would not have any qualitative effect on our results.

2.3 Two benchmark cases in autarky

In this subsection we extend the setting of LMD to general equilibrium and analyze the autarky equilibrium first for the case of an economy where no firms discriminate and then for one where all firms discriminate. In the next subsection we allow for international trade.

Our equilibrium concept is that of a sub-game perfect monopolistically competitive equilibrium (SPMCE), which is characterized as follows:

1. Each firm's b_z is a best response to the vectors of firms' and skilled workers' strategies, $\mathbf{b}$ and $\mathbf{a}$.
2. Each skilled worker's $a(\mathbf{b})$ is a best response to $\mathbf{b}$ and to $\mathbf{a}(\mathbf{b})$ of all other workers.
3. Each firm chooses q_z to maximize π_z.
4. Each agent chooses C_0 and the amount c_z consumed of each variety of C_M to maximize utility subject to the budget constraint and given prices P_M and p_z.
5. Relative supply of the M manufactured goods and of the numeraire good equals relative demand for these goods and the labor market clears.

Note that the large number of firms, skilled workers, and consumers ensures that $\mathbf{b}$, $\mathbf{a}$, C_M and P_M are neither sensitive to a firm's own bonus and quantity choice nor to a skilled worker's choice or a consumer's choice. When solving for the SPMCE of this game we will follow LMD and focus only on symmetric equilibria. "Symmetric" refers to an equilibrium in which all skilled workers choose the same application strategy.

We start with the case of no discrimination, therefore, we can temporarily suppress the subscript k. We are interested in the limiting case when N and S become very large but their ratio is still finite. A sophisticated firm will only be able to produce if it hires a manager. This occurs if and only if it receives at least one applicant. The probability it receives at least one applicant is $1-(1-a_z)^S$. When S and N are large, this converges to

$$1 - Pr\left(\lambda_z = 0\right) = 1-(1-a_z)^S \rightarrow 1-e^{-a_z S} = 1-e^{-\lambda_z}. \tag{9}$$

A firm's expected profits net of payments to a manager is:

$$E\left(\pi_z^{net}\right) = \left(1 - e^{-\lambda_z}\right)(\pi_z - b_z), \tag{10}$$

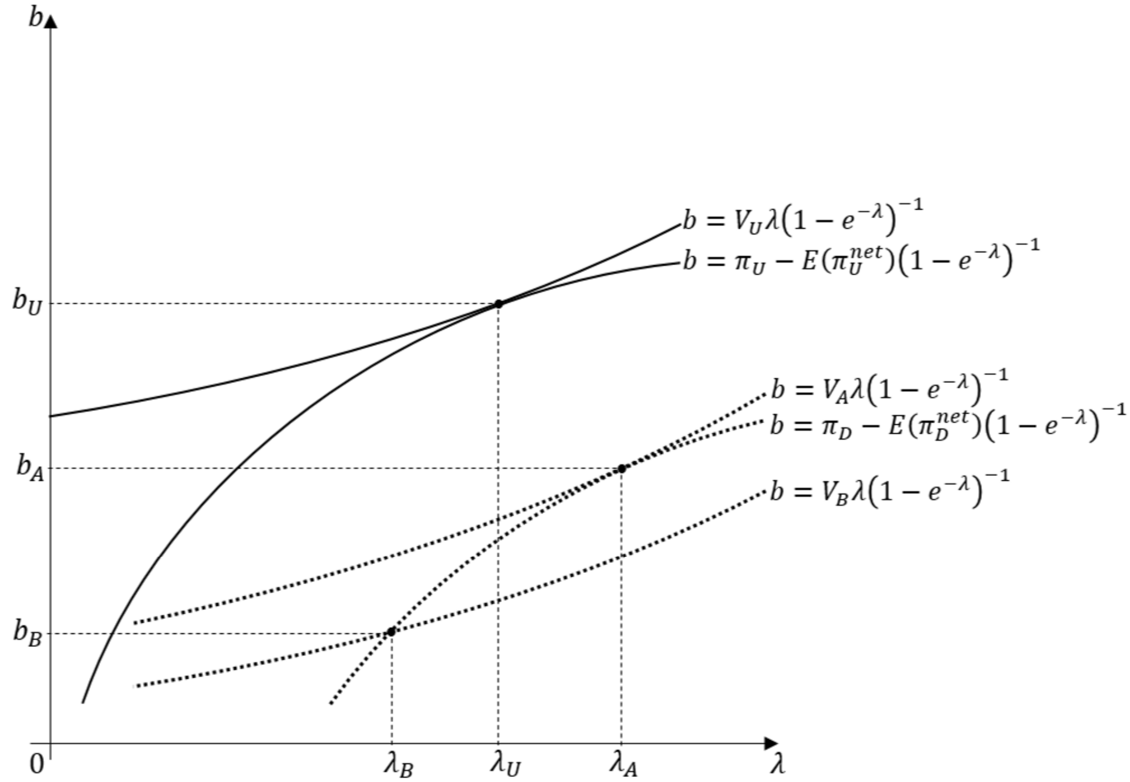
where b_z denotes the bonus to the manager. The equilibrium level of b_z, which maximizes $E\left(\pi_z^{net}\right)$, will be derived below.

The probability that an applicant is hired at a firm z is the product of the probability that there is at least one applicant times the probability that they are the chosen one. Hence, the probability (from the perspective of an applicant) that they are hired at a single firm z is:

$$Pr\,(hired) = h(\lambda_z) = \frac{1 - (1 - a_z)^S}{a_z S} \to \frac{1 - e^{-\lambda_z}}{\lambda_z}. \tag{11}$$

Thus, a skilled worker's expected bonus from applying to a firm z is given by $V_z = b_z h(\lambda_z)$. We assume that the total labor supply, L, is sufficiently large compared to the skilled labor supply, S, so that the expected bonus is larger than the wage, which is one, so that all skilled workers apply for a managerial position. The necessary condition on L and S is derived in the proof of proposition 1.

Figure 1: Benchmark autarky equilibria with and without discrimination



In figure 1, we illustrate the equilibria for the two benchmark cases.[12] First consider the case when all firms are label-blind or non-discriminatory. We use the subscript U to refer to this benchmark **unbiased** equilibrium without discrimination. From equation (10) we can write an equation for a firm's isoprofit function in (λ, b) space as $b = \pi_U - E\left(\pi_U^{net}\right)\left(1-e^{-\lambda}\right)^{-1}$. Firms prefer a larger arrival rate of applicants, λ, and a lower offered bonus, b, therefore, their profits are increasing toward the southeast of the figure. Inserting equation (11) into the expression for the expected bonus we can write the equation for a skilled applicant's indifference curve as $b = V_U\lambda\left(1-e^{-\lambda}\right)^{-1}$. A skilled applicant prefers a larger bonus and fewer co-applicants, i.e. the applicant prefers outcomes to the northwest of the figure.

The unbiased equilibrium is described by the point of tangency between the skilled applicant's indifference curve and a firm's isoprofit curve. We will summarize and prove this result with proposition 1. The idea of this result is as follows. Skilled workers only apply at firms where the

[12]In addition to providing an intuitive graphical explanation of the results in LMD, analysis of our figure 1 also illuminates the genesis of our idea on how to simultaneously include discriminatory and label-blind firms.

expected bonus is highest. As all workers follow the same application strategy, all firms offer the same bonus in equilibrium. Substituting $b_z = \frac{V_z}{h(\lambda_z)}$ into a firm's expected net profits (from equation 10) allows us to consider each firm as choosing the optimal expected number of applicants per firm, λ_U, which then yields the optimal bonus, b_U, and the expected number of matches, M_U, which are then used to determine the other equilibrium values.

We now consider the case in which all of the sophisticated firms are discriminatory, therefore, we must explicitly consider two labels of skilled workers. We use the subscripts A and B for variables pertaining to either label and the subscript D for aggregate values in the **discriminatory** equilibrium.

Firms can only post a single bonus (it is illegal to post label-dependent wages in most countries) and skilled workers can apply at most to only one firm. A firm that attracts at least one applicant at its posted wage will successfully hire a manager. If a single firm has more than one applicant of the same label, then it will choose randomly among those applicants, however, if it has applicants from both labels, then it will always hire an A-label. As mentioned above, firms prefer to have a match, but given a match, they prefer an A-label manager.

The case for A-label skilled workers is similar to the non-discriminatory case. Of course, the number of all skilled workers combined, S, is greater than the number of A-labels, S_A. Furthermore, the application strategies for the A-labels, a_A, will also differ from the probabilities, $a_z = \frac{1}{N}$, as given in the previous section.

The more interesting case is that of the B-labels. In the proof of proposition 1 we show that each discriminatory firm employs only A- or B-labels, but not both. The intuition is that any bonus that is large enough to attract A-labels would discourage B-labels from applying (because they know that an A-label would always be shown preference). In particular, any bonus that attracts both labels could be improved upon by one that is slightly lower and that attracts many more B-labels while only losing the few A-label applicants. The result is that some firms post a lower bonus and only attract B-labels and others post a larger one to attract only A-labels.

