The Long-Run Non-Neutrality of Monetary Policy: A General Statement in a Dynamic General Equilibrium Model

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Abstract
This paper provides an explanation of the long-run neutrality of monetary policy in a dynamic general equilibrium model with micro-foundations. If the rate of time preference is endogenous there is no natural rate of interest. Therefore, if the central bank follows an interest rate rule this will affect the real rate of interest in financial markets and thereby the real economy. In principle, there is a negative relationship between the real rate of interest and the rate of inflation. This turns out to be nothing other than the historical “forced savings effect”, or the twentieth century Mundell-Tobin effect.

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1. Introduction
The idea of the long-run neutrality of changes in monetary policy is part of the DNA of the “Classical” approach to economic theory, going back at least to Hume in 1752 (Humphrey 1998, 8-9). Of course, there have always been challenges to this position. Historically, for example, the so-called “forced savings effect” (Hayek 1932, 1939, Humphrey 1983, Smithin 2013, 2018) was often treated as a sort of exception that proves the rule to the general theoretical presumption of monetary neutrality. In the mid-twentieth century there was considerable discussion of the analogous “Mundell-Tobin effect” named after the contributions of Mundell (1963) and Tobin (1965) which also appeared to show non-neutrality (Begg 1980, 1982, Blanchard and Fisher 1989, Smithin 1980, 2013, 2018, Turnovsky 2000, Walsh 1998). However, whether in the nineteenth century, twentieth century, or now in the twenty-first century, these sorts of arguments have not been well received, to say the least, by the majority of economic theorists in the mainstream of the profession.4

One argument that has frequently been made in recent decades is that a correct understanding of the so-called “microfoundations of macroeconomics” will enable the theorist to confidently rule out anything like a forced savings result. Walsh (1998, 48-9), for example, has put forward a number of arguments against some of the twentieth century demonstrations of the Mundell-Tobin effect, the most important of which is the following:

the … behavioural relationships are *ad hoc* in the sense that they are not explicitly based on maximizing behaviour by the agents of the model. This limitation can lead to problems when we try to understand the effects of changes in the economic environment, such as changes in the rate of inflation. The effects will depend in part, on the way in which individual agents adjust, so we need to be able to predict how the demand function for money changes if the underlying time series behaviour of the inflation process were to change ... (d)oing so will ... highlight channels leading to quite different predictions than Tobin found ...

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4 See also Kam (2000, 2005), Kam and Moshin (2006), Kam and Smithin (2012a, 2012b) and Tabassum (2012).
Now this sort of appeal to the microfoundations is not, in fact, a generally valid argument from either the philosophical or methodological point of view. According to King (2012, 9), for example, there are two main problems with what he unhesitatingly calls the microfoundations dogma, namely, “the fallacy of composition and downward causation”. Therefore:

Since the microfoundations dogma is inconsistent with both of these principles, the dogma itself must be false. (Emphasis added)

Nonetheless as suggested in the quote from Walsh, and in very many other examples in the contemporary literature, the idea that an appeal to “the” microfoundations is decisive is now almost universally accepted among the relevant peer group of academic economists. In the current intellectual environment, this situation in itself provides an extremely difficult challenge for those trying to engage in meaningful debate. Therefore, Kam (2000, 2005) took a different approach to that of King in addressing the question of monetary non-neutrality. That project was to show that non-neutrality still applies even in a framework which arguably had impeccable microfoundations by the standards of orthodox neoclassical economics. The purpose of the exercise was essentially to communicate with those colleagues who may be very well-versed in mathematical techniques, but not necessarily in questions of ontology and epistemology.

Kam’s work was based on a modification of the well-known Sidrauski model (Sidrauski 1967) which at that time had been a staple of graduate-level textbooks for many years (Blanchard and Fisher 1989, Turnovsky 2000, Chiang and Wainwright 2005), and still is to this day. However, the canonical model in twenty-first century theoretical macroeconomics is now

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5 The intellectual roots of this general approach go even further back, to the works of the Cambridge mathematician and philosopher Frank Ramsey in the 1920s (Ramsey 1927, 1928). Rather obviously, Ramsey must have had his mathematical rather than his philosophical hat firmly on his head at the time when he made these contributions. Tellingly Keynes, who was the editor of the journal in which these papers were published (and also a mathematician and philosopher and colleague and friend of Ramsey) made no use whatsoever of Ramsey’s approach in either the Treatise on Money (Keynes 1930) or the General Theory (Keynes 1936).
one version or another of either the dynamic general equilibrium (DGE) model or the dynamic stochastic general equilibrium (DSGE) model (DeVroey 2016, King 2012, 1, Scarth 2014, Woodford 2010, 1-4). At this stage of the game it therefore also seems important (again for the purposes of communication) to make a more general statement about the issues in the context of a theoretical DGE model.