Using the result that discriminatory firms separate themselves we can depict the discriminatory equilibrium in figure 1. Although the firms separate, so that some post a high offered bonus to attract A-labels and some post a lower one to attract B-labels, their expected net profits are the same in equilibrium. For the net expected profits to be the same, the arrival rate of applicants at the A-label firms must be higher than at the B-label firms. Therefore, $\lambda_A = \frac{S_A}{N_A} > \lambda_U = \frac{S}{N} >$

$\lambda_B = \frac{S_B}{N_B}$, where $N_A + N_B = N$ and N_A and N_B are the numbers of A- and B-label attracting firms and $S_A + S_B = S$. Considering that the equilibrium for the A-labels occurs at a point of tangency between a firm's isoprofit and an A-label's indifference curve, the firms have higher profits in the discriminatory equilibrium and this is reflected by an iso-profit curve that lies to the southeast of the non-discriminatory equilibrium iso-profit curve. In the resulting discriminatory equilibrium the A-labels are on a lower indifference curve, with a lower bonus and a lower probability of finding a match (a larger λ). The B-labels are on an even lower indifference curve with an even lower bonus, but a greater probability of successfully finding a match. The firms' expected net profits are the same, regardless of whether they post a bonus to attract A- or B-label managers.

We provide a formal proof of these observations as well as a complete equilibrium depiction of both cases in proposition 1.

Proposition 1. *Only non-discriminatory firms. There exists a unique symmetric unbiased SPMCE in which all firms offer an identical bonus* $b_U = \frac{\pi_U \lambda_U}{e^{\lambda_U} - 1}$ *and all skilled workers adopt the same mixed application strategy in which they apply at each single firm with the same probability. A single skilled worker's expected bonus is given by* $V_U = \pi_U e^{-\lambda_U}$*, profits of each operating firm result as* $\pi_U = \frac{1}{\sigma - \alpha}\left(\alpha\left[\frac{L}{N(1-e^{-\lambda_U})} - 1\right] - \sigma f\right)$ *and expected profits of each firm net of bonus payments are given by* $E\left(\pi_U^{net}\right) = \left[1 - (1 + \lambda_U)\, e^{-\lambda_U}\right]\pi_U$*. National income results as* $I_U = \frac{\sigma}{\sigma - \alpha}\left[L - (1 + f)\, M_U\right]$ *and the number of operating firms is given by* $M_U = S\frac{1-e^{-\lambda_U}}{\lambda_U} = N(1 - e^{-\lambda_U})$.

Only discriminatory firms. There exists a unique symmetric SPMCE with discrimination. Firms separate so that a firm chooses a bonus that will attract either only A-label applicants or only B-label applicants, but not both. Bonuses are given by $b_A = \frac{\pi_A \lambda_A}{e^{\lambda_A} - 1}$ *and* $b_B = V_A(\mathbf{b})$ *and expected bonuses are given by* $V_A(\mathbf{b}) = \pi_D e^{-\lambda_A}$ *and* $V_B(\mathbf{b}) = \pi_D e^{-\lambda_A}\frac{1-e^{-\lambda_B}}{\lambda_B}$*. Profits of each operating firm result as* $\pi_D = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_D} - 1\right] - \sigma f\right)$*, where* $M_D = S_B\frac{1-e^{-\lambda_B}}{\lambda_B} + S_A\frac{1-e^{-\lambda_A}}{\lambda_A}$ *and expected profits net of bonus payments are given by* $E\left(\pi_A^{net}\right) = \left[1 - (1 + \lambda_A)\, e^{-\lambda_A}\right]\pi_D$ *and* $E\left(\pi_B^{net}\right) = \left(1 - e^{-\lambda_B}\right)\left(1 - e^{-\lambda_A}\right)\pi_D$*. National income results as* $I_D = \frac{\sigma}{\sigma-\alpha}\left[L - (1 + f)\, M_D\right]$*. Finally,* $\lambda_B < \lambda_U < \lambda_A$.

The proof of proposition 1 is in the appendix.

When comparing the discriminatory to the non-discriminatory equilibrium, the most important variables are the arrival rate of applicants at the firms and the number of successful matches. We define $\eta \equiv \frac{N_A}{N}$ and we note that $\lambda_U = \eta\lambda_A + (1 - \eta)\lambda_B = \frac{S}{N}$. The average unfilled vacancy rate in the discriminatory equilibrium can be written as:

$$\Psi(\eta) = \frac{N_A}{N} e^{-\lambda_A} + \frac{N_B}{N} e^{-\lambda_B} = \eta e^{-\frac{S_A}{\eta N}} + (1-\eta)\, e^{-\frac{S_B}{(1-\eta)N}}. \tag{12}$$

In the appendix we show that the vacancy rate is minimized when $\lambda_A = \lambda_B = \lambda_U$, so that the number of vacancies is smallest in the absence of discrimination.

Proposition 2. *The number of unfilled vacancies is larger, and the number of successful matches is smaller, in the discriminatory equilibrium.*

The proof of proposition 2 is in the appendix. Proposition 2 is an important result because it points to the inefficiency generated by discrimination: there are fewer successful matches. In our general equilibrium setting the number of successful matches has additional effects on the the firms' expected profits, the offered bonuses, and expected income. These additional effects are important in deriving our main results.

To see how these effects are realized in our benchmark cases denote with a subscript $e \in \{U, D\}$ the type of equilibrium we are considering and rewrite the realized profits of a successful firm as $\pi_e = \frac{\alpha}{\sigma-\alpha}\left(\frac{L}{M_e} - 1\right) - \frac{\sigma f}{\sigma-\alpha}$. After substituting income into the demand for a variety (from equation 7) we can write the equilibrium output of a successful firm in either type of equilibrium as $q_e = \frac{(\sigma-1)\alpha}{\sigma-\alpha}\left[\frac{L}{M_e} - (1+f)\right]$.

From proposition 2 we know that $M_D < M_U$, therefore, the realized profits and output of a successful firm are higher in the discriminatory equilibrium: $\pi_D > \pi_U$ and $q_D > q_U$. This result is intuitive. If there are less successful firms, then there is less competition and the profits of each producing firm are greater. In comparing expected profits in the discriminatory and non-discriminatory equilibrium note that $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$ in equilibrium. Hence, we only need to compare $E\left(\pi_A^{net}\right) = \left[1 - (1+\lambda_A)\, e^{-\lambda_A}\right]\pi_D$ in the discriminatory case to $E\left(\pi_U^{net}\right) = \left[1 - (1+\lambda_U)\, e^{-\lambda_U}\right]\pi_U$ from the non-discriminatory case. Now, $1 - (1+\lambda)\, e^{-\lambda}$ is increasing in λ and from proposition 1 we know that $\lambda_A > \lambda_U$. Hence, given that $\pi_D > \pi_U$ we know that the expected profits are also larger in a discriminatory equilibrium. We summarize these results in proposition 3.

Proposition 3. *Realized and expected firm profits and output of each variety are larger in the discriminatory equilibrium.*

The overall effect on skilled workers is not as easy to disentangle. The change in λ produces two opposing effects on skilled workers. First, with respect to A-label workers, note that holding π constant, b_A and $V_A(\mathbf{b})$ are both decreasing in λ. Hence, given that $\lambda_A > \lambda_U$, if π does not change, then the bonuses and expected incomes of A-labeled skilled workers are lower in the discriminatory equilibrium. Of course, as shown in proposition 3 the realized profits of each successful firm are higher in the discriminatory equilibrium and part of these profits are passed on to the managers in their bonuses. With respect to the B-label managers note that they have a lower bonus and expected bonus than A-labels. Their bonus is lower because $b_B = V_A(\mathbf{b}) = h(\lambda_A)\, b_A$ and $h(\lambda_A) < 1$. In addition, their expected bonus is lower since $V_B(\mathbf{b}) = h(\lambda_B)\, b_B = h(\lambda_B)\, V_A(\mathbf{b}) < V_A(\mathbf{b})$.

2.4 Two benchmark cases with trade

We now consider international trade. In order to analyze the pattern of trade, we begin by deriving the autarkic prices for the home economy, which is in a discriminatory equilibrium, and for the foreign economy, which is assumed to be in a label-blind equilibrium. Given that $L = L^*$ and $S = S^*$, we know from proposition 2 that the number of matches is lower in the home country. In particular, $M_D < M_U = M^*$. Hence, production of the sophisticated good is lower in the home country. Given that the vacancy rate is higher, and $L = L^*$, the production of the simple good must be larger in the home country. From equations (4) through (7) we can then write the relative autarkic prices in Home and Foreign as:

$$\frac{P_M}{P_0} = \frac{\alpha}{1-\alpha}\frac{C_0}{C_M} = M_D^{\frac{1}{1-\sigma}} p_z > (M^*)^{\frac{1}{1-\sigma}} p_z = \frac{P_M^*}{P_0}. \tag{13}$$

We have now established the following result.

Proposition 4. *The country in the discriminatory equilibrium has a comparative disadvantage in the sophisticated sector.*

Proposition 4 indicates that a greater degree of discrimination can cause a country to become an exporter of simple products and a net importer of products from the sophisticated sector. If production of sophisticated products generates larger improvements in the rate of economic growth, then international trade can magnify the inefficiencies generated by discrimination. In autarky, discrimination reduces the number of successful matches and reduces production of the more sophisticated good. As the more discriminatory country adjusts its production patterns in line with comparative advantage, trade magnifies this effect by the country's increased reliance on production of simple goods. In the introduction we note several recent papers that demonstrated the effect of discrimination on economic growth. The result of proposition 4 suggests that international trade enhances those effects for countries that suffer from a high level of labour market discrimination.

In the next section we extend the benchmark cases by allowing for the co-existence of discriminatory and label-blind firms and then analyzing international trade between two economies that differ in their ratio of label-blind to discriminatory firms.