2. A Neo-Wicksellian DGE model with a Representative Agent, Endogenous Money and a Constant Rate of Time Preference

A first step is to construct a benchmark DGE model in which, by analogy to the conventional Sidrauski model, long-run monetary neutrality holds. This will involve a neo-Wicksellian framework with a representative agent, endogenous money, and a constant rate of time preference (Smithin 2013, 125-32). The representative agent is thought of as a so-called “worker-consumer” and solves the following dynamic optimization problem by maximizing utility over an infinite time horizon:

\[
\text{Max} \sum \beta U(C_t), \quad U'(C_t) > 0, \quad U''(C_t) < 0
\]

Subject to:

\[
W - W_{-1} = Y + rD - C - \delta K, \quad 0 < \delta < 1
\]

\[
Y = F(K), \quad F'(K) > 0, \quad F''(K) < 0
\]

\[
W = K + D.
\]

Here, \(W\) is real wealth, \(D\) is the real value of an interest-bearing financial asset denominated in the unit of account (such as interest-bearing bank deposits), \(K\) is (supposedly) a measure of the

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\(^6\) The difference between these two approaches is the extent to which stochastic calculus and the theorems of statistical probability theory play a role in the analysis (Woodford 2010, 1). For the purposes of the present argument, dealing as it does primarily with long-run equilibrium issues, a consideration of the DGE model will suffice to make the point.

\(^7\) The seeming absurdity of these sorts of assumptions (that is, from the point of view of the non-economist and also even those of many heterodox economists) is actually not our main concern here. As mentioned, the objective is to conform as far as possible to the procedures typically followed in the standard literature.
“real” capital stock,\(^8\) \(Y\) is real GDP, \(r_D\) is the real interest rate on the financial asset, and \(\delta\) is the deprecation rate. The overall problem is:

\[
(5) \quad \text{Max} \sum \beta^t \{U(C_t) + \lambda_t[F(K_t) + r_D(W_t - K_t) - C_t - \delta K_t + W_{t+1} - W_t]\}
\]

noting that,

\[
(6) \quad \beta = \frac{1}{1 + \theta}.
\]

The term \(\beta\) is usually known as the “discount factor” where \(\theta\) in the denominator stands for the rate of time preference which is taken as given. In fact, the assumption of a constant rate of time preference is the precise modern equivalent of Wicksell’s (1898, xxv) concept of a “natural rate” of interest. It is this assumption rather than anything to do with the mathematical structure of the problem or the level of the analysis which ensures an eventual result of monetary neutrality. The first-order conditions for the solution to the optimization problem are:

\[
(7) \quad U'(C) - \lambda = 0 \\
(8) \quad F'(K) - r_D - \delta = 0 \\
(9) \quad \lambda r_D + \lambda_{t+1} \beta = 0
\]

And, therefore, the dynamic system reduces to:

\[
(10) \quad U'(C)r_D = -U'(C_{t+1})\beta \\
(11) \quad F'(K) - \delta = r_D.
\]

As shown by Kam (2000, 30-3), drawing on the literature in mathematical economics from the second half of the twentieth century, the dynamic properties of this type of model with only two

\(^8\) From the point of view of a \textit{realist} approach “\textit{per totam viam}” (Mendoza 2012) there would be yet another problem with this additional assumption. This is because the debates about capital theory, in the 1950s and 1960s, raised serious doubts about whether it is even possible to give any precise quantitative meaning to the notion of a physical capital stock (Cohen and Harcourt 2003, Harcourt 1969, Smithin 2018). However, and as is well-known, this problem is routinely simply ignored in the standard literature and has been so for the past fifty years or more. Therefore, given once again that a central purpose of this paper is communication with the mainstream of the economics profession, we waive any further discussion of the capital theory debate.
assets will generally involve a saddle-point. This particular specification is no exception.