3 Co-existence of discriminatory and label-blind firms in autarky

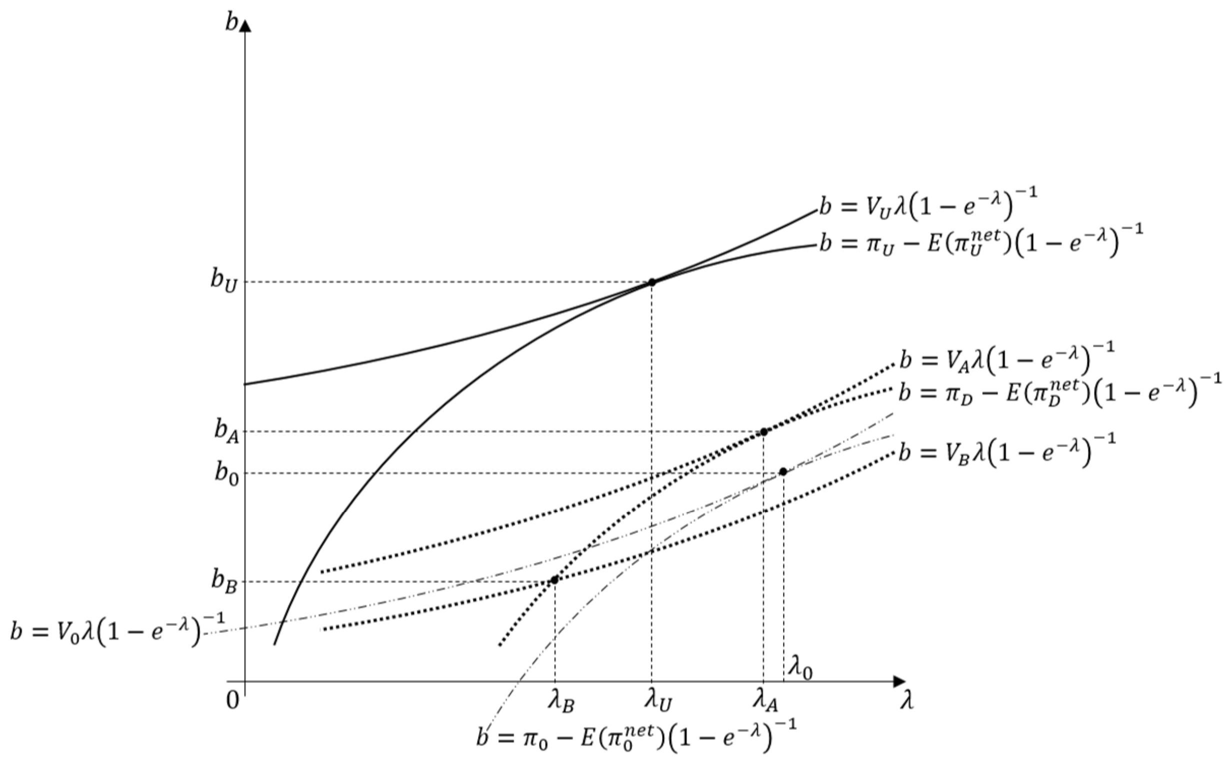
In this section we introduce $N_0 < \beta N$ label-blind firms into the home country. For these firms the label is irrelevant so that when faced with both an A-label and a B-label managerial applicant each applicant is hired with equal probability. In order to continue to have a clear conception of comparative advantage we assume that the total number of firms in each country is still $N = N^*$ so that $N_D = N - N_0$ is the number of discriminatory firms in the home country.[13] Although the result that the portion of discriminatory firms in each country could determine the pattern of trade is an interesting artifact of our model, it is not necessary for our main results. In order to allow comparative advantage to be determined in more traditional ways we also allow for the technology in producing the simple good to differ across countries. Whereas $\ell_0 = q_0$ in the home country, the technology in the foreign country is described as $\gamma \ell_0^* = q_0^*$, where $\gamma \geq 1$. If $\gamma > 1$, then Home has a technological comparative advantage in the sophisticated sector.

To develop the intuition for the results that are introduced in this section we refer the reader to the dotted lines in figure 1, which depict the equilibrium with only discriminatory firms. We see

[13] If we allowed for free entry, then we could no longer be certain of $N = N^*$ and the total number of matches, as well as the pattern of comparative advantage, would no longer be a simple mapping from the unfilled vacancy rate. It would also depend on the shape of the distribution driving firm heterogeneity, which would be necessary for co-existence of discriminatory and label-blind firms. Although (as will be seen below), a label-blind firm would have larger expected profits than a discriminatory one with equivalent costs, as long as there is some firm heterogeneity (in either fixed or variable costs), then the two types of firms would co-exist with free entry. In particular, the cutoff productivity level (fixed costs) would be lower (higher) so that the average costs of the marginal label-blind firm would be higher than those of the marginal discriminatory firm. Even though the pattern of comparative advantage does not admit a simple solution in such an environment we could make predictions on how the number of each type of firm responds to trade liberalization. We save this extension for future research.

there that the low bonus offered to the B-label applicants generates "too many" firms posting that low bonus in the attempt to attract a B-label manager.[14] The inefficiency illustrated in figure 1 suggests that a firm that is known not to discriminate could post a bonus (and a corresponding hiring probability) that would attract B and not A-label applicants. A discriminatory firm could not post such a bonus (and expect only B-label applicants) because it is known that they would show priority to A-label applicants. One possible such bonus, b_0, with corresponding expectation V_0, and expected profits $E(\pi_0^{net})$, is shown in figure 2.

Figure 2: The potential for label-blind firms



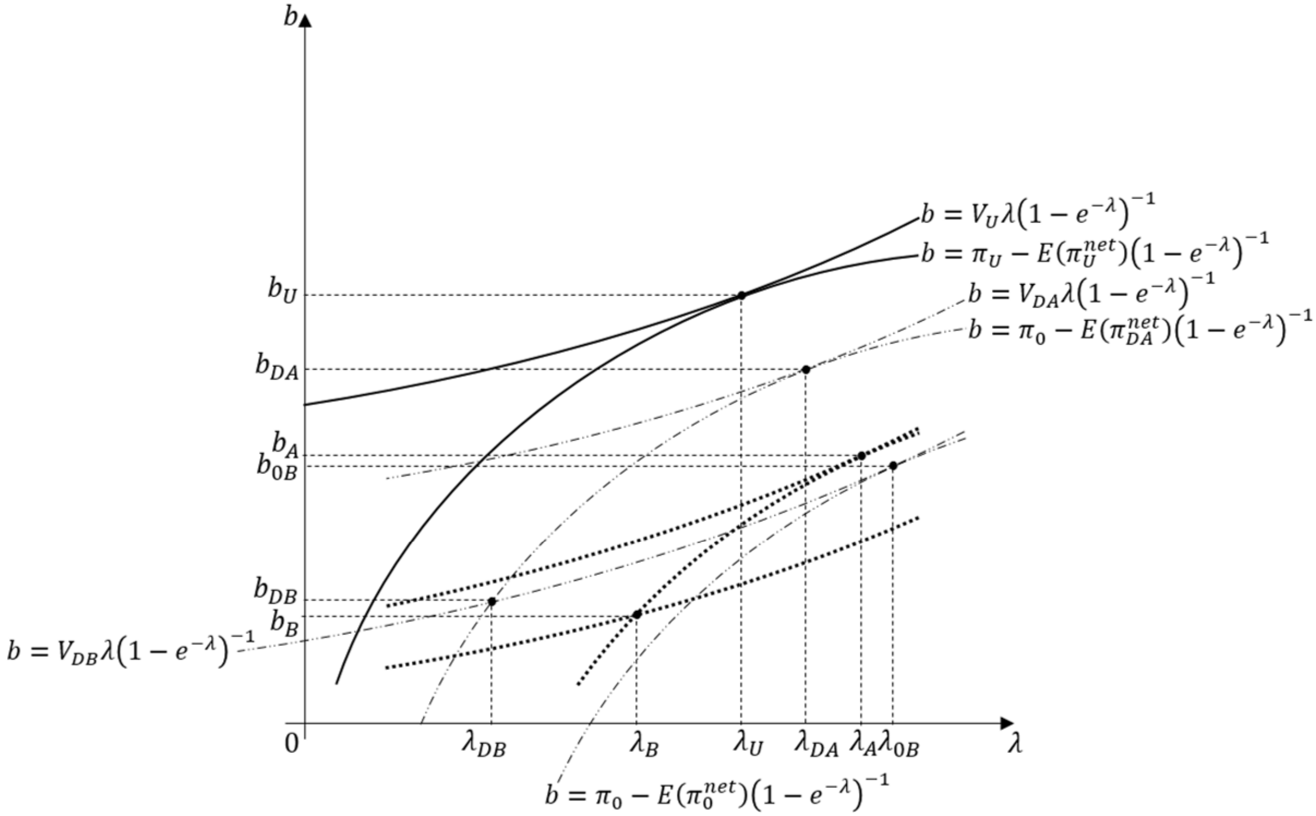
In figure 2, at b_0 the B-label applicants are on a higher indifference curve and the label-blind firms have larger profits than the discriminatory ones. Of course, figure 2 does not depict the new equi-

[14] $\lambda_B < \lambda_A$ implies $\frac{\beta S}{N_B} < \frac{(1-\beta)S}{N_A}$ or $\frac{\beta}{1-\beta} < \frac{N_B}{N_A}$, therefore, we say "too many" firms post to attract B-labels.

librium. In equilibrium, a B-label must be indifferent between a label-blind and a discriminatory firm, therefore, the bonus offered by the discriminatory firms to the B-labels must increase as well. In response to the higher bonus required to attract B-label applicants (and the resulting fewer applicants at discriminatory B-label firms) some discriminatory firms switch from attracting B-label applicants to attracting A-label applicants.

The new co-existence equilibrium is depicted by the dashed lines in figure 3 (along with the unbiased equilibrium with solid lines and the discriminatory equilibrium with dotted lines). The bonuses in the co-existence equilibrium are denoted as b_{0B} for those offered by the label-blind firms and b_{DB} and b_{DA} for those offered by the discriminatory firms (as the next proposition shows, the label-blind firms do not post a bonus to attract A-labels). We see in figure 3 that, compared to the discriminatory equilibrium, the expected profits for discriminatory firms are lower and the expected bonuses of both labels of applicants are larger in the co-existence equilibrium. Not only are the bonuses offered to both labels of applicants larger, but the applicant to position ratio is smaller for both types of discriminatory firms.

Figure 3: The autarkic equilibrium with co-existence of discriminatory and label-blind firms



Before proceeding with the formal analysis we introduce the following notation. Of the N_D firms N_{DA} will attract only A-labels and N_{DB} will attract only B-labels in the co-existence equilibrium. Similarly, N_{0A} and N_{0B} are the numbers of label-blind firms attracting only A and only B-labels in the co-existence equilibrium. Of the skilled workers, S_{DA} and S_{DB} are the numbers that apply to the discriminatory firms and S_{0A} and S_{0B} are the numbers that apply to the label-blind firms. The extension to λ_{DA}, λ_{DB}, λ_{0A}, and to λ_{0B} is straightforward. More generally, we can write N_{tk}, S_{tk}, and λ_{tk} where $t \in \{D, 0\}$ and k signifies the label of worker attracted by a type t firm. Similarly, b_{tk} is the bonus offered by a type t firm attracting a label-k manager and V_{tk} is the expected bonus.

It will prove useful to consider the skilled workers that apply to the discriminatory firms. The fraction of B-labels among all the applicants to discriminatory firms is $\beta_D = \frac{S_{DB}}{S_{DA}+S_{DB}}$ and the average arrival rate of applicants at discriminatory firms is $\lambda_D = \frac{S_{DA}+S_{DB}}{N_{DA}+N_{DB}}$.

We write M_0, π_0, and I_0 for the number of total matches, realized firm profits, and aggregate income in the co-existence equilibrium. We write $E\left(\pi_0^{net}\right)$ for the expected net profits of the label-blind firms and $E\left(\pi_{DA}^{net}\right)$ and $E\left(\pi_{DB}^{net}\right)$ for those of the discriminatory firms. In equilibrium the expected net profits of all discriminatory firms must be equal.

The restriction $N_0 < \beta N$ indicates that if the label-blind firms post to attract only B-labels and the discriminatory ones post to attract only A-labels, then we would have $\lambda_{0B} > \lambda_U > \lambda_{DA}$ and it would not replicate the unbiased equilibrium. We now establish the composition of the firms in any equilibrium with $N_0 < \beta N$ label-blind firms co-existing with $N_D = N - N_0$ discriminatory ones.

Proposition 5. *There exists a unique symmetric SPMCE with $N_0 < \beta N$ label-blind firms and $N_D = N - N_0$ discriminatory firms, and it has the following properties. All label-blind firms post the same bonus $b_{0B} > b_{DB}$, and attract only B-label applicants. Therefore, $N_{0B} = N_0$, $N_{0A} = 0$, $S_{DA} = S_A$ and $N_{DA} > 0$. Furthermore, $E\left(\pi_0^{net}\right) > E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$, $\lambda_{0A} = 0 < \lambda_{DB} < \lambda_B$, $0 < \lambda_{DA} < \lambda_A$, and $\lambda_{DB} < \lambda_{DA} < \lambda_{0B}$.*

The proof of proposition 5 is in the appendix. It shows that, because the label-blind firms only have an advantage with respect to the B-labels, they only attract B-labels in equilibrium. Note as well that in this equilibrium some of the discriminatory firms continue to attract B-labels, so that $N_{DB} > 0$. In addition, the profits of the label-blind firms are strictly larger than those of the discriminatory firms.

An additional facet of the equilibrium with N_0 label-blind firms is that, holding realized profits constant, we have $V_{0B} = V_{DB} > V_B$ and $V_{DA} > V_A$ as seen in figure 3. To see the first point, consider equation (18) from the appendix and note that $\lambda_{DB} < \lambda_B$ and that $\lambda_{DA} < \lambda_A$. To see the second point, note that $V_{DA} = \pi e^{-\lambda_{DA}}$ which is decreasing in λ_{DA}. Similarly, holding realized profits constant and noting that $E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$, that $E\left(\pi_{DA}^{net}\right)$ is increasing in λ, and that $\lambda_{DA} < \lambda_A$ we see that expected firm profits of the discriminatory firms are lower in the co-existence equilibrium than in the fully discriminatory equilibrium.

4 The effect of international trade on labour market discrimination

The introduction of some label-blind firms allows us to consider the effect of trade liberalization on the prevalence of discrimination. We begin by analyzing how the movement from autarky to free trade affects the expected profits of discriminatory and label-blind firms. We then examine the effect of liberalizing trade on the wage gap for skilled workers.

The important difference between the two types of firms is that the label-blind firms have larger expected profits. The realized profits of all successful firms in the co-existence equilibrium with trade, π_0^{trade}, are the same, however, a label-blind firm receives a greater proportion of those profits in expectation. Hence, the effect of trade liberalization on realized profits has a magnified effect on the expected net profits of label-blind firms.

Proposition 6. *In the movement from autarky to free trade the expected net profits of label-blind firms will change by more than than those of the discriminatory firms. Hence, trade liberalization will disproportionately affect the label-blind firms.*

The proof of proposition 6 is in the appendix. Although proposition 6 shows that trade liberalization has a greater effect on the label-blind firms, it says nothing about the direction of that effect. We now derive the realized free trade profits π_0^{trade} and then compare them to the realized autarky profits $\pi_0 = \frac{L-M(1+f)}{(1-\frac{\alpha}{\sigma})M}\frac{\alpha}{\sigma} - f$.

When foreign productivity in the simple sector is given by γ, the foreign wage becomes γ as well. The price of a foreign sophisticated good is then $p_z = \frac{\sigma-1}{\sigma}\gamma$, foreign profits per firm are $\left(\pi_0^{trade}\right)^* = \gamma\left(\frac{\left(q_0^{trade}\right)^*}{\sigma-1} - f\right)$, output per domestic firm is $q_0^{trade} = \frac{\alpha\left(I_0+I_0^*\right)}{M_0+M_0^*\gamma^{1-\sigma}}\frac{\sigma-1}{\sigma}$ and output per foreign firm is $\left(q_0^{trade}\right)^* = \frac{\alpha\left(I_0+I_0^*\right)\gamma^{-\sigma}}{M_0+M_0^*\gamma^{1-\sigma}}\frac{\sigma-1}{\sigma}$.

Following our analysis from the proof of proposition 1, substituting for realized profits, and solving simultaneously, income in the home and the foreign country are given by:

$$I_0^{trade} = \frac{\left[L - M_0(1+f)\right]\left[M_0 + M_0^*\gamma^{1-\sigma} - M_0^*\frac{\alpha}{\sigma}\gamma^{1-\sigma}\right] + M_0\frac{\alpha}{\sigma}\left[L - M_0^*(1+f)\right]\gamma}{\left(M_0 + M_0^*\gamma^{1-\sigma}\right)\left(1-\frac{\alpha}{\sigma}\right)}$$

$$\text{and } \left(I_0^{trade}\right)^* = \frac{\left[L - M_0^*(1+f)\right]\gamma\left[M_0 + M_0^*\gamma^{1-\sigma} - M_0\frac{\alpha}{\sigma}\right] + M_0^*\frac{\alpha}{\sigma}\left[L - M_0(1+f)\right]\gamma^{1-\sigma}}{\left(M_0 + M_0^*\gamma^{1-\sigma}\right)\left(1-\frac{\alpha}{\sigma}\right)}.$$

Thus, realized profits of a Home firm with trade are given by $\pi_0^{trade} = \frac{\left[L-M_0(1+f)\right]+\gamma\left[L-M_0^*(1+f)\right]}{\left(M_0+M_0^*\gamma^{1-\sigma}\right)\left(\frac{\sigma}{\alpha}-1\right)} - f$.

It is straightforward to verify that π_0^{trade} is increasing in γ and L and decreasing in M_0 and M_0^*. If $\gamma = 1$, then $\pi_0^{trade} = \frac{2L-(M_0+M_0^*)(1+f)}{(M_0+M_0^*)(\frac{\sigma}{\alpha}-1)} - f$, which is equal to $\pi_0^{autarky}$ if $M_0 = M_0^*$. If $\gamma > 1$ and $M_0 = M_0^*$, then Home has a comparative advantage in the sophisticated sector and realized gross profits of the sophisticated firms increase with trade liberalization. In a similar manner, if $M_0 > M_0^*$ and $\gamma = 1$, then $\pi_0^{trade} > \pi_0^{autarky}$. On the other hand, if $M_0 < M_0^*$, then $\pi_0^{trade} < \pi_0^{autarky}$. Hence, if Home is a net exporter of the sophisticated good (note that realized profits are a strictly increasing affine transformation of output), then realized profits increase with trade. Combining this result with that in proposition 6 we have established the following.

Proposition 7. *If home is a net exporter of the sophisticated (simple) goods, then trade liberalization increases (decreases) the expected profit differential between the label-blind and the discriminatory firms.*

Proposition 7 suggests two ways in which trade liberalization can affect labour market discrimination. First, it shows how opening to trade affects the additional expected profits enjoyed by label-blind firms. Although our model takes a short-run approach by assuming away free entry of firms, proposition 7 suggests that label-blind and discriminatory firms enter and exit at different rates in response to trade liberalization. The second way, to be analyzed below, points to the direction in which the realized and expected bonuses will be affected by trade.