Therefore, if a plausible transversality condition can be identified, the system will converge to the steady-state:

(12) \[ r = \theta \]

(13) \[ F'(K) - \delta = r_D \]

And the overall macroeconomic equilibrium can thus be characterized as:

(14) \[ r_D = \theta \]

(15) \[ F'(K^N) - \delta = \theta \]

(16) \[ Y^N = F(K^N) \]

where \( K^N \) is the equilibrium level of the capital stock, and \( Y^N \) is the equilibrium level of output.9

Dropping the “D” subscript on the real rate of interest, the political economy of these results can be expressed even more simply:

(17) \[ r = r^N (= \theta) \quad \text{(there is natural rate of interest)} \]

(18) \[ Y = Y^N \quad \text{(there is a natural level of output)} \]

In equilibrium “the” real rate of interest will be at its natural level (determined by the constant rate of time preference) and rate of output will also be at its natural level.

3. How to Handle Inflation in Neo-Wicksellian DGE models?

As has been shown above, it is actually a fairly straightforward exercise to derive the real equilibrium of the neo-Wicksellian model. The results conform to what would be expected. However, as previously discussed, for example, by Rogers (2006) and Smithin (2003, 2013) there turns out to be something of a problem for the neo-Wicksellian theorist in any attempt to

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9 As indicated in the text below the superscript “\( N \)” carries the connotation of the “natural rates” of real interest (au Wicksell) and output, to which the economy is always supposedly converging.
include an explanation of inflation in the analysis.\textsuperscript{10} To see why this is so note that, by definition in equilibrium, the real interest rate is given by the nominal interest rate less the equilibrium inflation rate. That is:

\begin{equation}
    r = i - p
\end{equation}

But then, from equation (17), it must also be true that:

\begin{equation}
    p = i - r^N
\end{equation}

The problem with this is that it implies that \textit{(e.g.)} an increase in the nominal interest rate will lead to an increase in the inflation rate. But this would not be a Wicksell-type result at all. It is entirely counter-intuitive from a Wicksellian point of view if not from an old-fashioned monetarist perspective (MacKinnon and Smithin 1993). The monetarists were always quite happy to insist that nominal interest rates would be positively correlated with inflation, with causality running from the growth rate of the money supply to inflation, and then to nominal interest rates. The Wicksell-type argument, on the contrary, would be that a \textit{lower} not a higher interest rate leads to \textit{higher} inflation.

To try to solve this problem suppose that, alternatively, we add a typical central bank reaction function to our model, the proto-type of which was the famous “Taylor rule” (Taylor 1993). It will be noted that at this point we have already had to postulate the existence of a second agent in the shape of a banking system of some kind to make the model work. This does not, however, compromise the attempt at providing microfoundations. Indeed, introducing a second agent of some kind is inevitable as soon as any attempt is made to introduce money into the process. Even in the original neoclassical money and growth models descended from

\textsuperscript{10} As far as we know this set of issues was first raised more than thirty years ago in a paper by McCallum (1986). This was published some years before the so-called “new consensus” model was widely accepted in the literature.
Sidrauski there was always at least implicitly a second agent present in the shape of a *deus ex machina* to actually *issue* the money (Harkness 1978, Smithin 1983). In the present case we can therefore similarly suppose that there is a “Wicksellian bank” (Smithin 2016c, 2018) in the system which adjusts the nominal interest rate according to the rule:

\[(21) \quad i = i_j + \gamma p \quad 0 < \gamma < 1, \quad i_j = i_1 \ldots \ldots \ i_n\]

where the \(i_j\) are the different possible values that could be chosen for the intercept in the reaction function. Then, from equation (20):

\[(22) \quad i_j + \gamma p - p = r^N\]

and;

\[(23) \quad p = \left[1/(1-\gamma)\right](i_j - r^N),\]

Unfortunately, as there is only a partial adjustment coefficient \((0 < \gamma < 1)\) in this case the Wicksell-type argument still does not work. As \([1/(1-\gamma)] > 1\), a setting of \(i_j\) higher than the natural rate continues to cause inflation to rise not fall and *vice versa*. This would be backwards from Wicksell’s point of view.