Proposition 7 also suggests that trade liberalization will make it more costly to discriminate in countries where there are fewer discriminatory firms and less costly where it is already more prevalent. In particular, if comparative advantage is driven by the number of matches instead of any technology difference, and if the home country has fewer matches because it has fewer label-blind firms, then the profit advantage of these label-blind firms will be diminished by opening up to international trade. In this way trade liberalization magnifies the good and the bad institutions that a country has in autarky.

Propositions 5 and 7 together provide some support and some limitations of the suggestion in Becker (1957) and Arrow (1972) that the market can ameliorate discrimination. First, proposition 5 shows that label-blind firms earn larger expected profits (the extra cost that discriminatory firms pay for their preferences are in the form of a reduced matching rate), which provides some support for Becker's hypothesis. On the other hand, proposition 7 shows that trade liberalization can reinforce a country's market imperfections (and perfections) and affect the expected profits of label-blind firms by more than those of discriminatory firms.

We now consider the effect of trade liberalization on the bonus gap for skilled workers. From proposition 5 we have that the realized and expected bonuses are $b_{DA} = \frac{\lambda_{DA}}{e^{\lambda_{DA}}-1}\pi_0 > b_{0B} = \frac{h(\lambda_{DB})}{h(\lambda_{0B})}e^{-\lambda_{DA}}\pi_0 > e^{-\lambda_{DA}}\pi_0 = b_{DB}$, where $h(\lambda) = \frac{1-e^{-\lambda}}{\lambda}$ is decreasing in λ and $V_{DA} = \pi_0 e^{-\lambda_{DA}} > \pi_0 \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}} e^{-\lambda_{DA}} = V_{DB} = V_{0B}$. Using these expressions, the reaction of realized and expected bonuses for skilled workers to changes in π is:

$$\frac{\partial b_{DA}}{\partial \pi} = \frac{\lambda_{DA}}{e^{\lambda_{DA}}-1} > \frac{\partial b_{0B}}{\partial \pi} = \frac{h(\lambda_{DB})}{h(\lambda_{0B})}\frac{\partial b_{DB}}{\partial \pi} > \frac{\partial b_{DB}}{\partial \pi} = h(\lambda_{DA})\frac{\partial b_{DA}}{\partial \pi};$$

$$\frac{\partial V_{0B}}{\partial \pi} = \frac{\partial V_{DB}}{\partial \pi} = h(\lambda_{DB})\frac{\partial V_{DA}}{\partial \pi} = h(\lambda_{DB})e^{-\lambda_{DA}}. \tag{14}$$

Hence, the realized and expected bonuses of both labels of skilled workers applying to the discriminatory firms are increasing in realized firm profits and, because $h(\lambda) < 1$, they are increasing faster for A-labels. Note as well that they are increasing faster for the B-label managers who are employed at the label-blind firms (because $\frac{h(\lambda_{DB})}{h(\lambda_{0B})} > 1$), although not as rapidly as for the A-labels. Still, the expected bonus of all B-labels is the same in equilibrium ($V_{0B} = V_{DB}$), so that the expected impact on B-labels applying to either type of firm is the same.

Equation (14) and proposition 7 together illustrate another way how the pattern of comparative advantage has a magnifying effect on the bonus for skilled workers. If the home country is a net exporter of the sophisticated good, then trade liberalization will increase all realized and expected bonuses, but also magnify the bonus gap between A- and B-labels. If, on the other hand, the home country has fewer matches and similar technologies, then increased globalization will decrease the bonus gap (and also the skilled workers' bonus). We summarize these results in proposition 8.

Proposition 8. *Trade liberalization has a larger impact on the realized and expected bonuses of A-label skilled workers. If home is a net exporter of the sophisticated (simple) goods, then trade liberalization increases (decreases) the bonus gap.*

Proposition 8 provides a provocative complement to proposition 7. If comparative advantage is driven by discrimination, i.e. the fewer number of home matches ($M_0 < M_0^*$) is more important than the foreign technology advantage ($\gamma > 1$) in determining the pattern of trade, then proposition 7 shows that trade liberalization reduces the profit advantage of the label-blind firms. In the profit dimension, trade magnifies a country's discriminatory (or non-discriminatory) tendencies. On the other hand, proposition 8 shows that increased trade has the opposite effect on the skilled worker wage gap. In particular, a more-discriminatory country will see a reduction in the skilled worker wage gap arising from discrimination and the already smaller gap increases in a less-discriminatory one.

More generally, propositions 7 and 8 suggest a way to understand the differing effects of trade liberalization on the wage gap and the survival of discriminatory firms. Countries that are exporters of simpler goods will see a decrease in the wage gap when liberalizing trade, but also a decrease in the profit advantage of label-blind firms. The opposite occurs in countries that are net exporters of more sophisticated goods. These contrasting results help shed light on why the wage gap has remained constant over the last fifteen years for women in the UK (The Economist, 2018) and in the US (Graf, Brown and Patten, 2018) and for the last thirty-five years for black and Hispanic men in the US (Patten, 2016), while at the same time the ability of firms to openly discriminate in hiring has been dramatically reduced.

5 Conclusion

We embed a competitive search model with labor market discrimination into a two-sector, two-country framework in order to analyze the relationship between international trade and labor market discrimination. Discrimination reduces the matching probability and output in the skill intensive differentiated-product sector so that discrimination-induced comparative advantage may overshadow technological comparative advantage in determining the pattern of trade. As countries alter their production mix in accordance with their comparative advantage, trade liberalization can then reinforce the negative effect of discrimination on development in the more discriminatory country and reduce its effect in the country with fewer discriminatory firms. Furthermore, globalization can increase the profit difference between label-blind and discriminatory firms in the less discriminatory country and can diminish it in the more discriminatory one. On the other hand, trade liberalization generates a reduction in the wage gap in the more discriminatory country and an expansion in the other one. The identification of these two opposing factors provides an explanation for the persistence of the wage gap over time even as the prevalence of discriminatory firms declines.

Appendix: proofs

Proposition 1. *Only non-discriminatory firms. There exists a unique symmetric unbiased SPMCE in which all firms offer an identical bonus $b_U = \frac{\pi_U \lambda_U}{e^{\lambda_U}-1}$ and all skilled workers adopt the same mixed application strategy in which they apply at each single firm with the same probability. A single skilled worker's expected bonus is given by $V_U = \pi_U e^{-\lambda_U}$, profits of each operating firm result as $\pi_U = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N(1-e^{-\lambda_U})}-1\right]-\sigma f\right)$ and expected profits of each firm net of bonus payments are given by $E\left(\pi_U^{net}\right) = \left[1-(1+\lambda_U)\, e^{-\lambda_U}\right]\pi_U$. National income results as $I_U = \frac{\sigma}{\sigma-\alpha}\left[L-(1+f)\, M_U\right]$ and the number of operating firms is given by $M_U = S\frac{1-e^{-\lambda_U}}{\lambda_U} = N(1-e^{-\lambda_U})$.*

Only discriminatory firms. There exists a unique symmetric SPMCE with discrimination. Firms separate so that a firm chooses a bonus that will attract only A-label applicants or only B-label applicants, bot not both. Bonuses are given by $b_A = \frac{\pi_A \lambda_A}{e^{\lambda_A}-1}$ and $b_B = V_A(\mathbf{b})$ and expected bonuses are given by $V_A(\mathbf{b}) = \pi_D e^{-\lambda_A}$ and $V_B(\mathbf{b}) = \pi_D e^{-\lambda_A}\frac{1-e^{-\lambda_B}}{\lambda_B}$. Profits of each operating firm result as $\pi_D = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_D}-1\right]-\sigma f\right)$, where $M_D = S_B\frac{1-e^{-\lambda_B}}{\lambda_B}+S_A\frac{1-e^{-\lambda_A}}{\lambda_A}$ and expected profits net of bonus payments are given by $E\left(\pi_A^{net}\right) = \left[1-(1+\lambda_A)\, e^{-\lambda_A}\right]\pi_D$ and $E\left(\pi_B^{net}\right) = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)\pi_D$. National income results as

$I_D = \frac{\sigma}{\sigma-\alpha}\left[L-(1+f)\, M_D\right]$. Finally, $\lambda_B < \lambda_U < \lambda_A$.

Proof. *Only non-discriminatory firms.* Since a skilled worker will only apply with positive probability at the firm(s) which offer(s) the highest expected bonus, the equilibrium expected bonus for a skilled worker is $V_U = \max_z\{V_z\}$, where $V_z = b_z\frac{1-e^{-\lambda_z}}{\lambda_z}$. Hence, in equilibrium, a firm will only receive applicants if it offers the highest expected bonus: $\lambda_z > 0$ and $V_z = V_U$ for $b_z \geq V_U$; $\lambda_z = 0$ and $V_z = b_z$ for $b_z < V_U$. Thus, for $b_z \geq V_U$ we have $\lambda_z = h^{-1}\left(\frac{V_U}{b_z}\right)$. Then, for any firm choosing $b_z \geq V_U$ the expected number of applicants is λ_z. In equilibrium the expected number of applicants to all firms is:

$$\sum_{z=1}^{N}\lambda_z = \sum_{z|b_z\geq V_U} h^{-1}\left(\frac{V_U}{b_z}\right) = S.$$

Note that h is strictly decreasing in λ_z. Therefore, h^{-1} is strictly decreasing in V_U and the number of terms in the summand is weakly decreasing in V_U. Hence, for a given vector of bonus offers $\mathbf{b}$ there exists a unique solution V_U to the above equation. Given V_U and the vector of bonus offers $\mathbf{b}$, each λ_z follows from $\lambda_z = h^{-1}\left(\frac{V_U}{b_z}\right)$. Notice that, from the perspective of a single firm, V_U is constant and independent of the firm's own bonus offer due to the large number of firms and skilled workers. Given this relationship between λ_z and b_z, we can now solve for the equilibrium of the entire wage-posting game by determining the firms' optimum bonus offers. From $V_z = b_z h(\lambda)$ we get $b_z = \frac{V_z}{h(\lambda_z)}$. Considering that $h(\lambda_z) = \frac{1-e^{-\lambda_z}}{\lambda_z}$, we can thus rewrite the expected profits net of payments to a manager as follows: $E\left(\pi_z^{net}\right) = \left(1 - e^{-\lambda_z}\right)\pi_z - \lambda_z V$. The value of λ_z which maximizes $E\left(\pi_z^{net}\right)$ results as $\lambda_z = \ln\left(\frac{\pi_z}{V(\mathbf{b})}\right)$. This latter expression can be transformed to $V(\mathbf{b}) = \frac{\pi}{e^\lambda}$. Considering that $V(\mathbf{b}) = bh(\lambda)$, we can derive the bonus which maximizes $E\left(\pi_z^{net}\right)$ by equating $\frac{\pi}{e^\lambda}$ with $bh(\lambda)$ and solving for b: $b = \frac{\pi\lambda}{e^\lambda - 1}$. As a consequence, we can rewrite the expected equilibrium profits of a firm z, net of payments to a manager, as $E\left(\pi_z^{net}\right) = \left[1 - (1+\lambda)e^{-\lambda}\right]\pi$. Since all firms offer an identical bonus in equilibrium, potential managers apply at all firms with an identical probability, therefore, $\lambda_U = \frac{S}{N}$. Thus, we can also solve for M_U: $M_U = S\frac{1-e^{-\lambda_U}}{\lambda_U} = N(1 - e^{-\lambda_U})$. The profit maximizing pricing rule for each single firm z is given by $p = \frac{\sigma}{\sigma-1}$. The consumers' utility maximizing consumption choices are given by the demand functions in equations (5)-(7).

In solving for market clearing, note that since all sophisticated firms charge an identical price in equilibrium, they all sell the same amount of their variety. Thus, demand for the numeraire good relative to demand for a single variety of the sophisticated good is given by: $\frac{C_0}{c} = M\frac{\sigma}{\sigma-1}\frac{1-\alpha}{\alpha}$. Labor market clearing implies that $L - Sh(\lambda) = L - M$ workers work as unskilled workers, and $M(q + f) = L_M$ of these unskilled workers work in the sophisticated sector. Hence, $C_0 = L - Sh(\lambda) - L_M = L - M(1+q+f)$. The total number of skilled workers is S, therefore, the number of skilled workers who work as unskilled is $S - M = S[1 - h(\lambda)]$. The condition that relative supply equals relative demand therefore becomes: $\frac{L-M(1+q+f)}{q} = M\frac{\sigma}{\sigma-1}\frac{1-\alpha}{\alpha}$. Thus, $q_U = \alpha\frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_U} - (1+f)\right]$.

National income is given as the sum of the wage bill plus expected profits plus the expected payment to the managers. The $L - S$ unskilled workers each receive a wage of one. The S skilled workers have an expected return of $V + \left(1 - \frac{M}{S}\right)$, where $\frac{M}{S}$ is the probability of a successful match. The profits of the M successful firms, $\pi - b$, are shared equally by all agents and in equilibrium $V = \frac{Mb}{S}$. Hence, total income is $I = L - S + \left[V + \left(1 - \frac{M}{S}\right)\right]S + M(\pi - b) = L + M(\pi - 1)$. Substituting from equation (8) for firm profits yields $I_U = \frac{\sigma}{\sigma-\alpha}\left[L - (1+f)M_U\right]$.

Since $\pi = \frac{q}{\sigma-1} - f$, profits of an operating firm result as: $\pi = \frac{\alpha}{\sigma-\alpha}\left[\frac{L}{M_U} - (1+f)\right] - f$. We can then use this expression for π_U and $\lambda_U = \frac{S}{N}$ to solve for $E\left(\pi_U^{net}\right)$, b_U, and V_U. Then we can solve for the aggregate price index P_M and consumption of the two aggregate goods C_0 and C_M.

Finally note that $V_U = \frac{e^{-\lambda_U}}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N(1-e^{-\lambda_U})} - 1\right] - \sigma f\right)$ which is increasing in L and decreasing in S. Hence, if L is sufficiently large compared to S, then $V_U > 1$ and since $V_U > 1$, all skilled workers search for a managerial job.

Only discriminatory firms. First, we will prove that firms separate. The probability that a firm receives at least one k-label applicant is $1-e^{-\lambda_k}$, while the probability that a firm receives no A-label applicants is $e^{-\lambda_A}$. Thus, if a firm z attracts both A-label and B-label applicants, the firm's expected net profits are:

$$E\left(\pi_z^{net}\right) = E\left(\pi_A^{net}\right) + E\left(\pi_B^{net}\right) = \left(1-e^{-\lambda_A}\right)(\pi_z - b_z) + e^{-\lambda_A}\left(1-e^{-\lambda_B}\right)(\pi_z - b_z).$$

The firm's optimal choice of bonus satisfies $\frac{\partial E(\pi_z^{net})}{\partial b_z} = 0$, or

$$e^{-\lambda_A} e^{-\lambda_B} - 1 + e^{-\lambda_B} e^{-\lambda_B}\ (\pi_z - b_z)\left(\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z}\right) = 0.$$

However, if $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z} < 0$, then $\frac{\partial E(\pi_z^{net})}{\partial b_z} < 0$. In this case, a firm choosing a bonus that is large enough to attract A- and B-label workers would want to lower the offered bonus and then only attract B-label workers. Notice that the condition $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z} < 0$ says that a reduction in the bonus would increase the number of B-level applicants by more than it would decrease the number of A-label applicants and no firm would ever choose a bonus that attracts both labels of potential managers.

To evaluate the sign of $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z}$, we rewrite the terms for the expected market bonuses to $V_B(\mathbf{b}) = b_z e^{-\lambda_A} h(\lambda_B)$ and $V_A(\mathbf{b}) = b_z h(\lambda_A)$, where $h(\lambda_k) = \frac{1-e^{-\lambda_k}}{\lambda_k}$. Totally differentiating $V_B = b_z e^{-\lambda_A}\frac{1-e^{-\lambda_B}}{\lambda_B}$ and $V_A = b_z\frac{1-e^{-\lambda_A}}{\lambda_A}$ with respect to the (common) bonus, considering that V_B and V_A are constant from a single firm's perspective and solving for $\frac{d\lambda_B}{db}$ and $\frac{d\lambda_A}{db}$ leads to:

$$\frac{d\lambda_A}{db} = -\frac{\left(1-e^{-\lambda_A}\right)\lambda_A}{b\left(e^{-\lambda_A}\lambda_A - 1 + e^{-\lambda_A}\right)}$$

$$\frac{d\lambda_B}{db} = -\frac{\lambda_B\left(1-e^{-\lambda_B}\right)\left(\lambda_A - 1 + e^{-\lambda_A}\right)}{\left(e^{-\lambda_B}\lambda_B - 1 + e^{-\lambda_B}\right) b\left(e^{-\lambda_A}\lambda_A - 1 + e^{-\lambda_A}\right)}.$$

Thus, we get:

$$\frac{d\lambda_A}{db} + \frac{d\lambda_B}{db} = -\frac{1}{b}\frac{\lambda_A\left(e^{\lambda_A}-1\right)}{\left(e^{\lambda_A}-\lambda_A-1\right)}\left[\frac{\lambda_B\left(e^{\lambda_B}-1\right)}{2\left(e^{\lambda_B}-\lambda_B-1\right)}\frac{2\left(\lambda_A e^{\lambda_A}-e^{\lambda_A}+1\right)}{\lambda_A\left(e^{\lambda_A}-1\right)} - 1\right].$$

$\frac{d\lambda_A}{db} + \frac{d\lambda_B}{db} < 0$ since $e^{\lambda_A} - \lambda_A - 1 > 0$, $\frac{\lambda_B(e^{\lambda_B}-1)}{2(e^{\lambda_B}-\lambda_B-1)} > 1$ and $\frac{2(\lambda_A e^{\lambda_A}-e^{\lambda_A}+1)}{\lambda_A(e^{\lambda_A}-1)} > 1$, and no firm would ever choose a bonus that attracts both labels of potential managers.

The derivations of b_A and V_A are identical to the derivations in the case without discrimination and are shown in the proof for the part with only non-discriminatory firms.

For the derivations of b_B and V_B note that for the firms that attract B-label applicants we must have $b_B \leq V_A(\mathbf{b})$ because A-label workers will apply if $b_B > V_A(\mathbf{b})$. If $b_B \leq V_A(\mathbf{b})$ then only B-labels will apply. Hence, $b_B = V_A(\mathbf{b})$. Then, $V_B(\mathbf{b}) = V_A(\mathbf{b})\, h(\lambda_B) = V_A(\mathbf{b}) \frac{1-e^{-\lambda_B}}{\lambda_B} = \frac{1-e^{-\lambda_B}}{\lambda_B} \pi_D e^{-\lambda_A}$ and $E\left(\pi_B^{net}\right) = \left(1-e^{-\lambda_B}\right)[\pi_D - V_A(\mathbf{b})] = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)\pi_D$.