4. Is This Where the “Taylor Principle” Comes In?

Smithin (2013, 130-1) has conjectured that the problems associated with correctly incorporating the inflation rate into neo-Wicksellian models may actually have played some role in the popularity of the disastrous policy fad known as the “Taylor Principle” (as opposed to the Taylor rule) in the early twenty-first century (Mankiw 2001, 2003, Davig and Leeper 2007, 2010, Woodford 2010a). This Taylor Principle was the suggestion that the central bank should always raise the nominal policy rate by *more* than one-for-one with the observed inflation rate, in a sort of “pre-emptive strike” against inflation. This advice turned out to be disastrous in the real world because it amounts to deliberately destabilizing real interest rates and hence financial markets.
This is just one (of many) examples where mathematically-trained neoclassical economic theorists have been at cross-purposes with more practically-oriented market participants or “market-watchers”. Nonetheless from the theorists’ standpoint, and regardless of its impact on the real economy, applying the “Taylor Principle” does actually solve some technical problems. It would give rise to a rule such as:

\[(24) \quad i = i_j + (1 + \gamma)p, \quad 0 < \gamma < 1\]

So that:

\[(25) \quad i = r_j + \gamma p. \quad r_j = r_1 \ldots \ldots r_n\]

Next, substitute back into (20) which yields:

\[(26) \quad r_j + \gamma p - p = r^N.\]

And finally, solving for inflation, we obtain;

\[(27) \quad p = \frac{1}{1-\gamma}(r^N - r_j)\]

where the \(r_j\) are the different values for the intercept that could be chosen in a “real interest rate rule” (Barrows and Smithin 2009, 258, Smithin 1994, 2003, 2013, 2018). This is a much more Wicksellian result. The argument now is that if the real intercept in the reaction function is consistently less than the natural rate (which, as mentioned, is effectively the rate of time preference) there will be inflation. After much mathematizing the equilibrium of the enhanced neo-Wicksellian model therefore finally comes down to:

\[(28) \quad Y = Y^N\]

\[(29) \quad p = \frac{1}{1-\gamma}(r^N - r_j), \quad 0 < \gamma < 1\]

The conclusion is that in such a model the level of output \(Y\) in equilibrium is always at its natural value \(Y^N\) which is also supposedly the same as that which would prevail in a barter exchange economy. Furthermore, if the “base real policy rate” as we might now call it, that is \(r_j\), is too low
relative to the natural rate $r^N$ there will be inflation, and *vice versa*. These are, in fact, exactly the results the neo-Wickellsian theorist would be looking for, never mind their applicability to an actual economy. Smithin’s (2013, 131-2) comment on all this was as follows:

The historically-minded reader will note that the model in … [(28)–(29)] … is only a marginal advance from the position already reached by Keynes (1930, 21-44) in chapter 10 of his *Treatise on Money*.

As argued by Kam and Smithin (2018, 13), this seems to be an unbelievably small reward for what has now been nine decades of intensive mathematical research in academia.

5. *Endogenous Time Preference*

The key move in the analysis by Kam (2000, 2005) was to endogenize the rate of time preference. It has been known at least since Uzawa (1968) that this would restore the property of monetary *non*-neutrality.

However, the particular specification used by Uzawa was always highly controversial (Kam 2000, 15-16). Uzawa had assumed that time preference depends positively on the level of current utility which itself is an increasing function of consumption. On this view, inflation raises the opportunity cost of holding real money balances and renders the initial equilibrium too costly. This increases the real interest rate and decreases the demand for real balances which increases savings and the capital stock. This specification does make the rate of time preference endogenous. However, the argument that if consumption *increases* the rate of time preference also *increases* is not at all convincing. In effect, the very act of consumption itself is supposed to make the representative agent impatient for still more consumption. This does not seem reasonable except perhaps in the pathological case of addictive substances. Persson and Svenson (1985, 45), for example, dismiss the Uzawa specification as "... arbitrary and even counter-intuitive". Blanchard and Fischer (1989, 71) go much further, and specifically warn off budding
economic theorists by stating that:

[although the] ... specification avoids the pathological results of the constant
discount rate ... the Uzawa function, with its assumption that the rate of
time preference increases in instantaneous utility is not ... attractive as a
description of preferences and is not recommended for general use.