Similar to the non-discriminatory case we can write total income as $I = L - S + \left[V_A + \left(1 - \frac{M_A}{S_A}\right)\right] S_A + \left[V_B + \left(1 - \frac{M_B}{S_B}\right)\right] S_B + M_A(\pi_D - b_A) + M_B(\pi_D - b_B) \ = \ L + M_D(\pi_D - 1)$. Substituting from equation (8) for firm profits yields $I_D = \frac{\sigma}{\sigma-\alpha}\left[L - (1+f) M_D\right]$.

The output of a single operating firm in the monopolistically competitive sector is then given as $q_D = \alpha \frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_D} - (1+f)\right]$ and the profits of an operating firm is $\pi_D = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_D} - 1\right] - \sigma f\right)$. We can then use this expression for π_D and $\lambda_k = \frac{S_k}{N_k}$ to solve for b_k and V_k for $k \in \{a, B\}$. Then we can solve for the aggregate price index P_M and consumption of the two (aggregate) goods C_0 and C_M.

Note that $V_B = \frac{1-e^{-\lambda_B}}{\lambda_B} \frac{e^{-\lambda_A}}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N_A(1-e^{-\lambda_A}) + N_B(1-e^{-\lambda_B})} - 1\right] - \sigma f\right)$ which is increasing in L and decreasing in S_A and in S_B. Hence, if L is sufficiently large compared to S, then $V_B > 1$ and, because $V_A > V_B$, we know that $V_A > 1$ so that all skilled workers will apply for a managerial position.

To show that the discriminatory equilibrium is unique, note first that $E\left(\pi_A^{net}\right)$ is increasing in λ_A and, therefore, is decreasing in N_A. Second, note that $E\left(\pi_B^{net}\right)$ is increasing in λ_B and decreasing in λ_A and, therefore, is decreasing in N_B and increasing in N_A. Hence, in equilibrium the number of firms attracting A- and B-label applicants will adjust until $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$. Third, note that using $\beta = \frac{S_B}{S_B+S_A}$ and $\lambda_U = \frac{S_A+S_B}{N}$, we can write: $\lambda_B = \frac{S_B}{N_B} = \beta \frac{\lambda_U N}{N_B} = \beta \frac{\lambda_U N}{N_B}\frac{\lambda_A}{\lambda_A} = \frac{\beta \lambda_U \lambda_A}{\frac{\lambda_A N_B}{N}}$, which we can further transform to: $\lambda_B = \frac{\beta\lambda_U\lambda_A}{\frac{\lambda_A(N-N_A)}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A - \frac{\lambda_A N_A}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A - \frac{\Lambda_A}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A - (1-\beta)\lambda_U}$. Fourth, note that from $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$ it follows that $1 - (1+\lambda_A)e^{-\lambda_A} = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)$, which we can transform to $e^{-\lambda_B}\left(1-e^{-\lambda_A}\right) = \lambda_A e^{-\lambda_A}$ and further to $\lambda_B = \ln\left(\frac{1-e^{-\lambda_A}}{\lambda_A e^{-\lambda_A}}\right)$. From $\lambda_B = \frac{\beta\lambda_U\lambda_A}{\lambda_A-(1-\beta)\lambda_U}$ we have $\frac{\partial\lambda_B}{\partial\lambda_A} = -\frac{\beta(1-\beta)\lambda_U^2}{\left[\lambda_A-(1-\beta)\lambda_U\right]^2}$, which is negative and defined as long as $\lambda_A \neq (1-\beta)\lambda_U$. In addition, $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} = \frac{2\beta\lambda_U^2(1-\beta)}{\left[\lambda_A-(1-\beta)\lambda_U\right]^3}$. Hence, $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} > 0$ if $\lambda_A > (1-\beta)\lambda_U$ and $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} < 0$ if $\lambda_A < (1-\beta)\lambda_U$. Considering $\lambda_B = \ln\left(\frac{1-e^{-\lambda_A}}{\lambda_A e^{-\lambda_A}}\right)$, we can derive the following: $\frac{\partial\lambda_B}{\partial\lambda_A} = \frac{\lambda_A - 1 + e^{-\lambda_A}}{(1-e^{-\lambda_A})\lambda_A} > 0$. This second equation for λ_B and its derivative are positive for all values of $\lambda_A \geq 0$ and the first equation is positive for $\lambda_A > (1-\beta)\lambda_U$. Note that $\lambda_A > (1-\beta)\lambda_U$ is equivalent to $N_A < N$ which must hold since $\beta \in (0,1)$. Hence, there is a unique solution for λ_A, λ_B where both are greater than zero.

Finally, the ordering $\lambda_B < \lambda_U < \lambda_A$ follows from $\lambda_B = \ln\left(\frac{1-e^{-\lambda_A}}{e^{-\lambda_A}\lambda_A}\right)$, which can be transformed to $\lambda_B - \lambda_A = \ln\left(\frac{1-e^{-\lambda_A}}{\lambda_A}\right) = \ln\left(h\left(\lambda_A\right)\right) < 0$. Thus, $\lambda_B = \frac{S_B}{N_B} < \frac{S_B+S_A}{N_B+N_A} = \lambda_U < \frac{S_A}{N_A} = \lambda_A$. □

Proposition 2. *The number of unfilled vacancies is larger, and the number of successful matches is smaller, in the discriminatory equilibrium.*

Proof. The proof proceeds by showing that $\Psi(\eta)$ is strictly convex in η, that $\Psi(\eta)$ attains its minimum at $\eta_{min} = \frac{S_A}{S_A+S_B}$ and that $\Psi(\eta_{min}) = e^{-\frac{S_A+S_B}{N}} = e^{-\lambda_U}$. The partial derivative of Ψ with respect to η results as: $\frac{\partial\Psi}{\partial\eta} = e^{-\frac{S_A}{\eta N}}\left(1+\frac{S_A}{\eta N}\right) - e^{-\frac{S_B}{(1-\eta)N}}\left(1+\frac{S_B}{(1-\eta)N}\right)$. Note that $\frac{\partial\Psi}{\partial\eta} = 0$ if $\eta = \frac{S_A}{S_A+S_B}$. To see that $\eta = \frac{S_A}{S_A+S_B}$ is, in fact, a minimizer of Ψ, note that $\frac{\partial^2\Psi}{\partial\eta^2} = e^{-\frac{SA}{\eta N_D}}\lambda_A^2\frac{N_D}{N_A} + e^{-\frac{SB}{(1-\eta)N_D}}\lambda_B^2\frac{N_D}{N_B} > 0$. Substitution then yields that $\Psi(\eta_{min}) = e^{-\frac{S_A+S_B}{N}}$, which equals the unfilled vacancy rate in the non-discriminatory case since $\frac{S_A+S_B}{N} = \lambda_U$. □

Proposition 5. *There exists a unique symmetric SPMCE with $N_0 < \beta N$ open-minded firms and $N_D = N - N_0$ discriminatory firms and it has the following properties. All label-blind firms post the same bonus $b_{0B} > b_{DB}$, and attract only B-label applicants. Therefore, $N_{0B} = N_0$, $N_{0A} = 0$, $S_{DA} = S_A$ and $N_{DA} > 0$. Furthermore, $E\left(\pi_0^{net}\right) > E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$, $\lambda_{0A} = 0 < \lambda_{DB} < \lambda_B$, $0 < \lambda_{DA} < \lambda_A$, and $\lambda_{DB} < \lambda_{DA} < \lambda_{0B}$.*

Proof. We start by showing that N_{DA}, N_{DB}, N_{0A}, and N_{0B} cannot all simultaneously be positive. First, note that $N_{tk} > 0$ in equilibrium if and only if they attract some skilled workers so that $S_{tk} > 0$, therefore, λ_{tk} would be strictly positive and finite. Second, note that if $N_{tk} > 0$ then any posted bonus by the non-discriminatory firms must leave label k applicants indifferent between the non-discriminatory and the discriminatory firms in equilibrium. For the A-label applicants this indifference implies $V_{0A} = V_{DA} = \pi_0 e^{-\lambda_{DA}}$. For the B-label applicants this indifference implies $V_{0B} = V_{DB} = \pi_0\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}e^{-\lambda_{DA}}$. Third, note that if both $N_{0B} > 0$ and $N_{0A} > 0$, then applicants must be indifferent between either group of label-blind firms. Putting $V_{0A} = V_{0B}$ and using the above relationships implies that $\pi_0 e^{-\lambda_{DA}} = \pi_0\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}e^{-\lambda_{DA}}$ or $1 = \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}$, which is impossible given that $1 - e^{-\lambda_{DB}} < \lambda_{DB}$ for any $\lambda_{DB} \in (0,\infty)$.

We now show that $N_{0B} > 0 = N_{0A}$. To see this point, note that the discriminatory firms that attract B-label applicants maximize expected profits subject to the constraint that the bonus for B-labels is no larger than the expected bonus of the A-labels. This constraint arises because an A-label applicant would always be hired instead of a B-label at any discriminatory firm. A label-blind firm does not face this constraint and because the derivative of expected profits with respect to the bonus is positive at $b_{0B} = V_{DA}(\mathbf{b}) < b_{DA}$, the non-discriminatory firms can increase profits by offering a higher bonus, $b_{0B} > b_{DB}$, to B-label applicants. On the other hand, when attracting A-label applicants, the bonus b_{DA} is profit maximizing. Hence, given that all discriminatory firms earn the same profits, that the non-discriminatory firms cannot earn higher profits than the discriminatory firms if they attract A-labels, that they can earn higher profits if they attract B-labels, and that $N_0 < \beta N < N_B$ (because $\lambda_B = \frac{\beta S}{N_B} < \frac{S}{N} = \lambda_U$), we must have $N_{0B} = N_0 > 0 = N_{0A}$. Furthermore, because these $N_0 = N_{0B}$ firms are identical and cannot coordinate their actions we have that they all choose the same $b_{0B} > b_{DB}$. Finally, given that $N_{0A} = 0$ and that $S_A > 0$ it must be the case that $N_{DA} > 0$.

The equilibrium is defined as follows. The expected profits of the discriminatory firms must be equal so that

$$E\left(\pi_{DA}^{net}\right) = \left[1 - (1 + \lambda_{DA})\, e^{-\lambda_{DA}}\right]\pi_0 = \left(1 - e^{-\lambda_{DB}}\right)\left(1 - e^{-\lambda_{DA}}\right)\pi_0 = E\left(\pi_{DB}^{net}\right). \tag{15}$$

If $N_{DB} = 0$, then equation (15) would become $\left[1 - (1 + \lambda_{DA})\, e^{-\lambda_{DA}}\right]\pi_0 < \left(1 - e^{-\lambda_{DA}}\right)\pi_0$ so that a single discriminatory firm could increase its expected profits by the choice of a hiring probability (and bonus) that would attract a B-label.

Note that equation (15) can be transformed to

$$\lambda_{DB} = \ln\left(\frac{1 - e^{-\lambda_{DA}}}{\lambda_{DA} e^{-\lambda_{DA}}}\right). \tag{16}$$

Next, using the expressions $\beta_D = \frac{S_{DB}}{S_{DA}+S_{DB}}$ and $\lambda_D = \frac{S_{DA}+S_{DB}}{N_{DA}+N_{DB}}$, we can derive:

$$\lambda_{DB} = \frac{\beta_D \lambda_D \lambda_{DA}}{\lambda_{DA} - (1 - \beta_D)\,\lambda_D}. \tag{17}$$

In addition, the B-label applicants must be indifferent between applying to a non-discriminatory and a discriminatory firm:

$$V_{0B} = b_{0B}\frac{1 - e^{-\lambda_{0B}}}{\lambda_{0B}} = \frac{1 - e^{-\lambda_{DB}}}{\lambda_{DB}} e^{-\lambda_{DA}}\pi_0 = V_{DB}. \tag{18}$$

From the first part of this proof we know that $b_{0B} > e^{-\lambda_{DA}}\pi_0 = b_{DB}$ and because $h(\lambda)$ is declining in λ, it is therefore seen that $\lambda_{0B} > \lambda_{DB}$. From the first part of this proof we also know that the label-blind firms do not face the same constraint as the discriminatory firms, and by increasing $b_{0B} > b_{DB} = V_{DA}$ they have larger expected profits than the discriminatory ones so that $E\left(\pi_0^{net}\right) > E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$.

For a given λ_{0B} (which is a function of the yet to be determined S_{0B} or, equivalently, β_D) the profit maximizing bonus of the non-discriminatory firms can be derived in a manner similar to that in proposition 1 as:

$$b_{0B} = \frac{\lambda_{0B}\pi_0}{e^{\lambda_{0B}} - 1}. \tag{19}$$

Equations (18) and (19) can be solved for λ_{0B} as a function of λ_{DB} and λ_{DA}:

$$\lambda_{0B} = \ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1 - e^{-\lambda_{DB}}}\right). \tag{20}$$

Equations (16), (17), and (20) jointly determine the three variables λ_{0B}, λ_{DA} and λ_{DB}. Equation (16) describes a positively sloped curve in a diagram with λ_{DB} and λ_{DA} on the axes. Equation (17) describes a negatively sloped and strictly convex curve in the same space as long as $\lambda_{DA} > (1 - \beta_D)\lambda_D$, which is equivalent to $\frac{S_A}{N_{DA}} > \frac{S_A}{N_{DA}+N_{DB}}$, which is true. Thus, λ_{DB} and λ_{DA} are uniquely defined for a given level of S_{DB}.

To determine how λ_{DB} and λ_{DA} depend on S_{DB}, we consider equations (16) and (17). First, note that equation (16) does not depend on S_{DB}. Second, consider the following partial derivative from equation (17):

$$\frac{\partial\lambda_{DB}}{\partial S_{DB}} = \frac{\frac{\partial\beta_D}{\partial S_{DB}}\left(\lambda_{DA} - \lambda_D\right)\lambda_D + \frac{\partial\lambda_D}{\partial S_{DB}}\beta_D\lambda_{DA}}{\left[\lambda_{DA} - (1 - \beta_D)\lambda_D\right]^2}\lambda_{DA},$$

which is positive since $\lambda_{DA} - \lambda_D > 0$, $\frac{\partial\beta_D}{\partial S_{DB}} > 0$ and $\frac{\partial\lambda_D}{\partial S_{DB}} > 0$. To see that $\lambda_{DA} - \lambda_D > 0$, refer to equation (16) and perform the same analysis as in proposition 1. Thus, the downward sloping curve shifts upward with S_{DB}, implying that λ_{DA} and λ_{DB} depend positively on S_{DB}.

We now show that S_{DB} is uniquely defined by equation (20). Considering that $\lambda_{0B} = \frac{S_{0B}}{N_0}$, we can rewrite equation (20) as follows: $\frac{S_B - S_{DB}}{N_0} = \ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1-e^{-\lambda_{DB}}}\right)$. Thus, the left hand side of equation (20) depends negatively on S_{DB}, while the right hand side of equation (20) depends positively on S_{DB}:

$$\frac{\partial}{\partial S_{DB}}\left(\ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1 - e^{-\lambda_{DB}}}\right)\right) = \frac{\left[1 - e^{-\lambda_{DB}}\left(1 + \lambda_{DB}\right)\right]\frac{\partial\lambda_{DB}}{\partial S_{DB}} + \lambda_{DB}\left(1 - e^{-\lambda_{DB}}\right)\frac{\partial\lambda_{DA}}{\partial S_{DB}}}{\lambda_{DB}\left(1 - e^{-\lambda_{DB}}\right)} > 0.$$

Thus, equation (20) defines a unique value of S_{DB}.

The number of successful matches can then be expressed as: $M_{0B} = S_{0B}\frac{1-e^{-\lambda_{0B}}}{\lambda_{0B}}$, $M_{DA} = S_{DA}\frac{1-e^{-\lambda_{DA}}}{\lambda_{DA}}$ and $M_{DB} = S_{DB}\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}} = (S_B - N_0\lambda_{0B})\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}$. Notice that $S_{DA} = S_A$. Given $M_0 = M_{DA} + M_{DB} + M_{0B}$ we can then solve for $I = L - S + \sum_{i=DA,DB,0B}\left[V_i + \left(1 - \frac{M_i}{S_i}\right)\right]S_i + \sum_{i=DA,DB,0B} M_i(\pi_0 - b_i) = L + M_0(\pi_0 - 1)$, which in turn yields $q_0 = \alpha\frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_0} - (1+f)\right]$ and $\pi_0 = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_0} - 1\right] - \sigma f\right)$. The expression for π_0 then allows us to solve for b_{0B}, b_{DA}, b_{DB} and the corresponding expected bonuses.

We now show that $\lambda_{DA} < \lambda_A$. To see this fact, suppose instead that $\lambda_{DA} \geq \lambda_A$, which implies that $N_{DA} \leq N_A$ (because S_A cannot decrease). But then $N_{DB} \geq N_B$ and, therefore, $\lambda_{DB} < \lambda_B$ so that $E\left(\pi_{DA}^{net}\right) > E\left(\pi_{DB}^{net}\right)$ which does not satisfy equation (15). Hence, $\lambda_{DA} < \lambda_A$. Finally, to see that $\lambda_{DB} < \lambda_B$, note from equation (17) that λ_{DB} is increasing in β_D and in λ_D. Hence, because some of the B-labels apply to the non-discriminatory firms, we must have $\beta_D < \beta$ and, because $S_{DA} = S_A$, we must also have $\lambda_D < \lambda_U$. Hence, $\lambda_{DB} < \lambda_B$.

To see that $\lambda_{DB} < \lambda_{DA}$ rewrite equation (16) as $\lambda_{DB} - \lambda_{DA} = \ln\left(\frac{1-e^{-\lambda_{DA}}}{\lambda_{DA}}\right) = \ln(h(\lambda_{DA})) < 0$. To see that $\lambda_{DA} < \lambda_{0B}$ substitute equation (19) into the left hand side of (18) and rewrite as $e^{-\lambda_{0B}} = \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}e^{-\lambda_{DA}}$ or $e^{\lambda_{0B}} = \frac{\lambda_{DB}}{1-e^{-\lambda_{DB}}}e^{\lambda_{DA}}$ or $e^{\lambda_{0B}} = \frac{1}{h(\lambda_{DB})}e^{\lambda_{DA}}$, so that $e^{\lambda_{0B}} > e^{\lambda_{DA}}$ because $h(\lambda_{DB}) < 1$. □

Proposition 6. *In the movement from autarky to free trade the expected net profits of label-blind firms will change by more than than those of the discriminatory firms. Hence, trade liberalization will disproportionately affect the label-blind firms.*

Proof. From proposition 5 we have that $E\left(\pi_0^{net}\right) > E\left(\pi_{DB}^{net}\right) = E\left(\pi_{DA}^{net}\right) = \left[1 - (1+\lambda_{DA})e^{-\lambda_{DA}}\right]\pi_0$. For all successful firms the realized gross profits π_0 are the same. Therefore, the label-blind firms receive a larger expected share of the realized gross profits and any change in the realized gross profits generates a larger change in the expected net profits of the label-blind firms. □

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