Kam (2000, 2005) however, building on a suggestion by Epstein and Hynes (1983), has put
forward an alternative and far more intuitively plausible method of making the rate of time
preference endogenous. The idea is simply to make time preference a positive function of total
real wealth rather than an increasing function of consumption itself. Because the wealth effect on
time preference is positive this amounts to reinstating the idea that there is some sort of
“propensity to consume” out of wealth as well as out of income.11 Therefore, adapting the
treatment in Kam (2005, 12) for use in the present neo-Wicksellian framework, let:

\[
\theta = \theta(W). \quad \theta'(W) > 0
\]

The first order conditions for our optimization problem will therefore now be:

\[
U'(C) - \lambda = 0
\]
\[
\lambda [F'(K) - \delta - I] + \lambda_+ \beta = 0
\]
\[
\lambda (r_D - I) + \lambda_+ \beta = 0
\]

Thus, the revised dynamic system (where the relevant real interest rate is once again labelled \(r_D\))
turns out to be:

\[
F'(K) - \delta = r_D
\]
\[
U'(C) [F'(K) - \delta] = -U'(C+1) \beta
\]

Once again this will be a saddlepoint. The steady-state of the system therefore becomes:

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11 In fact, this was also the move made in several of the supposedly “ad-hoc” macro models criticized by Walsh and
others (e.g. Begg 1980, 1982, Smithin 1980). Endogenizing the rate of time preference simply formalizes this
assumption in a way that is deemed acceptable within the microfoundations literature. See also Kam (2000) and
Smithin (2013).
(36) \[ F'(K) - \delta = r_D \]

(37) \[ F'(K) - \delta = \theta(W). \]

There is no longer any natural rate of interest in the equilibrium of this model. All of the rate of time preference, the net marginal product of capital, and the real rate of interest on money must conform to the standard set by the deliberate monetary policy of the Wicksellian bank (Smithin 1994, 2003, 2013, 2018). Comparing these results to those of Kam (2000, 2005), endogenizing the rate of time preference is thus proven to break the orthodox result of long-run monetary neutrality, regardless of whether the monetary policy instrument is the rate of growth of the money supply itself or an interest rate.

6. A Simple Theory of Banking and the Relationship between Commercial Banks and the Central Bank

One possible interpretation of the nature of the financial asset in the optimization problem above is as an interest-bearing deposit in (specifically) a commercial bank. In this case, logically speaking, there would therefore have to be at least one more agent in the system in addition to the worker-consumer and the single Wicksellian bank already posited. It now becomes necessary to distinguish between two separate components of the banking system, namely, the commercial bank and the central bank. Following Kam and Smithin (2012a) therefore, let the simplified balance sheet of the commercial bank be as follows. Here, $D$ stands for nominal deposits in the commercial bank, $S$ for any outstanding negative settlement balances of the commercial bank at the central bank, $R$ for nominal bank reserves, and $L$ for the nominal dollar amount of commercial bank loans outstanding:
Table 1: A Simplified Commercial Bank Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>Deposits</td>
</tr>
<tr>
<td>Loans</td>
<td>Settlement Balances</td>
</tr>
<tr>
<td>$R$</td>
<td>$D$</td>
</tr>
<tr>
<td>$L$</td>
<td>$S$</td>
</tr>
</tbody>
</table>

The optimization problem for the commercial bank is therefore:

\[
\text{(38)} \quad \max \Pi = i_L L - i_D D - i_0 \sigma (S - R) - \mu L
\]

where \( \Pi \) stands for money profit, \( i_L \) is the nominal prime lending rate, \( i_D \) is the nominal deposit rate, \( i_0 \) is the nominal policy rate (for example, the overnight rate in Canada) and \( \sigma \) is the subjective probability, as assessed by bank officials themselves, of the commercial bank being out of the money at the clearing house. Substituting in from the bank balance sheet we obtain:

\[
\text{(39)} \quad \max \Pi = i_L L - i_D (L + R - S) - i_0 \sigma (S - R) - \mu L
\]

If we were to use a standard notation from statistical probability theory (recalling fn. 1 above) then we could write down something like:

\[
\text{(40)} \quad \sigma = \int_{0}^{\infty} f(x) dx
\]

Meanwhile, the expression \( \mu \) might be interpreted as the average cost per dollar (or euro or yen) for making bank loans. However, a difficulty with this interpretation is that there is no precise analogue to a textbook physical production function in the field of banking (Dow and Smith 1999). It is probably safer simply to say that \( \mu \) must be high enough to cover costs and also earn a normal rate of return for the banks given existing institutional arrangements, market structure, banking legislation, regulations, etc. It is ultimately determined by these four sets of
conditions. Next, substituting in from the balance sheet, the optimization problem becomes:

\[(41) \quad \text{Max: } II = i_L L - i_D (L + R - S) - i_0 \sigma (S - R) - \mu L \]

where the choice variables are the volume of loans granted and the quantity of precautionary reserves banks choose to hold. The first order conditions are obtained by differentiating with respect to \(L\) and \(R\) and setting the results equal to zero:

\[(42) \quad i_L - i_D = \mu \]
\[(43) \quad i_D = \sigma i_0.\]

The mark-up between commercial bank lending rates and deposit rates is therefore equal to \(\mu\), and the deposit rate in the commercial bank \(i_D\), is a mark-down from the central bank's setting of the policy rate, \(i_0\). The degree of the mark-down thus depends on the subjective assessment of the "risk" (as this is called in neoclassical economics - a true Keynesian would prefer to call it uncertainty) if the representative commercial bank does not "[keep] in step" (Keynes 1930, 23) with its rivals. In the past, a similar sort of result has sometimes been called the "two-for-one" rule (Rogers and Rymes 2000, 259). However, to get a value of exactly \(\sigma = 0.5\) would depend on making the twin assumptions of ergodicity and a normal distribution which are unlikely both to hold in practice. Empirically, the value of the \(\sigma\) term seems to be much higher than 0.5 (say around 0.8) but still less than unity (see Collis 2018).

Combining equations (42) and (43) we can see that there is a linear relationship between the policy rate and the commercial bank lending rate thus providing an account of how changes in the central bank policy rate are transmitted to interest rates in general. That is:

\[(44) \quad i_L = \mu + \sigma i_0\]

Next, subtract the observed inflation rate, \(p\), from both sides of equation (44):
Here the term \( r_0 \) is the inflation-adjusted real policy rate of interest, that is, the nominal policy rate adjusted for the currently observed inflation rate, or \( r_0 = i_0 - p \). This result gives some further insight into the several discussions over the years by Smith in (1994, 2003, 2007, 2009, 2013, 2016a, 2016b, 2018) about a "real interest rate rule" for monetary policy. As a practical matter such a rule would have to involve a target for the inflation-adjusted policy rate (as defined above) simply because the true expected inflation rate is not known. The question now, therefore, is whether the similar inflation-adjusted real commercial bank lending rate in equation (45) can also be taken as a “proxy” (Taylor 1993) for the real lending rate itself. If so, and in the absence of any other indicator on which the borrowers can base their estimates, equation (45) may be re-written as:

\[
(46) \quad r = \mu + \sigma r_0 - (1-\sigma)p
\]

Where term \( r \) now stands for the real interest rate actually involved in economic decision-making such as, for example, the interest rate in an investment function or in an “IS curve” in a macro model. Equation (46) thus shows how central bank activities do indeed have an influence over the real rate of interest in the market-place, and thereby on the real economy in general. Notice particularly, that there is negative theoretical relationship between inflation and the real rate of interest in this situation. This is nothing other than the forced saving (or Mundell-Tobin) effect, as already discussed.

### 7. A Real Interest Rate Rule for Monetary Policy?

From equations (42) and (43), we can see that it also must be the case that:

\[
(47) \quad r_D = \sigma r_0 - (1-\sigma)p
\]

This formulation raises the possibility that at least in principle the central bank could pursue a
feedback rule intended to fix the real rate of return on the financial asset (the bank deposits) actually held by the representative worker-consumer in our model. To set \( r_D = r' \), for example, the central bank must follow the rule:

\[
(48) \quad r_0 = (1/\sigma) r' + [(1-\sigma)/\sigma] p
\]

In and of itself this rule is quite complicated and in actual practice any rule followed by the central bank is presumably going to have to be much more straightforward, such as the various suggestions put forward in Smithin (2009, 2013, 2016a, 2016b). In the real world central bankers will not be able to cover every contingency and therefore their main objective should be not to add to instability (unlike in the case of their adherence to the Taylor Principle). Nonetheless, if we are prepared to allow that, in principle, the central bank could follow such a rule (that is, given an exact knowledge of the parameters) this would greatly simplify the theoretical model and thereby help us to better understand some of that model’s basic features (see Kam and Smithin 2012b and Smithin 2013, 2018). This, therefore, will be the underlying assumption in the theoretical discussion to follow.

8. **Is there a “User Cost” of Producing Capital Goods rather than Consumption Goods?**

There is still one loose-end to be tied up. As we have now abandoned the Taylor Principle *per se*, the system no longer determines the inflation rate. We are therefore back to the dilemma originally faced by theorists of the new consensus in their “models without money” (Rogers 2006, Smithin 2009, Woodford 1998) at the end of the twentieth century and the beginning of the twenty-first, discussed above. This failing, however can be remedied merely by introducing some frictions into the problem of our representative worker-consumer. Evidently one of the main choices that the agent faces is whether to allocate current output to investment goods (that

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\[^{12}\text{Smithin (2016a, 2016b, 2018) has also shown that a real rate rule will ensure inflation stability.}\]
is, to increase the capital stock) or to consumption. We can therefore reasonably suppose that there is some sort of lump-sum user cost that must be incurred when making these changes.

Let \( V \) be the nominal user cost and \( P \) the price level. According to the usual logic of profit maximization or cost minimization there must therefore be a further marginal condition for the representative worker-consumer as follows:

\[
V/P = F'(K)
\]

(49)

Next suppose that nominal user costs, in what we have supposed to be necessarily a money-using system, evolve according to:

\[
V = V_0P^{-1}
\]

(50) \hspace{1cm} V_0 > 1

Substituting (48) into (47) we obtain:

\[
V_0(P^{-1})/P = F'(K)
\]

(51) \hspace{1cm} which implies:

\[
V_0/F'(K) = (1 + p)
\]

(52)

This therefore suggests a positive relationship between the level of real output and inflation due to the frictions associated with switching production from consumer goods to capital goods in a monetary economy. The underlying reason for this is that the user costs have to be paid in terms of money (\(i.e.,\) from bank deposits).

9. Formal Results

Drawing now on each of equations (11), (37) and (52) reported above, the solution system for the complete macroeconomic model is as follows:

\[
F'(K) - \delta = r'
\]

(53)

\[
\theta(K + D) = r'
\]

(54)

\[
V_0/F'(K) = (1 + p)
\]

(55)
Totally differentiating:

\[ F''(K)dK = dr' \]  
(56)

\[ \theta'(W)dK + \theta'(W)dD = dr' \]  
(57)

\[ -V_0F''(K)dK/[F'(K)]^2 = dp \]  
(58)

Therefore, the results for changes in the target real rate of interest set by the central bank will be:

\[ dK/dr' = [I/F''(K)], \quad < 0, \]  
(59)

\[ dD/dr' = [1 - \theta'(W)]/\theta'(W), \quad > 0, \]  
(60)

\[ dp/dr' = -V_0/[F'(K)]^2 \quad < 0. \]  
(61)

In short, a tight money policy (which in the analytical context outlined above means that the central bank is in fact effectively able to set a higher target for the real rate of interest actually charged in the market-place) will indeed succeed in permanently reducing the inflation rate. However, at the same time, it will also permanently reduce the steady-state capital stock and level of output. Monetary policy is definitively non-neutral even in the long run. Meanwhile as the physical capital stock declines the total real holdings of the financial asset will actually increase. This may be a somewhat unfamiliar result from the point of view of the academic economist but it is really not all that surprising, given that the relevant financial asset (bank deposits) is itself assumed to be interest-bearing. In effect, what is going on is that society’s resources are being transferred from the creation of real assets to the accumulation of financial assets. To the layperson this may seem just to be the commonsense/expected result of a policy of deliberately increasing the rate of return to financial assets. However, in the past it has been extraordinarily difficult to establish the existence of this sort of effect within the framework of formal mathematical economics.

These are the same type of results as those found, for example, in Atesoglu and Smithin
(2006, 2007), Collis (2018) Kam (2000, 2005), Mackinnon and Smithin (1993), Smithin (2003, 2009, 2013, 2018) and Tabassum (2012) and they are therefore robust across a wide variety of different model specifications. From the point of view of the mainstream economist, the chief thing that should be interesting about them is that, in this treatment, the so-called “microfoundations” have been provided in detail. It is therefore no longer possible to simply dismiss the non-neutrality findings on a priori methodological grounds.

10. Conclusion

This paper has provided an explanation of the long-run non-neutrality of monetary policy in the context of a neo-Wicksellian DGE model with microfoundations. If the rate of time preference in endogenous there is no “natural rate” of interest. Therefore, if the central bank pursues a real interest rate rule this will influence the real levels of both the lending and deposit rates in the commercial banks and will affect the real economy via this route. There is a negative relationship between the inflation-adjusted real lending rate of the commercial banks and the rate of inflation itself. This is nothing other than the old “forced saving” effect or the twentieth century Mundell-Tobin effect.
References


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